Name ____ Date ____________

Big and Small

Dilating Triangles to Create Similar Triangles

Vocabulary

Define the term in your own words.

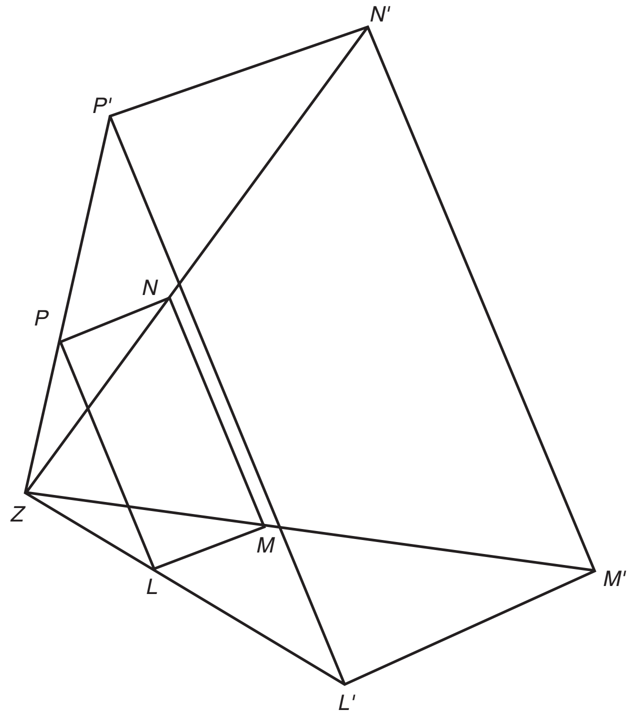
1. similar triangles

Triangles in which all corresponding angles are congruent and all corresponding sides are proportional.

Problem Set

Rectangle $L'M'N'P'$ is a dilation of rectangle $LMNP$. The center of dilation is point Z. Use a metric ruler to determine the actual lengths of $\overline{ZL}$, $\overline{ZN}$, $\overline{ZM}$, $\overline{ZP}$, $\overline{ZL'}$, $\overline{ZN'}$, $\overline{ZM'}$, and $\overline{ZP'}$ to the nearest tenth of a centimeter. Then, express the ratios $\frac{ZL'}{ZL}$, $\frac{ZN'}{ZN}$, $\frac{ZM'}{ZM}$, and $\frac{ZP'}{ZP}$ as decimals.

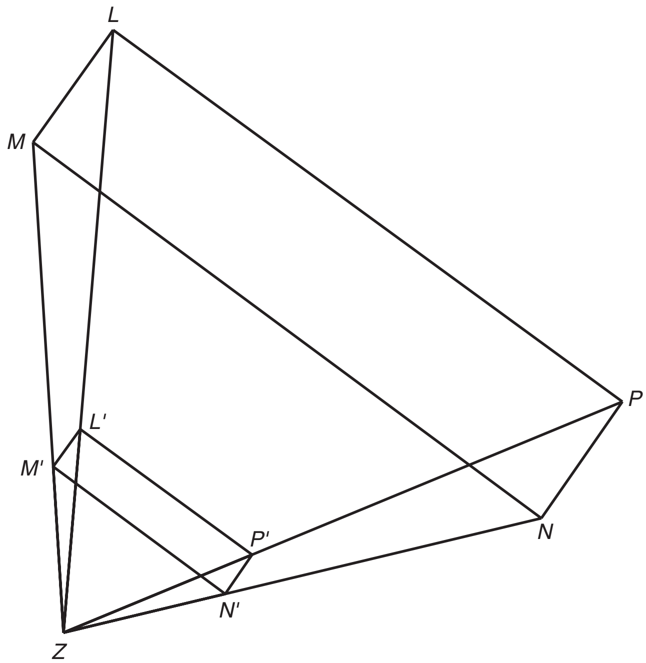
1.



