Name ________________________________ Date ____________

A Little Dash of Logic
Foundations for Proof

Vocabulary

Define each term in your own words.

1. induction
   Induction is reasoning that involves using specific examples to make conclusions.

2. deduction
   Deduction is reasoning that involves using a general rule to make a conclusion.

3. counterexample
   A counterexample is a specific example that shows that a general statement is not true.

4. conditional statement
   A counterexample is a specific example that shows that a general statement is not true.

5. propositional form
   A conditional statement is in propositional form if it is written in the form "if *p*, then *q*."

6. propositional variables
   In a conditional statement in propositional form, the variables *p* and *q* are called the propositional variables.

7. hypothesis
   The hypothesis of a conditional statement is the variable *p*.

8. conclusion
   The conclusion of a conditional statement is the variable *q*.

9. truth value
   The truth value of a conditional statement is whether the statement is true or false.

10. truth table
    A truth table is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$.

Problem Set

Identify the specific information, the general information, and the conclusion for each problem situation.

1. You read an article in the paper that says a high-fat diet increases a person's risk of heart disease. You know your father has a lot of fat in his diet, so you worry that he is at higher risk of heart disease.
   Specific information: Your father has a lot of fat in his diet.
   General information: High-fat diets increase the risk of heart disease.
   Conclusion: Your father is at higher risk of heart disease.

2. You hear from your teacher that spending too much time in the sun without sunblock increases the risk of skin cancer. Your friend Susan spends as much time as she can outside working on her tan without sunscreen, so you tell her that she is increasing her risk of skin cancer when she is older.
   Specific information: Susan spends a lot of time in the sun without sunscreen.
   General information: A lot of exposure to the sun without sunblock increases the risk of skin cancer.
   Conclusion: Susan is at higher risk of skin cancer.

3. Janice tells you that she has been to the mall three times in the past week, and every time there were a lot of people there. "It's always crowded at the mall," she says.
   Specific information: There have been a lot of people at the mall when Janice has been there.
   General information: The problem does not include any general information.
   Conclusion: It's always crowded at the mall.

4. John returns from a trip out West and reports that it was over 100 degrees every day. "It's always hot out West," he says.
   Specific information: It was over 100 degrees out West every day that John was there.
   General information: The problem does not include any general information.
   Conclusion: It's always hot out West.

5. Mario watched 3 parades this summer. Each parade had a fire truck lead the parade. He concluded "A fire truck always leads a parade."
   Specific information: Mario watched 3 parades this summer with each having a fire truck in the lead.
   General information: The problem does not have any general information.
   Conclusion: A fire truck always leads a parade.

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6. Ava read an article that said eating too much sugar can lead to tooth decay and cavities. Ava noticed that her little brother Phillip eats a lot of sugar. She concludes that Phillip's teeth will decay and develop cavities.

   Specific information: Philip eats a lot of sugar.

   General information: Eating too much sugar can lead to tooth decay and cavities.

   Conclusion: Phillip's teeth will decay and develop cavities.

Determine whether inductive reasoning or deductive reasoning is used in each situation. Then determine whether the conclusion is correct and explain your reasoning.

7. Jason sees a line of 10 school buses and notices that each is yellow. He concludes that all school buses must be yellow.

   It is inductive reasoning because he has observed specific examples of a phenomenon—the color of school buses—and come up with a general rule based on those specific examples.

   The conclusion is not necessarily true. It may be the case, for example, that all or most of the school buses in this school district are yellow, while another school district may have orange school buses.

8. Caitlyn has been told that every taxi in New York City is yellow. When she sees a red car in New York City, she concludes that it cannot be a taxi.

   It is deductive reasoning because she has taken a general rule—all New York City taxis are yellow—and made a logical conclusion about the status of a particular car.

   Her conclusion is correct, if the rule she was told is true. The only way the red car could be a taxi would be if the general rule she had been told was wrong. If the rule she was told is false, then her conclusion would be incorrect as well.

9. Miriam has been told that lightning never strikes twice in the same place. During a lightning storm, she sees a tree struck by lightning and goes to stand next to it, convinced that it is the safest place to be.

   It is deductive reasoning because she has taken a general rule about lightning and applied it to this particular situation.

   Her conclusion is not correct because she was given incorrect information. It is a myth that lightning never strikes twice in the same place.

2

10. Jose is shown the first six numbers of a series of numbers: 7, 11, 15, 19, 23, 27. He concludes that the general rule for the series of numbers is $a_n = 4n + 3$.

It is inductive reasoning because he has reasoned from a number of specific examples to a general rule.

Yes. This is the correct formula for the series.

11. Isabella sees 5 red fire trucks. She concludes that all fire trucks are red.

It is inductive reasoning because she has observed specific examples of a phenomenon—the color of fire trucks—and come up with a general rule based on those specific examples.

The conclusion is not necessarily true. It may be the case, for example, that all or most fire trucks are red, but other communities may have orange or yellow fire trucks.

12. Carlos is told that all garter snakes are not venomous. He sees a garter snake in his backyard and concludes that it is not venomous.

It is deductive reasoning because he has taken a general rule—all garter snakes are not venomous—and made a logical deduction about a particular snake.

His conclusion is correct, if the rule he was told is true. The only way the snake could be venomous would be if the general rule he had been told was wrong. If the rule he was told is false, then his conclusion would be incorrect as well.

In each situation, identify whether each person is using inductive or deductive reasoning. Then compare and contrast the two types of reasoning.

13. When Madison babysat for the Johnsons for the first time, she was there 2 hours and was paid \$30. The next time she was there for 5 hours and was paid \$75. She decided that the Johnsons were paying her \$15 per hour. The third time she went, she stayed for 4 hours. She tells her friend Jennifer that she makes \$15 per hour babysitting. So, Jennifer predicted that Madison made \$60 for her 4-hour babysitting job.

Madison used inductive reasoning to conclude that the Johnsons were paying her at a rate of \$15 per hour. From that general rule, Jennifer used deductive reasoning to conclude that 4 hours of babysitting should result in a payment of \$60. The inductive reasoning looks at evidence and creates a general rule from the evidence. By contrast, the deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance.

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14. When Holly was young, the only birds she ever saw were black crows. So, she told her little brother Walter that all birds are black. When Walter saw a bluebird for the first time, he was sure it had to be something other than a bird.

Holly used inductive reasoning to conclude that all birds are black because every bird she had ever seen had been black. (The conclusion was obviously incorrect.) From that general rule, Walter used deductive reasoning to conclude that the bluebird could not be a bird. Inductive reasoning looks at evidence and creates a general rule from the evidence. As can be seen here, it is easy to come up with an incorrect rule if the examples you see are limited. Because deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance, it is dependent upon the general rule being correct.

15. Tamika is flipping a coin and recording the results. She records the following results: heads, tails, heads, tails, heads, tails, heads. She tells her friend Javon that the coin alternates between heads and tails for each toss. Javon tells her that the next time the coin is flipped, it will definitely be tails.

Tamika used inductive reasoning to conclude that the coin flipping was following a pattern of heads, then tails, then heads, etc. Then Javon used deductive reasoning to conclude that the next flip would land tails. Inductive reasoning looks at specific examples and creates a general rule from the evidence. Because of the limited number of specific examples it is easy to create an incorrect rule. Because deductive reasoning makes a prediction based on given general rule the accuracy of the prediction is dependent on the rule being correct.

16. John likes to watch the long coal trains moving past his house. Over the weeks of watching he notices that every train going east is filled with coal, but the trains heading west are all empty. He tells his friend Richard that all trains heading east have coal and all trains heading west are empty. When Richard hears a train coming from the west, he concludes that it will certainly be filled with coal.

John used inductive reasoning to conclude that all trains heading east have their cars filled with coal and all trains heading west have empty cars. When Richard hears a train coming from the west, he used deductive reasoning to conclude before he ever saw the train that its cars would be filled with coal. Inductive reasoning depends upon things that have already been observed, such as the trains John had seen over the many weeks, while deductive reasoning can be applied to something that has not yet been observed.

2

17. Vance earned \$60 mowing 5 lawns last weekend for the Greenvalley Homeowners Association. Vance concluded that he earned \$12 for each lawn. Vance told Sherwin that he planned to mow 7 lawns for Greenvalley next weekend. Sherwin concluded that Vance would earn \$84 mowing the 7 lawns.

Vance used inductive reasoning to conclude that he was paid \$12 per lawn by the Greenvalley Homeowners Association. From that general rule, Sherwin used deductive reasoning to conclude that mowing 7 lawns should result in a payment of \$84. The inductive reasoning looks at evidence and creates a general rule from the evidence. By contrast, the deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance.

18. As a child, the only frogs Emily ever saw were green. Emily told Juan that all frogs are green. When Juan visited a zoo and saw a blue poison dart frog he concluded that it must be something other than a frog.

Emily used inductive reasoning to conclude that all frogs are green because every frog she had ever seen had been green. (The conclusion was obviously incorrect.) From that general rule, Juan used deductive reasoning to conclude that the blue poison dart frog could not be a frog. Inductive reasoning looks at evidence and creates a general rule from the evidence. As can be seen here, it is easy to come up with an incorrect rule if the examples you see are limited. Because deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance, it is dependent upon the general rule being correct.

Write each statement in propositional form.

19. The measure of an angle is 90°. So, the angle is a right angle.

If the measure of an angle is 90°, then the angle is a right angle.

20. Three points are all located on the same line. So, the points are collinear points.

If three points are all located on the same line, then they are collinear points.

21. Two lines are not on the same plane. So, the lines are skew.

If two lines are not on the same plane, then they are skew.

22. Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

If the sum of two angle measures is equal to 180°, then they are supplementary angles.

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23. Two angles share a common vertex and a common side. So, the angles are adjacent angles.
    If two angles share a common vertex and a common side, then they are adjacent angles.

24. A ray divides an angle into two congruent angles. So, the ray is an angle bisector.
    If a ray divides an angle into two congruent angles, then it is an angle bisector.

Identify the hypothesis and the conclusion of each conditional statement.

25. If two lines intersect at right angles, then the lines are perpendicular.
    The hypothesis is "Two lines intersect at right angles."
    The conclusion is "The lines are perpendicular."

26. If the sum of two angles is 180°, then the angles are supplementary.
    The hypothesis is that the sum of two angles is 180°.
    The conclusion is that the angles are supplementary.

27. If the sum of two adjacent angles is 180°, then the angles form a linear pair.
    The hypothesis is that the sum of two adjacent angles is 180°.
    The conclusion is that the angles form a linear pair.

28. If the measure of an angle is 180°, then the angle is a straight angle.
    The hypothesis is "The measure of an angle is 180°."
    The conclusion is "The angle is a straight angle."

29. If two lines are located in the same plane, then the lines are coplanar lines.
    The hypothesis is "Two lines are located in the same plane."
    The conclusion is "The lines are coplanar lines."

30. If the sum of two angle measures is equal to 90°, then the angles are complementary angles.
    The hypothesis is "The sum of two angle measures is equal to 90°."
    The conclusion is "The angles are complementary angles."

2

Answer each question about the given conditional statement.

31. Conditional statement: If the measure of angle *ABC* is 45° and the measure of angle *XYZ* is 45°, then $\angle ABC = \angle XYZ$.

What does it mean if the hypothesis is false and the conclusion is true, and then what is the truth value of the conditional statement?

If the hypothesis is false and the conclusion is true, then the measure of angle *ABC* is not 45 degrees and the measure of angle *XYZ* is not 45 degrees, and angles *ABC* and *XYZ* are congruent. The truth value of the conditional statement is true, because the angles could have measures that are equal, but different than 45 degrees.

32. Conditional statement: If the measure of angle *XYZ* is less than 90°, then angle *XYZ* is acute.

What does it mean if the hypothesis is true and the conclusion is false, and then what is the truth value of the conditional statement?

If the hypothesis is true and the conclusion is false, then the measure of angle *XYZ* is less than 90 degrees, and the angle is not acute. The truth value of the conditional statement is false.

33. Conditional statement: If $\angle 1$ and $\angle 2$ are two nonadjacent angles formed by two intersecting lines, then they are vertical angles.

What does it mean if the hypothesis is true and the conclusion is true, and then what is the truth value of the conditional statement?

If the hypothesis is true and the conclusion is true, then angles 1 and 2 are nonadjacent angles formed by two intersecting lines, and the angles are vertical angles. The truth value of the conditional statement is true.

34. Conditional statement: If the measure of $\angle LMN$ is 180°, then $\angle LMN$ is a straight angle.

What does it mean if the hypothesis is false and the conclusion is false, and then what is the truth value of the conditional statement?

If the hypothesis is false and the conclusion is false, then the measure of angle *LMN* is not 180 degrees, and the angle is not a straight angle. The truth value of the conditional statement is true.

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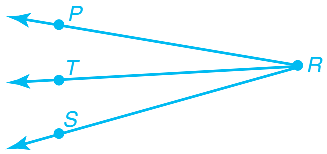
For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

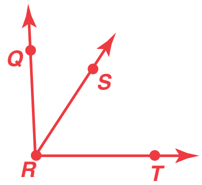
35. If $\overrightarrow{RT}$ bisects $\angle PRS$, then $\angle PRT$ and $\angle SRT$ are adjacent angles.



Given: $\overrightarrow{RT}$ bisects $\angle PRS$

Prove: $\angle PRT$ and $\angle SRT$ are adjacent angles

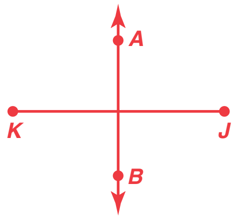
36. If $\angle QRS$ and $\angle SRT$ are complementary angles, then $m\angle QRS + m\angle SRT = 90°$.



Given: $\angle QRS$ and $\angle SRT$ are complementary angles

Prove: $m\angle QRS + m\angle SRT = 90°$

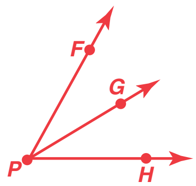
37. If $\overleftrightarrow{AB} \perp \overline{KJ}$ and $\overleftrightarrow{AB}$ bisects $\overline{KJ}$, then $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{KJ}$.



Given: $\overleftrightarrow{AB} \perp \overline{KJ}$ and $\overleftrightarrow{AB}$ bisects $\overline{KJ}$

Prove: $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{KJ}$

38. If $\overrightarrow{PG}$ bisects $\angle FPH$, then $\angle FPG \cong \angle GPH$.



Given: $\overrightarrow{PG}$ bisects $\angle FPH$

Prove: $\angle FPG \cong \angle GPH$

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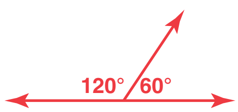
And Now From a New Angle
Special Angles and Postulates

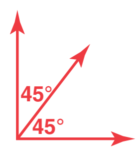
Vocabulary

Draw a figure to illustrate each term.

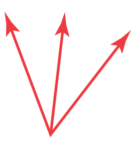
1. supplementary angles



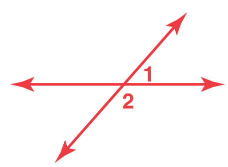
2. complementary angles



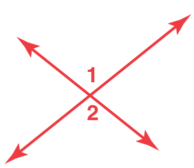
3. adjacent angles



4. linear pair



5. vertical angles



Define each term in your own words.

6. postulate
   a statement that is accepted without proof
7. theorem
   a statement that can be proven
8. Euclidean geometry
   a system of geometry developed by Euclid

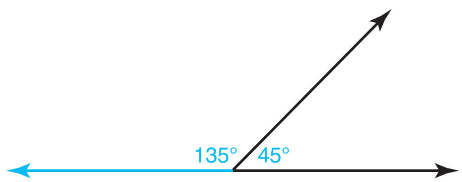
State each postulate.

9. Linear Pair Postulate
   If two angles are a linear pair, then the angles are supplementary.
10. Segment Addition Postulate
    If point B is on $\overline{AC}$ and between points A and C then $AB + BC = AC$.
11. Angle Addition Postulate
    If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.

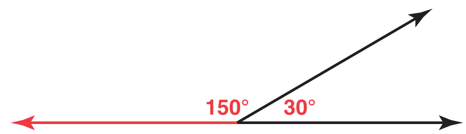
Problem Set

Use a protractor to draw an angle that is supplementary to each given angle. Draw the angle so it shares a common side with the given angle. Label the measure of each angle.

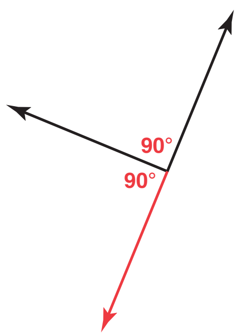
1.



2.



3.



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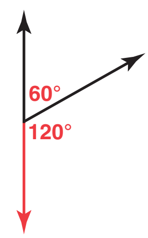
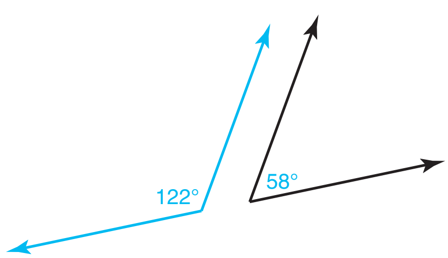


Use a protractor to draw an angle that is supplementary to each given angle. Draw the angle so it does not share a common side with the given angle. Label the measure of each angle.

5.



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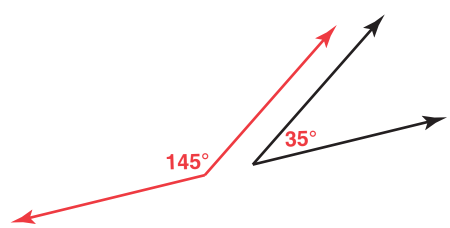

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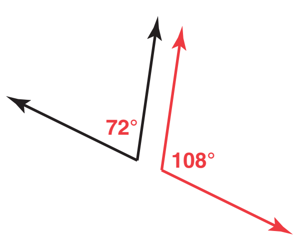

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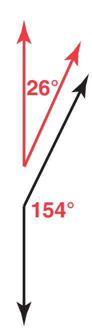
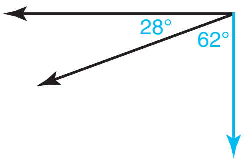


Name ____ Date _______________

Use a protractor to draw an angle that is complementary to each given angle. Draw the angle so it shares a common side with the given angle. Label the measure of each angle.

9.


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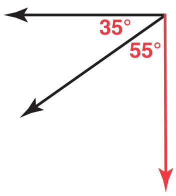

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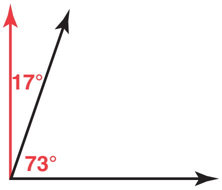

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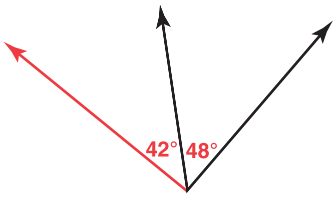
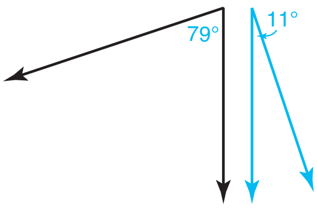


Use a protractor to draw an angle that is complementary to each given angle. Draw the angle so it does not share a common side with the given angle. Label the measure of each angle.

13.


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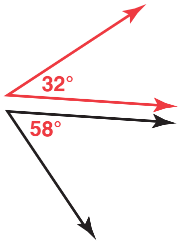

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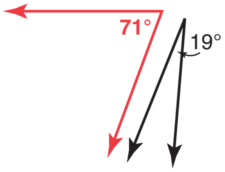

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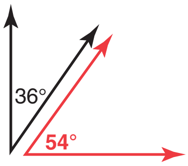
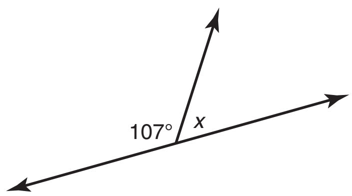


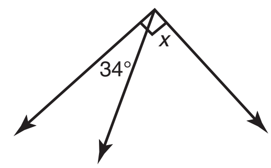
Solve for x.

17.



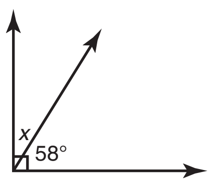
$x = 180° - 107° = 73°$

18.



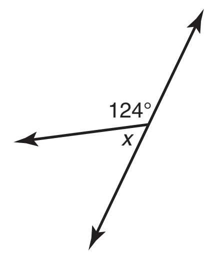
$x = 90° - 34° = 56°$

19.



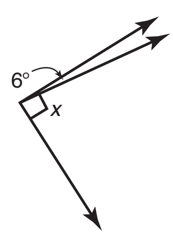
$x = 90° - 58° = 32°$

20.