$ZL = 2$, $ZN = 3$, $ZM = 3$, $ZP = 2$, $ZL' = 5$, $ZN' = 7.5$, $ZM' = 7.5$, $ZP' = 5$

$\frac{ZL'}{ZL} = \frac{5}{2} = 2.5$, $\frac{ZN'}{ZN} = \frac{7.5}{3} = 2.5$, $\frac{ZM'}{ZM} = \frac{7.5}{3} = 2.5$, $\frac{ZP'}{ZP} = \frac{5}{2} = 2.5$

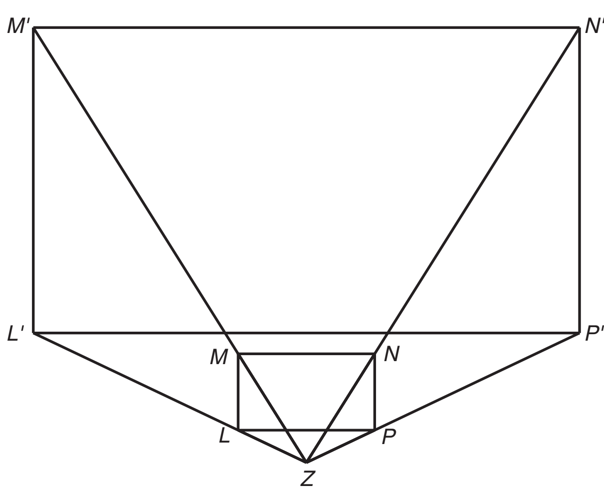
2.



$ZL = 8$, $ZN = 6.5$, $ZM = 6.5$, $ZP = 8$, $ZL' = 2.7$, $ZN' = 2.2$, $ZM' = 2.2$, $ZP' = 2.7$

$\frac{ZL'}{ZL} = \frac{2.7}{8} = 0.34$, $\frac{ZN'}{ZN} = \frac{2.2}{6.5} = 0.34$, $\frac{ZM'}{ZM} = \frac{2.2}{6.5} = 0.34$, $\frac{ZP'}{ZP} = \frac{2.7}{8} = 0.34$

3.



$ZL = 1$, $ZN = 1.7$, $ZM = 1.7$, $ZP = 1$, $ZL' = 4$, $ZN' = 6.8$, $ZM' = 6.8$, $ZP' = 4$

$\frac{ZL'}{ZL} = \frac{4}{1} = 4$, $\frac{ZN'}{ZN} = \frac{6.8}{1.7} = 4$, $\frac{ZM'}{ZM} = \frac{6.8}{1.7} = 4$, $\frac{ZP'}{ZP} = \frac{4}{1} = 4$

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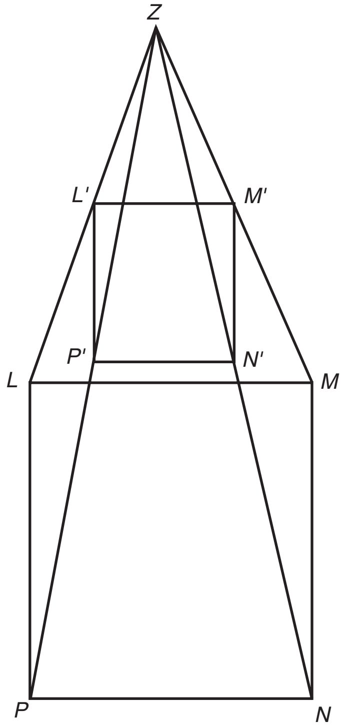
4.

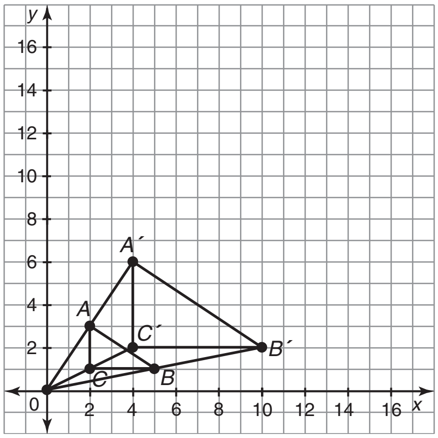


$ZL = 5$, $ZN = 9$, $ZM = 5$, $ZP = 9$, $ZL' = 2.5$, $ZN' = 4.5$, $ZM' = 2.5$, $ZP' = 4.5$

$\frac{ZL'}{ZL} = \frac{2.5}{5} = 0.5$, $\frac{ZN'}{ZN} = 4.5/9 = 0.5$, $\frac{ZM'}{ZM} = \frac{2.5}{5} = 0.5$, $\frac{ZP'}{ZP} = \frac{4.5}{9} = 0.5$

Given the image and pre-image, determine the scale factor.