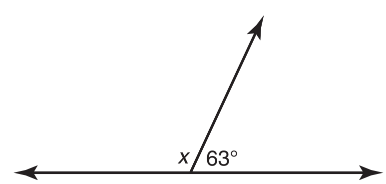
$x = 180° - 124° = 56°$

21.



$x = 90° - 6° = 84°$

22.



$x = 180° - 63° = 117°$

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2

Use the given information to determine the measures of the angles in each pair.

23. The measure of the complement of an angle is three times the measure of the angle. What is the measure of each angle?

$x + 3x = 90$

$4x = 90$

$x = 22.5$

The measure of the angle is 22.5° and the measure of the complement is 67.5°.

24. The measure of the supplement of an angle is one fourth the measure of the angle. What is the measure of each angle?

$x + 0.25x = 180$

$1.25x = 180$

$x = 144$

The measure of the angle is 144° and the measure of the supplement is 36°.

25. The measure of the supplement of an angle is twice the measure of the angle. What is the measure of each angle?

$x + 2x = 180$

$3x = 180$

$x = 60$

The measure of the angle is 60° and the measure of the supplement is 120°.

26. The measure of the complement of an angle is one fifth the measure of the angle. What is the measure of each angle?

$x + 0.2x = 90$

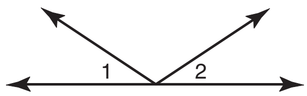
$1.2x = 90$

$x = 75$

The measure of the angle is 75° and the measure of the complement is 15°.

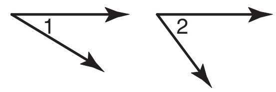
For each diagram, determine whether angles 1 and 2 are adjacent angles.

27.



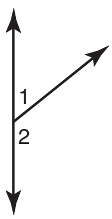
The angles are not adjacent.

28.



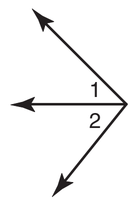
The angles are not adjacent.

29.



The angles are adjacent.

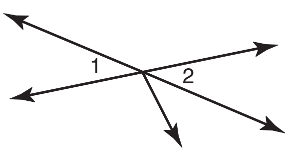
30.



The angles are adjacent.

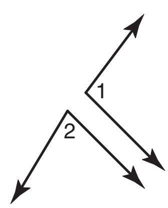
For each diagram, determine whether angles 1 and 2 form a linear pair.

31.



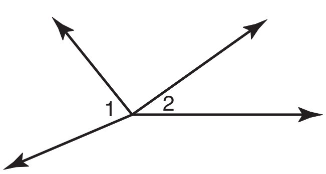
The angles do not form a linear pair.

32.



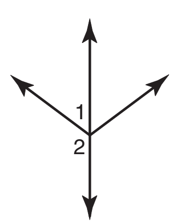
The angles do not form a linear pair.

33.



The angles do not form a linear pair.

34.

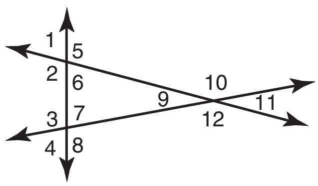


The angles form a linear pair.

Name ____ Date ______________

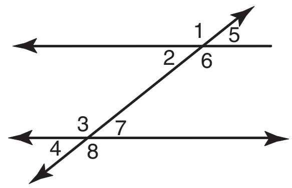
Name each pair of vertical angles.

35.



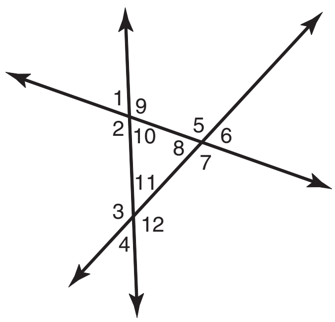
∠1 and ∠6, ∠2 and ∠5, ∠3 and ∠8,
∠4 and ∠7, ∠9 and ∠11, ∠10 and ∠12

36.



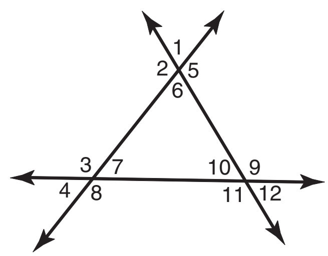
∠1 and ∠6, ∠2 and ∠5, ∠3
and ∠8, ∠4 and ∠7

37.



∠1 and ∠10, ∠2 and ∠9, ∠3 and ∠12,
∠4 and ∠11, ∠6 and ∠8, ∠5 and ∠7

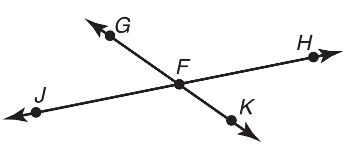
38.



∠1 and ∠6, ∠2 and ∠5,
∠9 and ∠11, ∠3 and ∠8,
∠4 and ∠7, ∠10 and ∠12

Write the postulate that confirms each statement.

39. Angles *GFH* and *KFH* are supplementary angles.

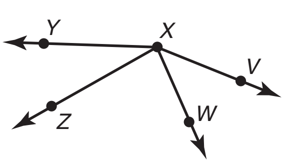


Linear Pair Postulate

40. $m\overline{RS} + m\overline{ST} = m\overline{RT}$

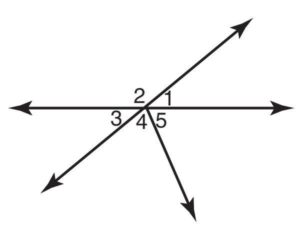
Segment Addition Postulate

41. $m\angle WXZ + m\angle ZXY = m\angle WXY$



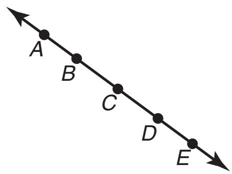
Angle Addition Postulate

42. $m\angle 1 + m\angle 2 = 180°$



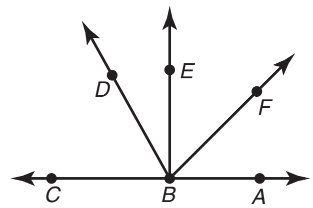
Linear Pair Postulate

43. $BC + CD = BD$



Segment Addition Postulate

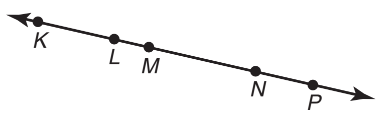
44. $m\angle DBE + m\angle EBF = m\angle DBF$



Angle Addition Postulate

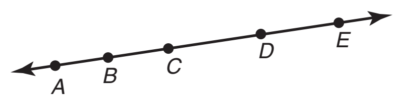
Complete each statement. The write the postulate you used.

45. $m\overline{LM} + m\overline{MN} = m\overline{LN}$



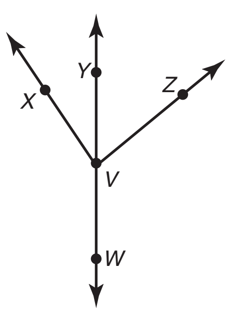
Segment Addition Postulate

46. $m\overline{AB} + m\overline{BC} = m\overline{AC}$



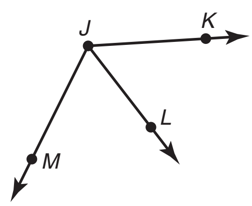
Segment Addition Postulate

47. $m\angle YVZ + m\angle ZVW = 180°$



Linear Pair Postulate

48. $m\angle MJL + m\angle LJK = m\angle MJK$

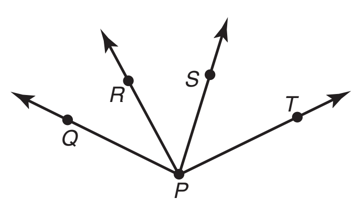


Angle Addition Postulate

49. $m\overline{FG} + m\overline{GI} = m\overline{FI}$

Segment Addition Postulate

50. $m\angle SPT + m\angle RPS = m\angle RPT$



Angle Addition Postulate

Name ______________________________ Date ____________

Forms of Proof
Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof

Vocabulary

Match each definition to its corresponding term.

1. If a is a real number, then $a = a$. i
2. If a, b, and c are real numbers, $a = b$, and $b = c$, then $a = c$. e
3. If a, b, and c are real numbers and $a = b$, then $a + c = b + c$. a
4. a proof in which the steps and corresponding reasons are written in complete sentences b
5. If a and b are real numbers and $a = b$, then a can be substituted for b. g
6. a proof in which the steps are written in the left column and the corresponding reasons in the right column h
7. a proof in which the steps and corresponding reasons are written in boxes f
8. If a, b, and c are real numbers and $a = b$, then $a - c = b - c$. d
9. a proof that results from creating an object with specific properties using only a compass and straightedge c

a. Addition Property of Equality
b. paragraph proof
c. construction proof
d. Subtraction Property of Equality
e. Transitive Property
f. flow chart proof
g. Substitution Property
h. two-column proof
i. Reflexive Property

Problem Set

Identify the property demonstrated in each example.

1. $m\angle ABC = m\angle XYZ$
   $m\angle ABC - m\angle RST = m\angle XYZ - m\angle RST$
   Subtraction Property of Equality

2. $m\overline{QT} = m\overline{TU}$
   $m\overline{QT} + m\overline{WX} = m\overline{TU} + m\overline{WX}$
   Addition Property of Equality

3. $\angle JKL \cong \angle JKL$
   Reflexive Property

4. $GH = MN$ and $MN = OP$, so $GH = OP$
   Transitive Property

5. $m\overline{XY} = 4$ cm and $m\overline{BC} = 4$ cm, so $m\overline{XY} = m\overline{BC}$
   Substitution Property

6. $\overline{PR} \cong \overline{PR}$
   Reflexive Property

7. $GH = JK$
   $GH - RS = JK - RS$
   Subtraction Property of Equality

8. $m\angle 1 = 134°$ and $m\angle 2 = 134°$, so $m\angle 1 = m\angle 2$
   Substitution Property

9. $m\angle ABC = m\angle DEF$
   $m\angle ABC + m\angle QRS = m\angle DEF + m\angle QRS$
   Addition Property of Equality

10. $GH = GH$
    Reflexive Property

11. $ED = 3$ in. and $PQ = 3$ in., so $ED = PQ$
    Substitution Property

12. $\angle EFG \cong \angle LMN$ and $\angle LMN \cong$ SPT, so $\angle EFG \cong \angle SPT$
    Transitive Property

Write a statement that fits each given description.

13. Write a segment statement using the Reflexive Property.
    Sample Answer: $\overline{XY} \cong \overline{XY}$

14. Write angle statements using the Addition Property of Equality.
    Sample Answer: $m\angle D = m\angle E$
    $m\angle D + m\angle F = m\angle E + m\angle F$

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15. Write angle statements using the Substitution Property.