5.



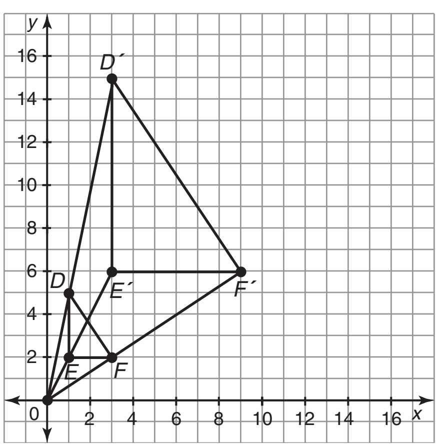
The scale factor is 2.

Each coordinate of the image is two times the corresponding coordinate of the pre-image.

$\triangle ABC$ has vertex coordinates $A(2, 3)$, $B(5, 1)$, and $C(2, 1)$.

$\triangle A'B'C'$ has vertex coordinates $A'(4, 6)$, $B'(10, 2)$, and $C'(4, 2)$.

6.



The scale factor is 3.

Each coordinate of the image is three times the corresponding coordinate of the pre-image.

$\triangle DEF$ has vertex coordinates $D(1, 5)$, $E(1, 2)$, $F(3, 2)$.

$\triangle D'E'F'$ has vertex coordinates $D'(3, 15)$, $E'(3, 6)$, $F'(9, 6)$.

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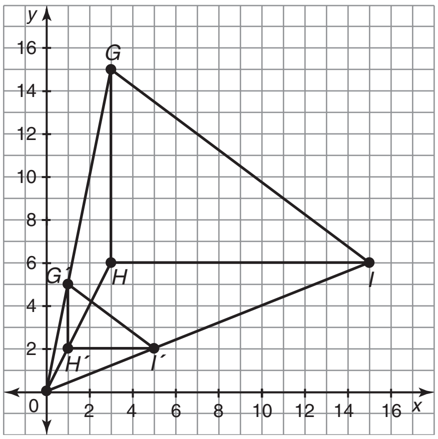
7.



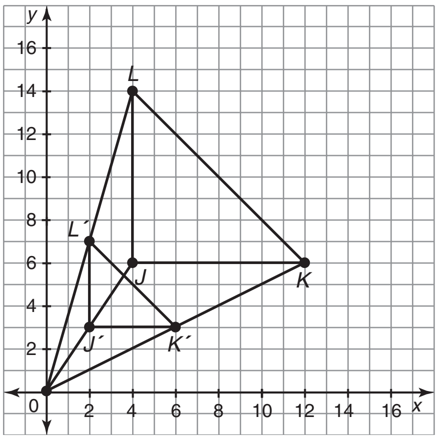
The scale factor is $\frac{1}{3}$.

Each coordinate of the image is one-third of the corresponding coordinate of the pre-image.

$\triangle GHI$ has vertex coordinates $G(3, 15)$, $H(3, 6)$, $I(15, 6)$.

$\triangle G'H'I'$ has vertex coordinates $G'(1, 5)$, $H'(1, 2)$, $I'(5, 2)$.

8.



The scale factor is $\frac{1}{2}$.

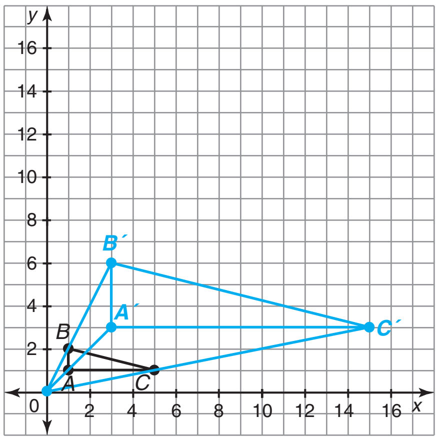
Each coordinate of the image is one-half of the corresponding coordinate of the pre-image.

$\triangle JKL$ has vertex coordinates $J(4, 6)$, $K(12, 6)$, $L(4, 14)$.

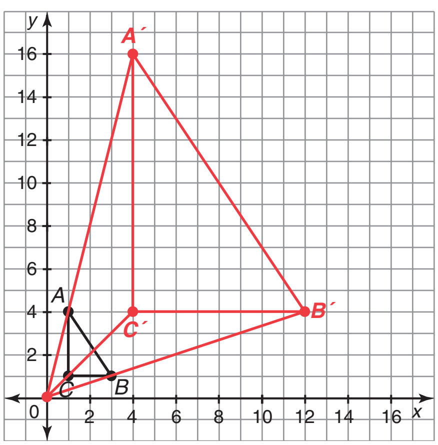
$\triangle J'K'L'$ has vertex coordinates $J'(2, 3)$, $K'(6, 3)$, $L'(2, 7)$.

Given the pre-image, scale factor, and center of dilation, use a compass and straight edge to graph the image.

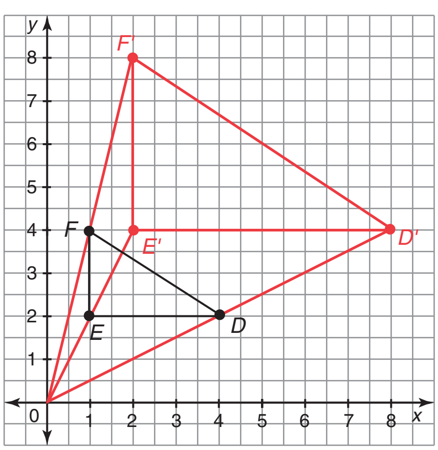
9. The scale factor is 3 and the center of dilation is the origin.



10. The scale factor is 4 and the center of dilation is the origin.



11. The scale factor is 2 and the center of dilation is the origin.



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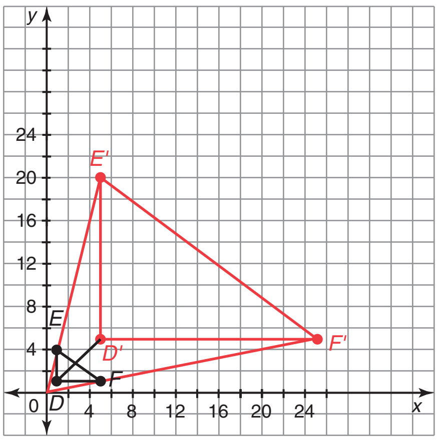
12. The scale factor is 5 and the center of dilation is the origin.



Use coordinate notation to determine the coordinates of the image.

13. △*ABC* has vertices *A*(1, 2), *B*(3, 6), and *C*(9, 7). What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

A(1, 2) → *A*′(4(1), 4(2)) = *A*′(4, 8)

B(3, 6) → *B*′(4(3), 4(6)) = *B*′(12, 24)

C(9, 7) → *C*′(4(9), 4(7)) = *C*′(36, 28)

14. △*DEF* has vertices *D*(8, 4), *E*(2, 6), and *F*(3, 1). What are the vertices of the image after a dilation with a scale factor of 5 using the origin as the center of dilation?

D(8, 4) → *D*′(5(8), 5(4)) = *D*′(40, 20)

E(2, 6) → *E*′(5(2), 5(6)) = *E*′(10, 30)

F(3, 1) → *F*′(5(3), 5(1)) = *F*′(15, 5)

15. △*GHI* has vertices *G*(0, 5), *H*(4, 2), and *I*(3, 3). What are the vertices of the image after a dilation with a scale factor of 9 using the origin as the center of dilation?

G(0, 5) → *G*′(9(0), 9(5)) = *G*′(0, 45)

H(4, 2) → *H*′(9(4), 9(2)) = *H*′(36, 18)

I(3, 3) → *I*′(9(3), 9(3)) = *I*′(27, 27)

16. △*JKL* has vertices *J*(6, 2), *K*(1, 3), and *L*(7, 0). What are the vertices of the image after a dilation with a scale factor of 12 using the origin as the center of dilation?

J(6, 2) → *J*′(12(6), 12(2)) = *J*′(72, 24)

K(1, 3) → *K*′(12(1), 12(3)) = *K*′(12, 36)

L(7, 0) → *L*′(12(7), 12(0)) = *L*′(84, 0)

17. $\triangle ABC$ has vertices $A(8, 4)$, $B(14, 16)$, and $C(6, 10)$. What are the vertices of the image after a dilation with a scale factor of $\frac{1}{2}$ using the origin as the center of dilation?