Sample Answer: $m\angle P = 10°$
$m\angle Q = 10°$
$m\angle P = m\angle Q$

16. Write segment statements using the Transitive Property.

Sample Answer: $\overline{AB} \cong \overline{JK}$
$\overline{JK} \cong \overline{ST}$
$\overline{AB} \cong \overline{ST}$

17. Write segment statements using the Subtraction Property of Equality.

Sample Answer: $m\overline{CD} = m\overline{GH}$
$m\overline{CD} - m\overline{JK} = m\overline{GH} - m\overline{JK}$

18. Write an angle statement using the Reflexive Property.

Sample Answer: $m\angle R = m\angle R$

Rewrite each conditional statement by separating the hypothesis and conclusion. The hypothesis becomes the "Given" information and the conclusion becomes the "Prove" information.

19. Conditional statement: If $\angle 2 \cong \angle 1$, then $\angle 2 \cong \angle 3$.

Given: $\angle 2 \cong \angle 1$

Prove: $\angle 2 \cong \angle 3$

20. Conditional statement: $\overline{RT} \cong \overline{LM}$, if $\overline{RT} \cong \overline{AB}$

Given: $\overline{RT} \cong \overline{AB}$

Prove: $\overline{RT} \cong \overline{LM}$

21. Conditional statement: If $m\angle ABC = m\angle LMN$ then $m\angle ABC = m\angle XYZ$.

Given: $m\angle ABC = m\angle LMN$

Prove: $m\angle ABC = m\angle XYZ$

22. Conditional statement: $AB + RS = CD + RS$, if $AB = CD$

Given: $AB = CD$

Prove: $AB + RS = CD + RS$

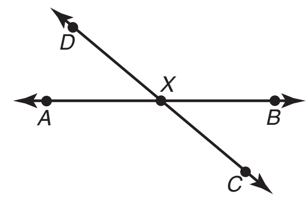
Use the indicated form of proof to prove each statement.

23. Prove the statement using a two-column proof.

Given: $m\overline{AX} = m\overline{CX}$

Given: $m\overline{BX} = m\overline{DX}$

Prove: $m\overline{AB} = m\overline{CD}$

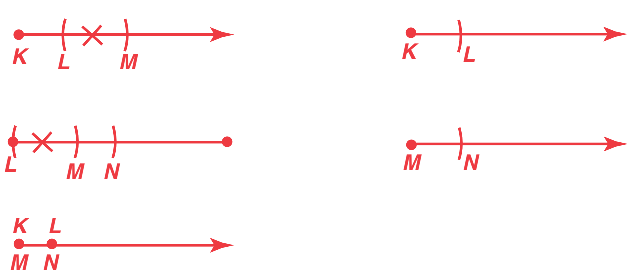


Statements	Reasons
1. $m\overline{AX} = m\overline{CX}$	1. Given
2. $m\overline{BX} = m\overline{DX}$	2. Given
3. $m\overline{AX} + m\overline{BX} = m\overline{CX} + m\overline{BX}$	3. Addition Property of Equality
4. $m\overline{AX} + m\overline{BX} = m\overline{CX} + m\overline{DX}$	4. Substitution Property
5. $m\overline{AX} + m\overline{BX} = m\overline{AB}$	5. Segment Addition Property
6. $m\overline{CX} + m\overline{DX} = m\overline{CD}$	6. Segment Addition Postulate
7. $m\overline{AB} = m\overline{CD}$	7. Substitution Property

24. Prove the statement using a construction proof.

Given: $\overline{KM} \cong \overline{LN}$

Prove: $\overline{KL} \cong \overline{MN}$

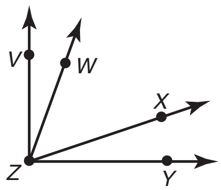


Name ______________________________ Date ____________

25. Prove the statement using a simple paragraph proof.

Given: $\angle VZW \cong \angle XZY$

Prove: $\angle VZX \cong \angle WZY$



If $m\angle VZW \cong \angle XZY$ then $m\angle VZW = m\angle XZY$ by the definition of congruent angles. Add the same angle measure, $m\angle WZX$ to both angles. By the Addition Property of Equality, $m\angle VZW + m\angle WZX = m\angle XZY + m\angle WZX$. By the Angle Addition Postulate, the angles can be renamed such that $m\angle VZW + m\angle WZX = m\angle VZX$ and $m\angle XZY + m\angle WZX = m\angle WZY$. Then $m\angle VZX = m\angle WZY$ because if you add the same angle to two angles of equal measure, the resulting angles remain equal in measure. Therefore, $\angle VZX \cong \angle WZY$.

26. Prove the statement using a flow chart proof.

Given: $\angle ABC$ and $\angle XYZ$ are straight angles.

Prove: $\angle ABC \cong \angle XYZ$

$\angle ABC$ is a straight angle
Given

$\angle XYZ$ is a straight angle
Given

$m\angle ABC = 180°$
Definition of straight angles

$m\angle XYZ = 180°$
Definition of straight angles

$m\angle ABC = m\angle XYZ$
Substitution Property

$\angle ABC \cong \angle XYZ$
Definition of congruent angles

27. Prove the statement using a simple paragraph proof.

Given: $\angle A$ is supplementary to $\angle B$

Given: $\angle C$ is supplementary to $\angle D$

Given: $\angle A \cong \angle D$

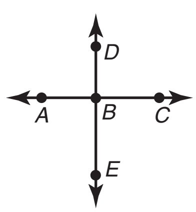
Prove: $\angle B \cong \angle C$

If $\angle A$ is supplementary to $\angle B$, $\angle C$ is supplementary to $\angle D$, and $\angle A \cong \angle D$, then $m\angle A = m\angle D$ by the definition of congruent angles. By the definition of supplementary angles, $m\angle A + m\angle B = 180°$ and $m\angle C + m\angle D = 180°$. $m\angle A + m\angle B = m\angle C + m\angle D$ by the Substitution Property. Then $m\angle B = m\angle C$ by the Subtraction Property of Equality. So, $\angle B \cong \angle C$ by the definition of congruent angles.

28. Prove the statement using a two-column proof.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{DE}$

Prove: $\angle ABD \cong \angle CBD$



Statements	Reasons
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{DE}$	1. Given
2. $m\angle ABD = 90°$	2. Definition of angles formed by perpendicular lines
3. $m\angle CBD = 90°$	3. Definition of angles formed by perpendicular lines
4. $m\angle ABD = m\angle CBD$	4. Transitive Property
5. $\angle ABD \cong \angle CBD$	5. Definition of congruent angles

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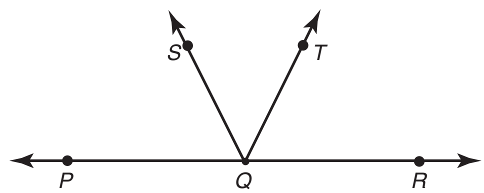
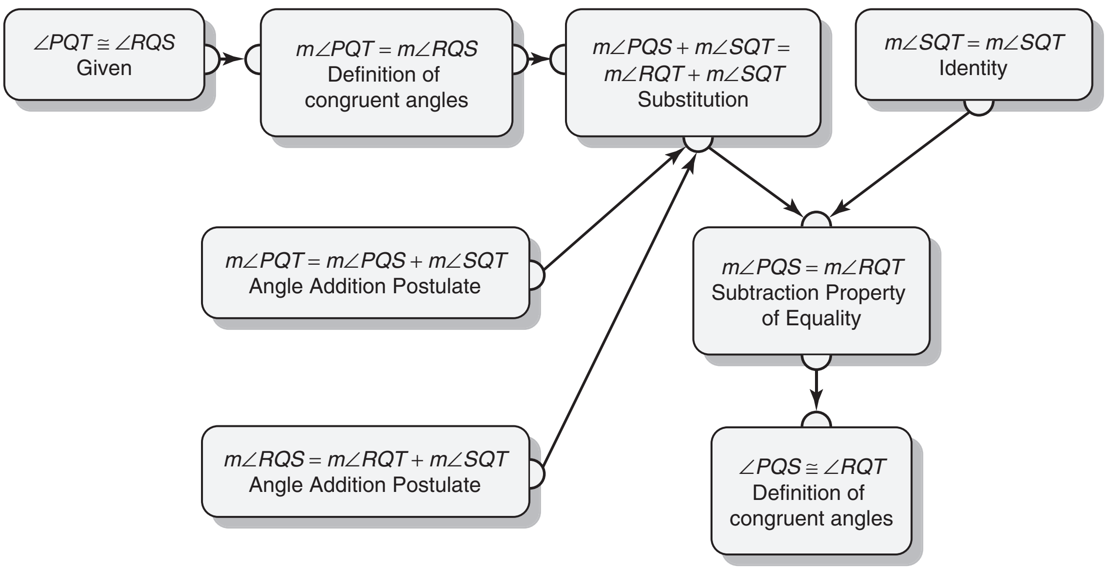
Write each given proof as the indicated proof.