$A(8, 4) \rightarrow A'\left(\frac{1}{2}(8), \frac{1}{2}(4)\right) = A'(4, 2)$

$B(14, 16) \rightarrow B'\left(\frac{1}{2}(14), \frac{1}{2}(16)\right) = B'(7, 8)$

$C(6, 10) \rightarrow C'\left(\frac{1}{2}(6), \frac{1}{2}(10)\right) = C'(3, 5)$

18. $\triangle DEF$ has vertices $D(25, 25)$, $E(15, 10)$, and $F(20, 10)$. What are the vertices of the image after a dilation with a scale factor of $\frac{1}{5}$ using the origin as the center of dilation?

$D(25, 25) \rightarrow D'\left(\frac{1}{5}(25), \frac{1}{5}(25)\right) = D'(5, 5)$

$E(15, 10) \rightarrow E'\left(\frac{1}{5}(15), \frac{1}{5}(10)\right) = E'(3, 2)$

$F(20, 10) \rightarrow F'\left(\frac{1}{5}(20), \frac{1}{5}(10)\right) = F'(4, 2)$

19. $\triangle$GHI has vertices $G(0, 20)$, $H(16, 24)$, and $I(12, 12)$. What are the vertices of the image after a dilation with a scale factor of $\frac{3}{4}$ using the origin as the center of dilation?

$G(0, 20) \rightarrow G'\left(\frac{3}{4}(0), \frac{3}{4}(20)\right) = G'(0, 15)$

$H(16, 24) \rightarrow H'\left(\frac{3}{4}(16), \frac{3}{4}(24)\right) = H'(12, 18)$

$I(12, 12) \rightarrow I'\left(\frac{3}{4}(12), \frac{3}{4}(12)\right) = I'(9, 9)$

20. $\triangle JKL$ has vertices $J(8, 2)$, $K(6, 0)$, and $L(4, 10)$. What are the vertices of the image after a dilation with a scale factor of $\frac{5}{2}$ using the origin as the center of dilation?

$J(8, 2) \rightarrow J'\left(\frac{5}{2}(8), \frac{5}{2}(2)\right) = J'(20, 5)$

$K(6, 0) \rightarrow K'\left(\frac{5}{2}(6), \frac{5}{2}(0)\right) = K'(15, 0)$

$L(4, 10) \rightarrow L'\left(\frac{5}{2}(4), \frac{5}{2}(10)\right) = L'(10, 25)$

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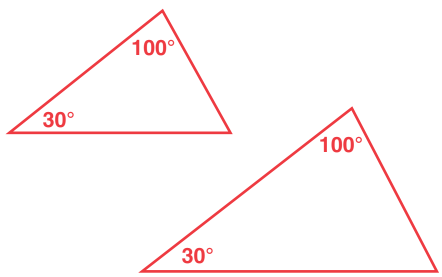
Similar Triangles or Not?
Similar Triangle Theorems

Vocabulary

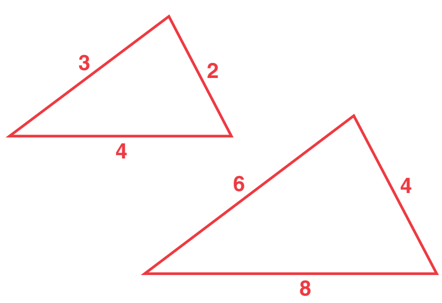
Give an example of each term. Include a sketch with each example.

1. Angle-Angle Similarity Theorem



One angle in each triangle measures 100°, and one angle in each triangle measures 30°. The triangles are similar because two angles of one triangle are congruent to two angles of the other triangle.

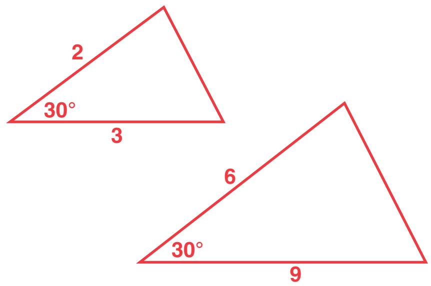
2. Side-Side-Side Similarity Theorem



The ratios of the corresponding side lengths are equal: $\frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

The triangles are similar because the corresponding sides are proportional.

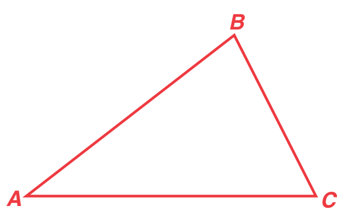
3. Side-Angle-Side Similarity Theorem



The ratios of two pairs of corresponding side lengths are equal: $\frac{2}{6} = \frac{3}{9}$.

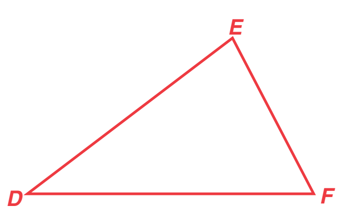
Also, the corresponding angles between those sides each have a measure of 30°. The triangles are similar because two of the corresponding sides of the two triangles are proportional and the included angles are congruent.

4. included angle



$\angle A$ is included by $\overline{AB}$ and $\overline{AC}$. So, $\angle A$ is an included angle.

5. included side



$\overline{DE}$ is included by $\angle D$ and $\angle E$. So, side DE is an included side.

4

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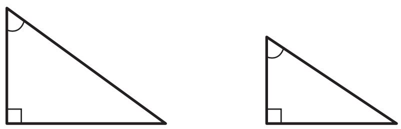
Problem Set

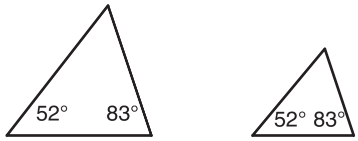
Explain how you know that the triangles are similar.

1.



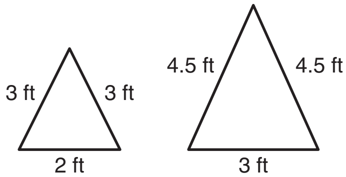
The triangles are congruent by the Angle-Angle Similarity Theorem. Two corresponding angles are congruent.

2.



The triangles are congruent by the Angle-Angle Similarity Theorem. Two corresponding angles are congruent.

3.



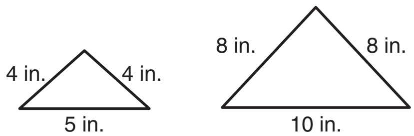
The triangles are congruent by the Side-Side-Side Similarity Theorem. All corresponding sides are proportional.

$$\frac{3}{4.5} = \frac{2}{3}$$

$$\frac{3}{4.5} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

4.



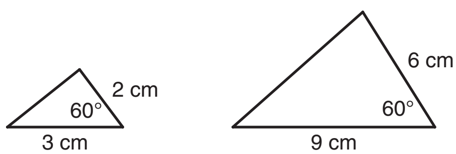
The triangles are congruent by the Side-Side-Side Similarity Theorem. All corresponding sides are proportional.

$\frac{4}{8} = \frac{1}{2}$

$\frac{4}{8} = \frac{1}{2}$

$\frac{5}{10} = \frac{1}{2}$

5.



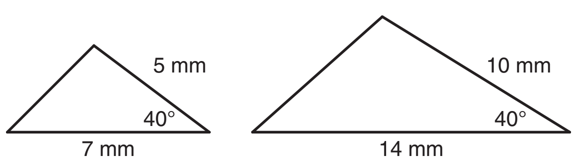
4

The triangles are congruent by the Side-Angle-Side Similarity Theorem. Two of the corresponding sides are proportional and the included angles are congruent.

$\frac{3}{9} = \frac{1}{3}$

$\frac{2}{6} = \frac{1}{3}$

6.



The triangles are congruent by the Side-Angle-Side Similarity Theorem. Two of the corresponding sides are proportional and the included angles are congruent.

$\frac{5}{10} = \frac{1}{2}$

$\frac{7}{14} = \frac{1}{2}$

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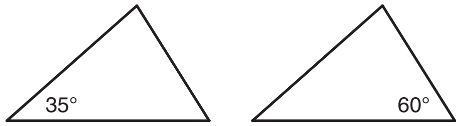
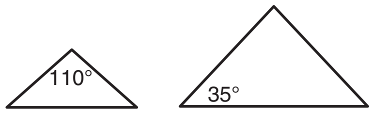
Determine what additional information you would need to prove that the triangles are similar using the given theorem.