29. Write the flow chart proof as a two-column proof.

Given: $\angle PQT \cong \angle RQS$

Prove: $\angle PQS \cong \angle RQT$





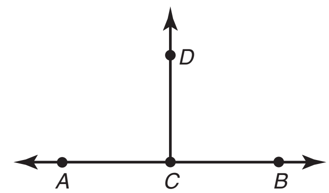
Statements	Reasons
1. $\angle PQT \cong \angle RQS$	1. Given
2. $m\angle PQT = m\angle RQS$	2. Definition of congruent angles
3. $m\angle PQT = m\angle PQS + m\angle SQT$	3. Angle Addition Postulate
4. $m\angle RQS = m\angle RQT + m\angle SQT$	4. Angle Addition Postulate
5. $m\angle PQS + m\angle SQT = m\angle RQT + m\angle SQT$	5. Substitution
6. $m\angle SQT = m\angle SQT$	6. Identity
7. $m\angle PQS = m\angle RQT$	7. Subtraction Property of Equality
8. $\angle PQS \cong m\angle RQT$	8. Definition of congruent angles

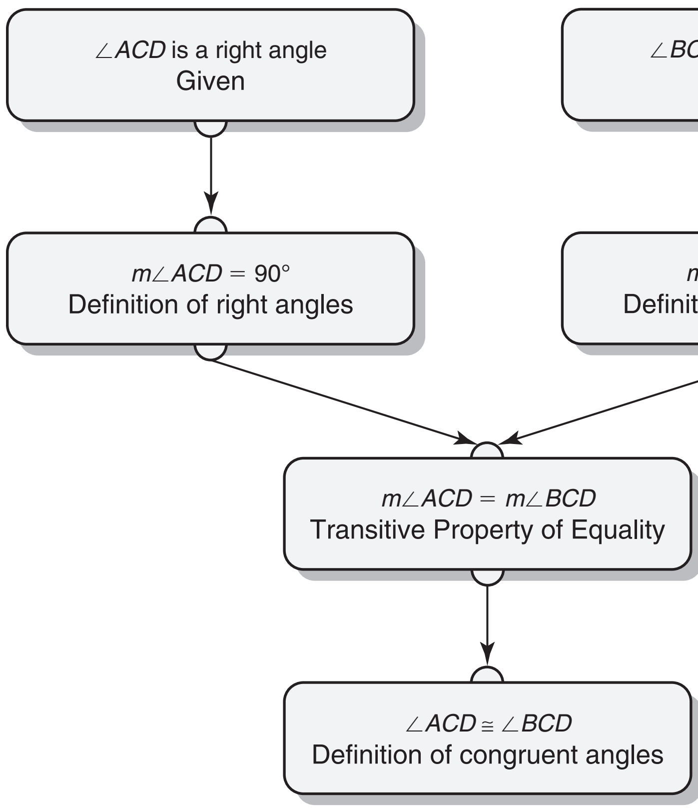
2

30. Write the flow chart proof of the Right Angle Congruence Theorem as a two-column proof.

Given: Angles *ACD* and *BCD* are right angles.

Prove: $\angle ACD \cong \angle BCD$





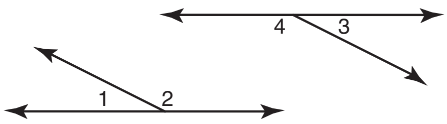
Statements	Reasons
1. $\angle ACD$ is a right angle	1. Given
2. $\angle BCD$ is a right angle	2. Given
3. $m\angle ACD = 90°$	3. Definition of right angles
4. $m\angle BCD = 90°$	4. Definition of right angles
5. $m\angle ACD = m\angle BCD$	5. Transitive Property of Equality
6. $\angle ACD \cong \angle BCD$	6. Definition of congruent angles

Name ______________________________ Date ____________

31. Write the two-column proof of the Congruent Supplement Theorem as a paragraph proof.

Given: $\angle 1$ is supplementary to $\angle 2$,
$\angle 3$ is supplementary to $\angle 4$,
and $\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$



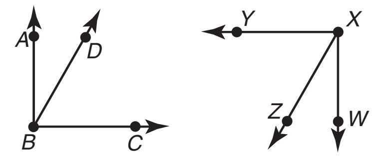
Statements	Reasons
1. $\angle 1$ is supplementary to $\angle 2$	**1.** Given
2. $\angle 3$ is supplementary to $\angle 4$	**2.** Given
3. $\angle 2 \cong \angle 4$	**3.** Given
4. $m\angle 2 = m\angle 4$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 = 180°$	**5.** Definition of supplementary angles
6. $m\angle 3 + m\angle 4 = 180°$	**6.** Definition of supplementary angles
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**7.** Substitution Property
8. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**8.** Substitution Property
9. $m\angle 1 = m\angle 3$	**9.** Subtraction Property of Equality
10. $\angle 1 \cong \angle 3$	**10.** Definition of congruent angles

If angles 1 and 2 are supplementary, then $m\angle 1 + m\angle 2 = 180°$ by the definition of supplementary angles. Likewise, if angles 3 and 4 are supplementary, then $m\angle 3 + m\angle 4 = 180°$ by the definition of supplementary angles. You can use the Substitution Property to write $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$. You are given that $\angle 2 \cong \angle 4$, so $m\angle 2 = m\angle 4$ by the definition of congruent angles. Then, you can use the Substitution Property to substitute $\angle 2$ for $\angle 4$ into the equation $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ to get $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$. By the Subtraction Property of Equality $m\angle 1 = m\angle 3$. So, $\angle 1 \cong \angle 3$ by the definition of congruent angles.

32. Write the two-column proof of the Congruent Complement Theorem as a paragraph proof.

Given: Angles ABD and DBC are complementary, angles WXZ and ZXY are complementary, and $\angle DBC \cong \angle ZXY$

Prove: $\angle ABD \cong \angle WXZ$



Statements	Reasons
1. $\angle ABD$ is complementary to $\angle DBC$	**1.** Given
2. $\angle WXZ$ is complementary to $\angle ZXY$	**2.** Given
3. $\angle DBC \cong \angle ZXY$	**3.** Given
4. $m\angle ABD + m\angle DBC = 90°$	**4.** Definition of complementary angles
5. $m\angle WXZ + m\angle ZXY = 90°$	**5.** Definition of complementary angles
6. $m\angle DBC = m\angle ZXY$	**6.** Definition of congruent angles
7. $m\angle ABD + m\angle DBC = m\angle WXZ + m\angle ZXY$	**7.** Substitution Property
8. $m\angle ABD + m\angle DBC = m\angle WXZ + m\angle DBC$	**8.** Substitution Property
9. $m\angle ABD = m\angle WXZ$	**9.** Subtraction Property of Equality
10. $\angle ABD \cong \angle WXZ$	**10.** Definition of congruent angles

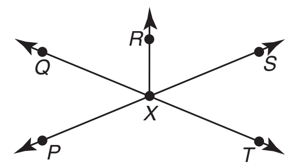
If angles ABD and DBC are complementary, then $m\angle ABD + m\angle DBC = 90°$ by the definition of complementary angles. Likewise, if angles WXZ and ZXY are complementary, then $m\angle WXZ + m\angle ZXY = 90°$ by the definition of complementary angles. You can use the Substitution Property to write $m\angle ABD + m\angle DBC = m\angle WXZ + m\angle ZXY$. You are given that $\angle DBC \cong \angle ZXY$, so $m\angle DBC = m\angle ZXY$ by the definition of congruent angles. Then, you can use the Substitution Property to substitute $m\angle DBC$ for $m\angle ZXY$ into the equation $m\angle ABD + m\angle DBC = m\angle WXZ + m\angle ZXY$ to get $m\angle ABD + m\angle DBC = m\angle WXZ + m\angle DBC$. By the Subtraction Property of Equality, $m\angle ABD = m\angle WXZ$. So, $\angle ABD \cong \angle WXZ$ by the definition of congruent angles.

Name ____ Date ____________

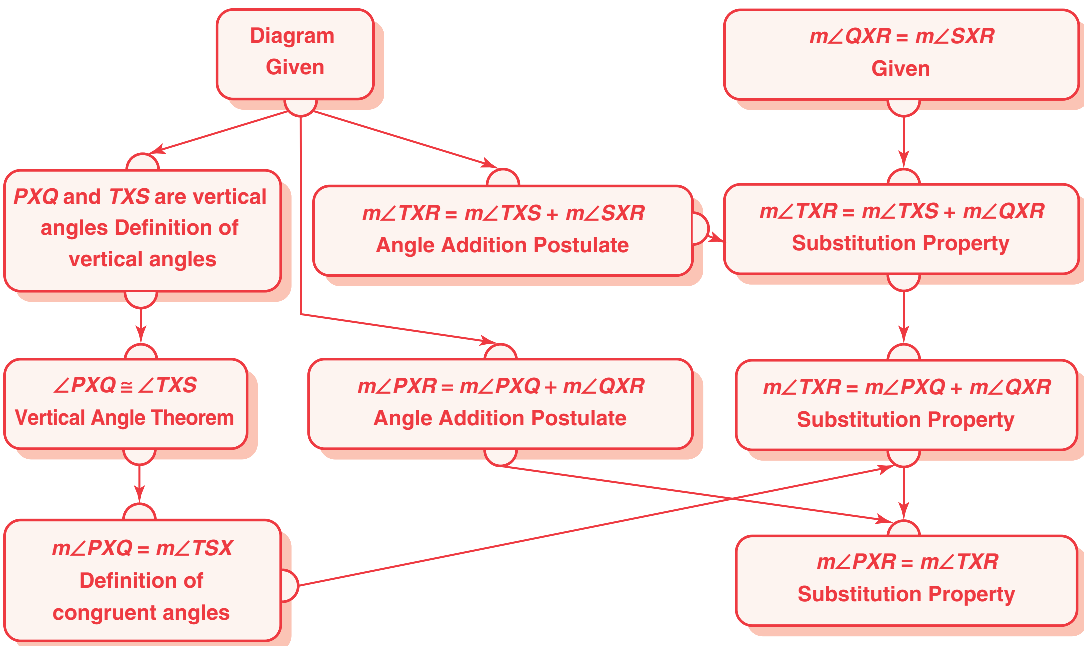
33. Write the paragraph proof as a flow chart proof.

Given: $m\angle QXR = m\angle SXR$

Prove: $m\angle PXR = m\angle TXR$



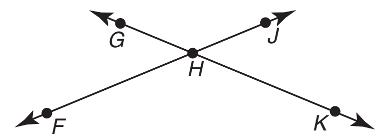
By the Angle Addition Postulate, $m\angle TXR = m\angle TXS + m\angle SXR$. It is given that $m\angle QXR = m\angle SXR$, so by substitution, $m\angle TXR = m\angle TXS + m\angle QXR$. Angles *PXQ* and *TXS* are vertical angles by the definition of vertical angles. Vertical angles are congruent by the Vertical Angle Theorem, so $\angle PXQ \cong \angle TXS$, and by the definition of congruent angles, $m\angle PXQ = m\angle TXS$. Using substitution, you can write $m\angle TXR = m\angle PXQ + m\angle QXR$. By the Angle Addition Postulate, $m\angle PXR = m\angle PXQ + m\angle QXR$. So, you can use substitution to write $m\angle PXR = m\angle TXR$.



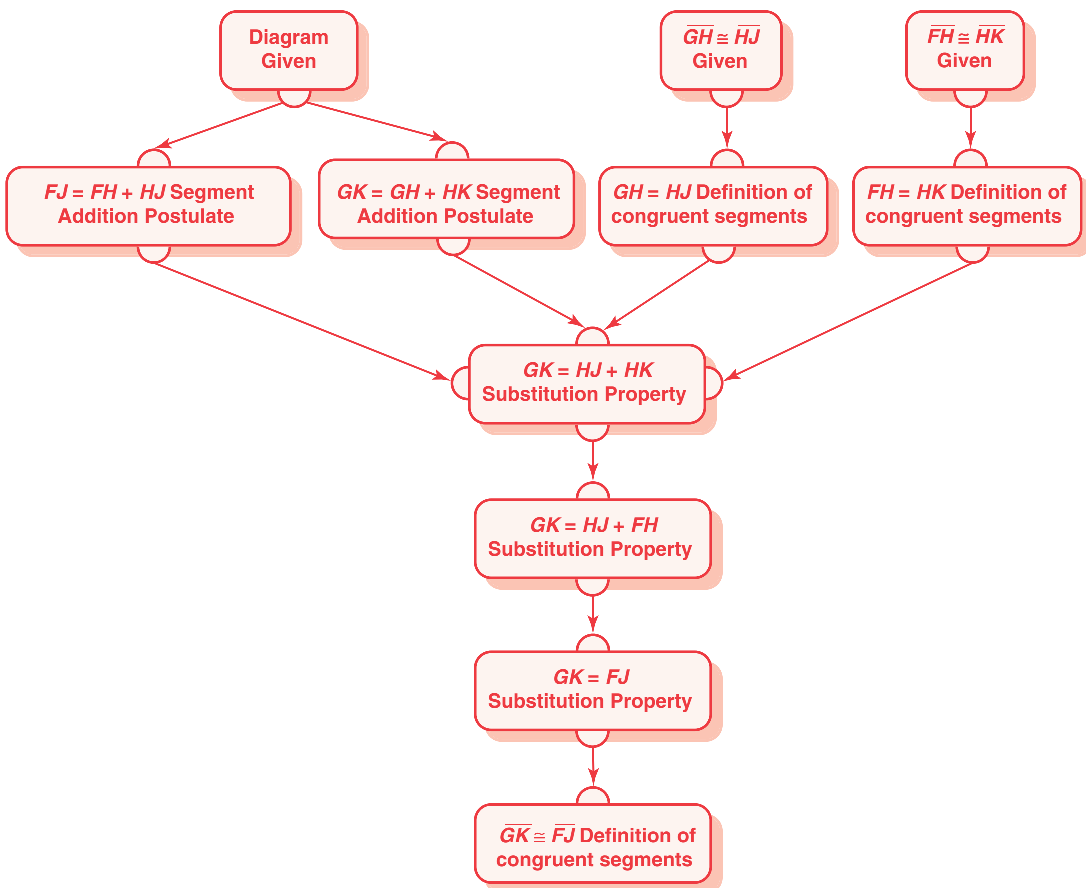
34. Write the paragraph proof as a flow chart proof.

Given: $\overline{GH} \cong \overline{HJ}$ and $\overline{FH} \cong \overline{HK}$

Prove: $\overline{GK} \cong \overline{FJ}$



By the Segment Addition Postulate, $GK = GH + HK$. You are given that $\overline{GH} = \overline{HJ}$, so $GH = HJ$ by the definition of congruent segments, and you can use substitution to write $GK = HJ + HK$. You are also given that $\overline{FH} \cong \overline{HK}$, so $FH = HK$ by the definition of congruent segments, and you can use substitution to write $GK = HJ + FH$. By the Segment Addition Postulate, $FJ = FH + HJ$. So, you can use substitution to write $GK = FJ$. By the definition of congruent segments, $\overline{GK} \cong \overline{FJ}$.



Name ____________________ Date __________

What's Your Proof?
Angle Postulates and Theorems

Vocabulary

Define each theorem in your own words.

1. Alternate Interior Angle Theorem
   If two parallel lines are intersected by a transversal then alternate interior angles are congruent.

2. Alternate Exterior Angle Theorem
   If two parallel lines are intersected by a transversal then alternate exterior angles are congruent.

3. Same-Side Interior Angle Theorem
   If two parallel lines are intersected by a transversal then interior angles on the same side of the transversal are supplementary.

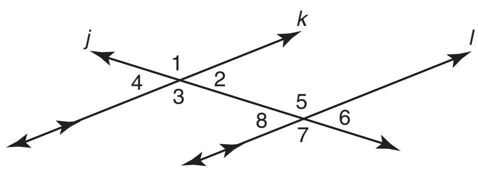
4. Same-Side Exterior Angle Theorem
   If two parallel lines are intersected by a transversal then exterior angles on the same side of the transversal are supplementary.

5. Corresponding Angle Postulate
   If two parallel lines are intersected by a transversal, then corresponding angles are congruent.

Problem Set

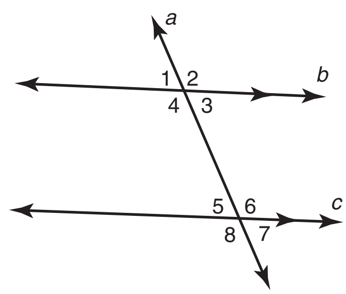
Write congruence statements for the pairs of corresponding angles in each figure.

1.



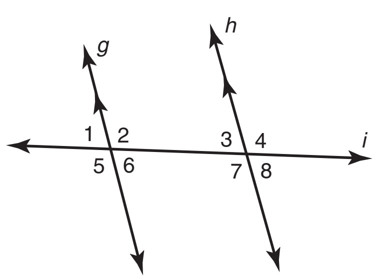
∠1 ≅ ∠5, ∠2 ≅ ∠6,
∠3 ≅ ∠7, ∠4 ≅ ∠8

2.



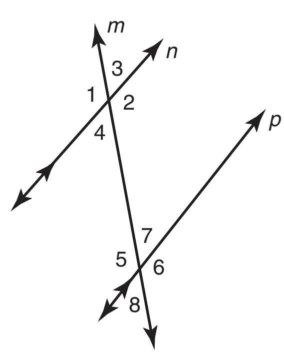
∠1 ≅ ∠5, ∠2 ≅ ∠6,
∠4 ≅ ∠8, ∠3 ≅ ∠7

3.



∠1 ≅ ∠3, ∠5 ≅ ∠7,
∠2 ≅ ∠4, ∠6 ≅ ∠8

4.



∠1 ≅ ∠5, ∠3 ≅ ∠7,
∠4 ≅ ∠8, ∠2 ≅ ∠6

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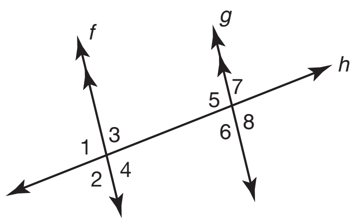
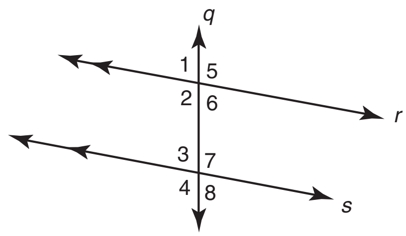
Make a conjecture to explain why each statement is true.

5. $\angle 3 \cong \angle 6$



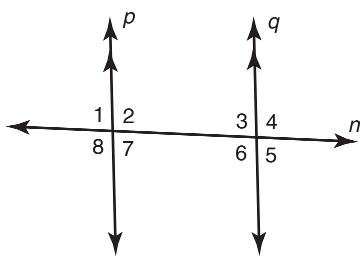
Alternate interior angles are congruent.

6. $m\angle 1 + m\angle 4 = 180°$



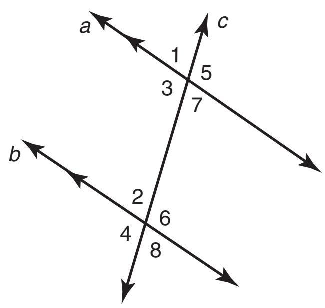
Same-side exterior angles are supplementary.

7. $\angle 1 \cong \angle 5$



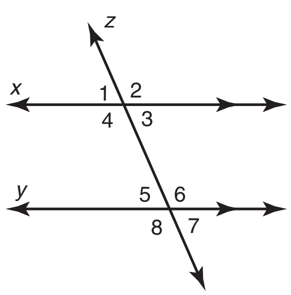
Alternate exterior angles are congruent.

8. $\angle 4 \cong \angle 6$



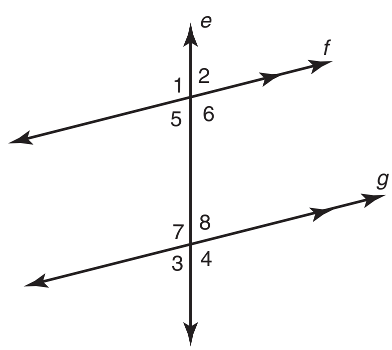
Vertical angles are congruent.

9. $m\angle 4 + m\angle 5 = 180°$



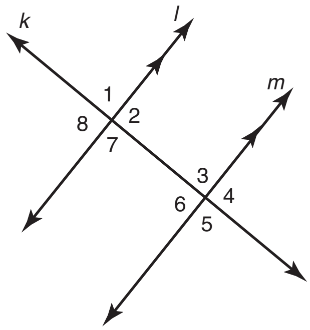
Same-side interior angles are congruent.

10. $\angle 5 \cong \angle 8$



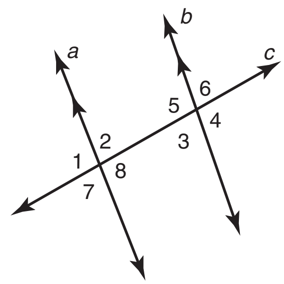
Alternate interior angles are congruent.

11. $\angle 6 \cong \angle 8$



Corresponding angles are congruent.

12. $\angle 6 \cong \angle 7$

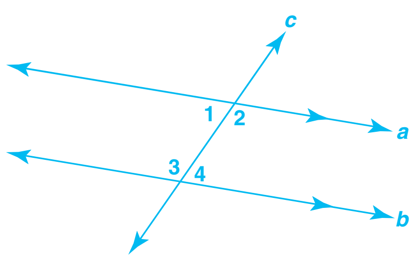


Alternate exterior angles are congruent.

Draw and label a diagram to illustrate each theorem.

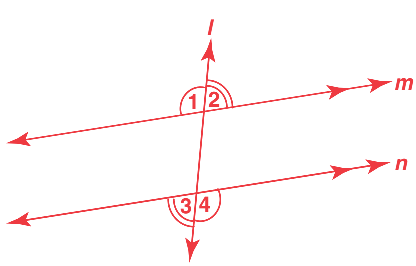
13. Same-Side Interior Angle Theorem

$\angle 1$ and $\angle 3$ are supplementary or $\angle 2$ and $\angle 4$ are supplementary



14. Alternate Exterior Angle Theorem

$\angle 1 \cong \angle 4$ or $\angle 2 \cong \angle 3$



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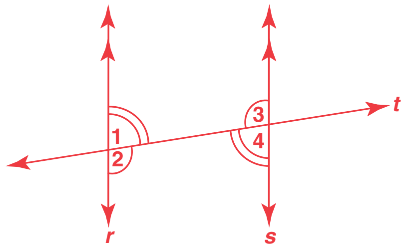
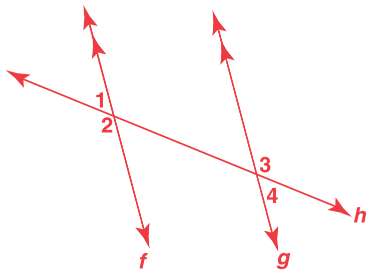
15. Alternate Interior Angle Theorem

$\angle 1 \cong \angle 4$ or $\angle 2 \cong \angle 3$



16. Same-Side Exterior Angle Theorem

$\angle 1$ and $\angle 3$ are supplementary or $\angle 2$ and $\angle 4$ are supplementary

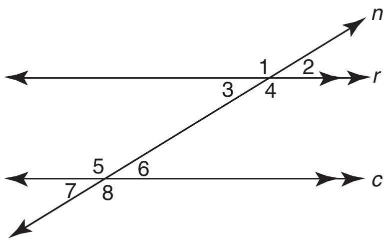


Use the diagram to write the "Given" and "Prove" statements for each theorem.

17. If two parallel lines are cut by a transversal, then the exterior angles on the same side of the transversal are supplementary.

Given: $r \parallel c$, n is a transversal

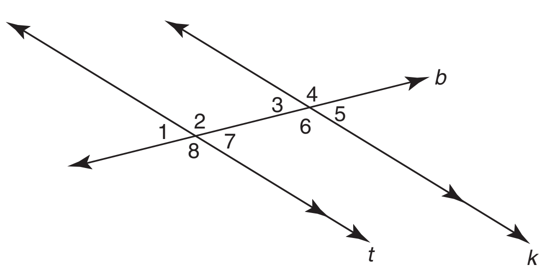
Prove: $\angle 1$ and $\angle 7$ are supplementary or $\angle 2$ and $\angle 8$ are supplementary



18. If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

Given: $t \parallel k$, b is a transversal

Prove: $\angle 1 \cong \angle 5$ or $\angle 8 \cong \angle 4$



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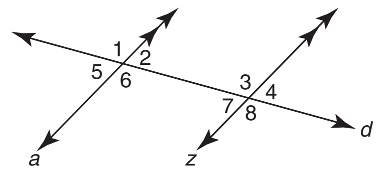
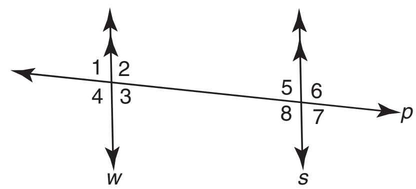
19. If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.



Given: $a \parallel z$, d is a transversal

Prove: $\angle 2 \cong \angle 7$ or $\angle 6 \cong \angle 3$

20. If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.

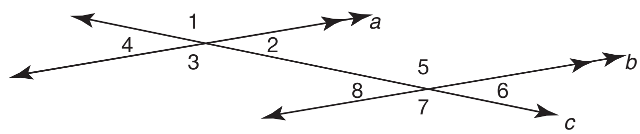


Given: $w \parallel s$, p is a transversal

Prove: $\angle 2$ and $\angle 5$ are supplementary or $\angle 3$ and $\angle 8$ are supplementary

Prove each statement using the indicated type of proof.

21. Use a paragraph proof to prove the Alternate Interior Angles Theorem. In your proof, use the following information and refer to the diagram.

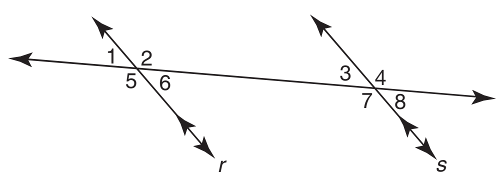


Given: $a \parallel b$, c is a transversal

Prove: $\angle 2 \cong \angle 8$

You are given that lines a and b are parallel and line c is a transversal, as shown in the diagram. Angles 2 and 6 are corresponding angles by definition, and corresponding angles are congruent by the Corresponding Angles Postulate. So, $\angle 2 \cong \angle 6$. Angles 6 and 8 are vertical angles by definition, and vertical angles are congruent by the Vertical Angles Congruence Theorem. So, $\angle 6 \cong \angle 8$. Since $\angle 2 \cong \angle 6$ and $\angle 2 \cong \angle 8$, by the Transitive Property, $\angle 2 \cong \angle 8$.

22. Use a two-column proof to prove the Alternate Exterior Angles Theorem. In your proof, use the following information and refer to the diagram.



Given: $r \parallel s$, t is a transversal

Prove: $\angle 4 \cong \angle 5$

Statements	Reasons
1. $r \parallel s$, t is a transversal	1. Given
2. $\angle 4 \cong /2$	2. Corresponding Angles Postulate
3. $\angle 2 \cong /5$	3. Vertical Angles Congruence Theorem
4. $\angle 4 \cong /5$	4. Transitive Property

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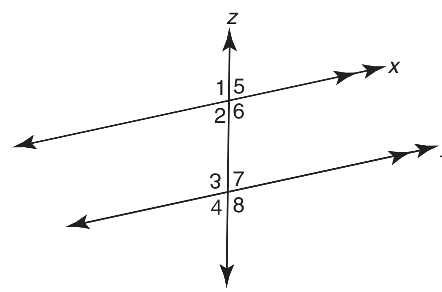
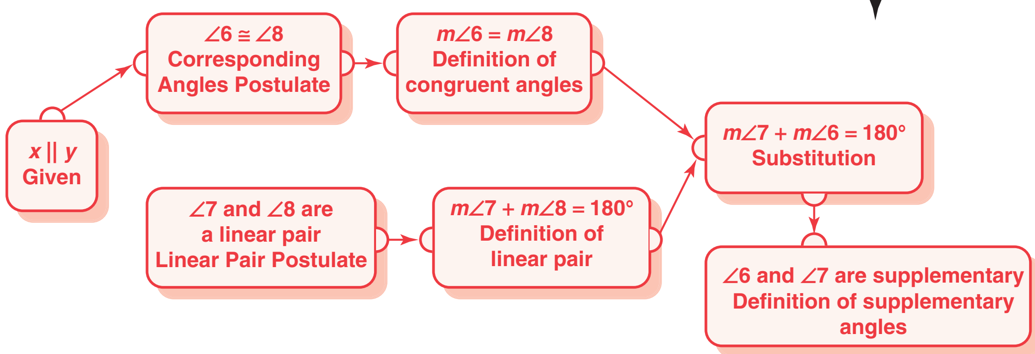
23. Use a flow chart proof to prove the Same-Side Interior Angles Theorem. In your proof, use the following information and refer to the diagram.

Given: $x \parallel y$, z is a transversal

Prove: $\angle 6$ and $\angle 7$ are supplementary

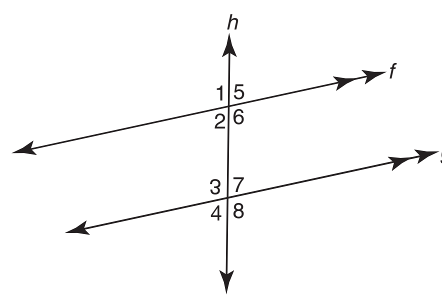




24. Use a two-column proof to prove the Same-Side Exterior Angles Theorem. In your proof, use the following information and refer to the diagram.

Given: $f \parallel g$, h is a transversal

Prove: $\angle 1$ and $\angle 4$ are supplementary



Statements	Reasons
1. $f \parallel g$, h is a transversal	1. Given
2. $\angle 1$ and $\angle 2$ are a linear pair	2. Linear Pair Postulate
3. $m\angle 1 + m\angle 2 = 180°$	3. Definition of linear pair
4. $\angle 2 \cong \angle 4$	4. Corresponding Angle Postulate
5. $m\angle 2 = m\angle 4$	5. Definition of congruent angles
6. $m\angle 1 + m\angle 4 = 180°$	6. Substitution
7. $\angle 1$ are $\angle 4$ supplementary	7. Definition of supplementary angles

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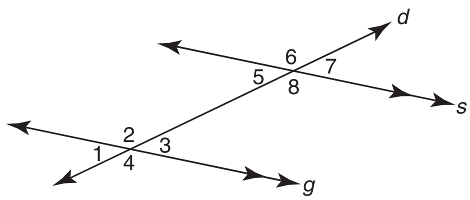
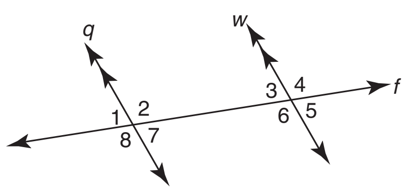
Write the theorem that is illustrated by each statement and diagram.

25. $\angle 4$ and $\angle 7$ are supplementary



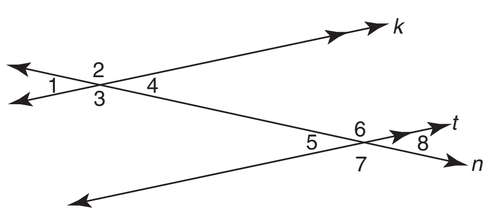
Same-Side Exterior Angles Theorem

26. $\angle 2 \cong \angle 6$



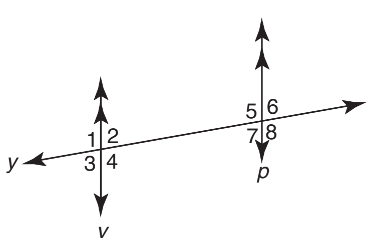
Alternate Interior Angles Theorem

27. $\angle 1 \cong \angle 8$



Alternate Exterior Angles Theorem

28. $\angle 2$ and $\angle 5$ are supplementary



Same-Side Interior Angles Theorem

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A Reversed Condition
Parallel Line Converse Theorems

Vocabulary

Answer the following question.

1. What is the converse of a statement?

   Sample: In a statement "if *p* then *q*," the converse is the statement, "If *q* then *p*."

Problem Set

Write the converse of each postulate or theorem.

1. Alternate Interior Angle Theorem:

   If a transversal intersects two parallel lines, then the alternate interior angles formed are congruent.

   If alternate interior angles formed by two lines and a transversal are congruent, then the two lines are parallel.

2. Alternate Exterior Angle Theorem:

   If a transversal intersects two parallel lines, then the alternate exterior angles formed are congruent.

   If alternate exterior angles formed by two lines and a transversal are congruent, then the two lines are parallel.

3. Same-Side Interior Angle Theorem:

   If a transversal intersects two parallel lines, then the interior angles on the same side of the transversal formed are supplementary.

   If same-side interior angles formed by two lines and a transversal are supplementary, then the two lines are parallel.

4. Same-Side Exterior Angle Theorem:

   If a transversal intersects two parallel lines, then the exterior angles on the same side of the transversal formed are supplementary.

   If same-side exterior angles formed by two lines and a transversal are supplementary, then the two lines are parallel.

Write the converse of each statement.

5. If a triangle has three congruent sides, then the triangle is an equilateral triangle.

   Converse: If a triangle is an equilateral triangle, then the triangle has three congruent sides.

6. If a figure has four sides, then it is a quadrilateral.

   Converse: If a figure is a quadrilateral, then it has four sides.

7. If a figure is a rectangle, then it has four sides.

   Converse: If a figure has four sides, then it is a rectangle.

8. If two angles are vertical angles, then they are congruent.

   Converse: If two angles are congruent, then they are vertical angles.

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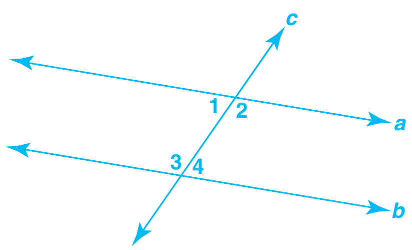
9. If two angles in a triangle are congruent, then the triangle is isosceles.
Converse: If a triangle is isosceles, then two angles in the triangle are congruent.

10. If two intersecting lines form a right angle, then the lines are perpendicular.
Converse: If two intersecting lines are perpendicular, then the lines form a right angle.

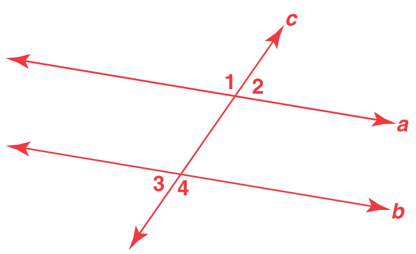
Draw and label a diagram to illustrate each theorem.

11. Same-Side Interior Angle Converse Theorem
Given: $\angle 1$ and $\angle 3$ are supplementary or $\angle 2$ and $\angle 4$ are supplementary



Conclusion: Lines *a* and *b* are parallel.

12. Alternate Exterior Angle Converse Theorem
Given: $\angle 1 \cong \angle 4$ or $\angle 2 \cong \angle 3$



Conclusion: Lines *a* and *b* are parallel.

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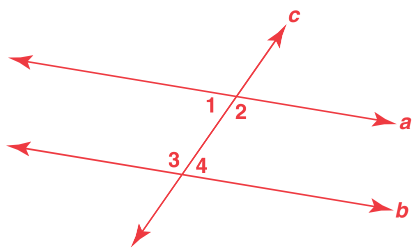
13. Alternate Interior Angle Converse Theorem

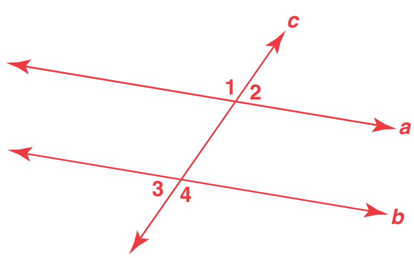
Given: $\angle 1 \cong \angle 4$ or $\angle 2 \cong \angle 3$



Conclusion: Lines *a* and *b* are parallel.

14. Same-Side Exterior Angle Converse Theorem

Given: $\angle 1$ and $\angle 3$ are supplementary or $\angle 2$ and $\angle 4$ are supplementary



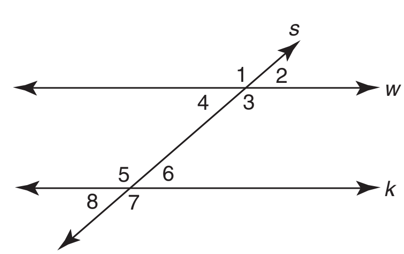
Conclusion: Lines *a* and *b* are parallel.

Use the diagram to write the "Given" and "Prove" statements for each theorem.

15. If two lines, cut by a transversal, form same-side exterior angles that are supplementary, then the lines are parallel.

Given: *s* is a transversal; $\angle 1$ and $\angle 8$ are supplementary or $\angle 2$ and $\angle 7$ are supplementary

Prove: $w \parallel k$

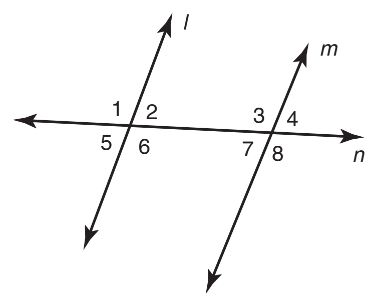


Name ______________________________ Date ______________

16. If two lines, cut by a transversal, form alternate exterior angles that are congruent, then the lines are parallel.

Given: n is a transversal; $\angle 1 \cong \angle 8$ or $\angle 5 \cong \angle 4$

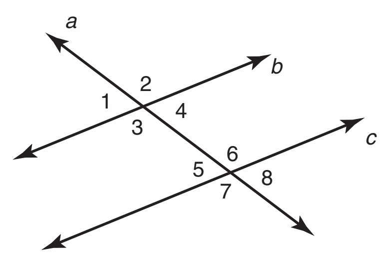
Prove: $l \parallel m$



17. If two lines, cut by a transversal, form alternate interior angles that are congruent, then the lines are parallel.

Given: a is a transversal; $\angle 3 \cong \angle 6$ or $\angle 4 \cong \angle 5$

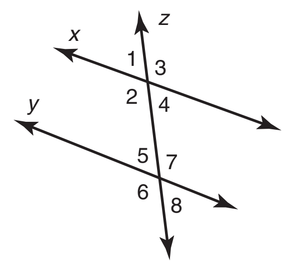
Prove: $b \parallel c$



18. If two lines, cut by a transversal, form same-side interior angles that are supplementary, then the lines are parallel.

Given: z is a transversal; $\angle 2$ and $\angle 5$ are supplementary or $\angle 4$ and $\angle 7$ are supplementary

Prove: $x \parallel y$

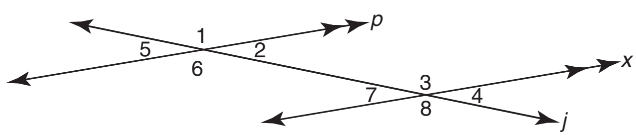


Prove each statement using the indicated type of proof.

19. Use a paragraph proof to prove the Alternate Exterior Angles Converse Theorem. In your proof, use the following information and refer to the diagram.

Given: $\angle 4 \cong \angle 5$, j is a transversal

Prove: $p \parallel x$

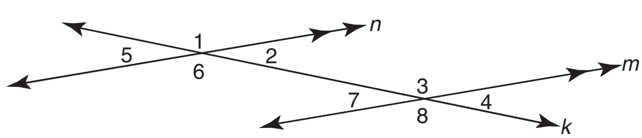


You are given that $\angle 4 \cong \angle 5$ and line j is a transversal, as shown in the diagram. Angles 5 and 2 are vertical angles by definition, and vertical angles are congruent by the Vertical Angles Congruence Theorem. So, $\angle 5 \cong \angle 2$. Since $\angle 4 \cong \angle 5$ and $\angle 5 \cong \angle 2$, by the Transitive Property, $\angle 4 \cong \angle 2$. Angles 4 and 2 are corresponding angles by definition, and they are also congruent, so by the Corresponding Angles Converse Postulate, $p \parallel x$.

20. Use a two-column proof to prove the Alternate Interior Angles Converse Theorem. In your proof, use the following information and refer to the diagram.

Given: $\angle 2 \cong \angle 7$, k is a transversal

Prove: $m \parallel n$



Statements	Reasons
1. $\angle 2 \cong \angle 7$ and line k is a transversal	1. Given
2. Angles 5 and 2 are vertical angles	2. Definition of vertical angles
3. $\angle 5 \cong \angle 2$	3. Vertical Angles Congruence Theorem
4. $\angle 5 \cong \angle 7$	4. Transitive Property
5. Angles 5 and 7 are corresponding angles	5. Definition of corresponding angles
6. $m \parallel n$	6. Corresponding Angles Converse Postulate

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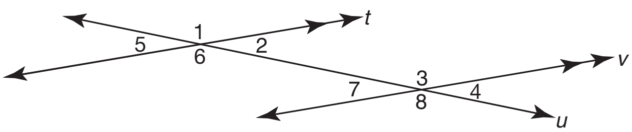
21. Use a two-column proof to prove the Same-Side Exterior Angles Converse Theorem. In your proof, use the following information and refer to the diagram.

Given: $\angle 1$ and $\angle 4$ are supplementary, u is a transversal

Prove: $t \parallel v$

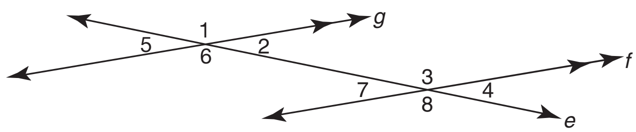


Statements	Reasons
1. $\angle 1$ and $\angle 4$ are supplementary and line u is a transversal	1. Given
2. $\angle 1$ and $\angle 2$ are a linear pair	2. Definition of linear pair
3. $\angle 1$ and $\angle 2$ are supplementary	3. Linear Pair Postulate
4. $\angle 2 \cong \angle 4$	4. Supplements of the same angle are congruent
5. Angles 2 and 4 are corresponding angles	5. Definition of corresponding angles
6. $t \parallel v$	6. Corresponding Angles Converse Postulate

22. Use a flow chart to prove the Same-Side Interior Angles Converse Theorem. In your proof, use the following information and refer to the diagram.

Given: $\angle 6$ and $\angle 7$ are supplementary, e is a transversal

Prove: $f \parallel g$



$\angle 6$ and $\angle 7$ are supplementary
Given

$\angle 5$ and $\angle 6$ are a linear pair
Definition

$\angle 5 \cong \angle 7$
Supplements of same angle are congruent

$\angle 5$ and $\angle 6$ are supplementary
Linear Pair Postulate

$\angle 5$ and $\angle 7$ are corresponding angles
Definition

$g \parallel f$
Corresponding Angles Converse Postulate