7. What information would you need to use the Angle-Angle Similarity Theorem to prove that the triangles are similar?



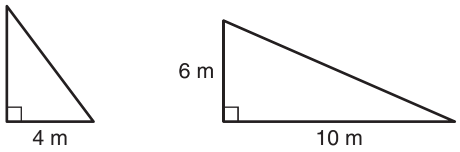
To prove that the triangles are similar using the Angle-Angle Similarity Theorem, the first triangle should have a corresponding 60 degree angle and the second triangle should have a corresponding 35 degree angle.

8. What information would you need to use the Angle-Angle Similarity Theorem to prove that the triangles are similar?



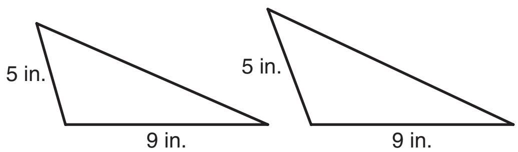
To prove that the triangles are similar using the Angle-Angle Similarity Theorem, the first triangle should have a corresponding 35 degree angle and the second triangle should have a corresponding 110 degree angle.

9. What information would you need to use the Side-Angle-Side Similarity Theorem to prove that the triangles are similar?



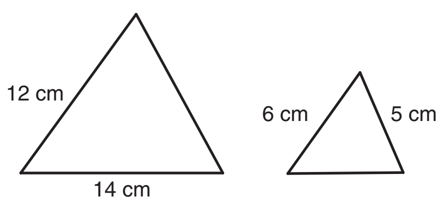
To prove that the triangles are similar using the Side-Angle-Side Similarity Theorem, the triangles both pairs of corresponding legs must be proportional.

10. What information would you need to use the Side-Angle-Side Similarity Theorem to prove that the triangles are similar?



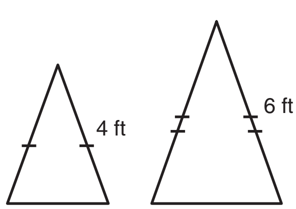
To prove that the triangles are similar using the Side-Angle-Side Similarity Theorem, the included angles between the known side lengths of both triangles should be congruent.

11. What information would you need to use the Side-Side-Side Similarity Theorem to prove that these triangles are similar?



To prove that the triangles are similar using the Side-Side-Side Similarity Theorem, all corresponding side lengths between the two triangles should be proportional.

12. What information would you need to use the Side-Side-Side Similarity Theorem to prove that these triangles are similar?



To prove that the triangles are similar using the Side-Side-Side Similarity Theorem, all corresponding side lengths between the two triangles should be proportional.

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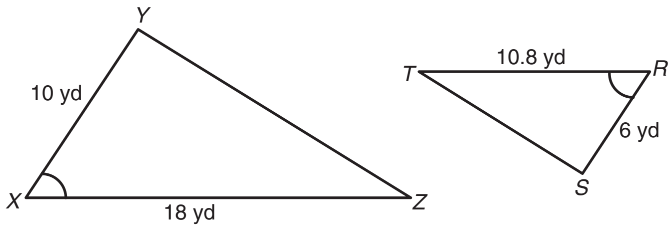
Determine whether each pair of triangles is similar. Explain your reasoning.

13.

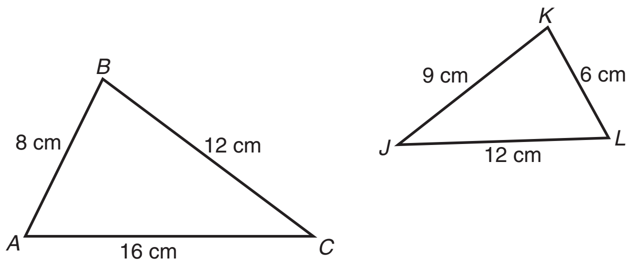


The triangles are similar by the Side-Angle-Side Similarity Theorem because the included angles in both triangles are congruent and the corresponding sides are proportional.

$\frac{XY}{RS} = \frac{10}{6} = \frac{5}{3}$

$\frac{XZ}{RT} = \frac{18}{10.8} = \frac{5}{3}$

14.



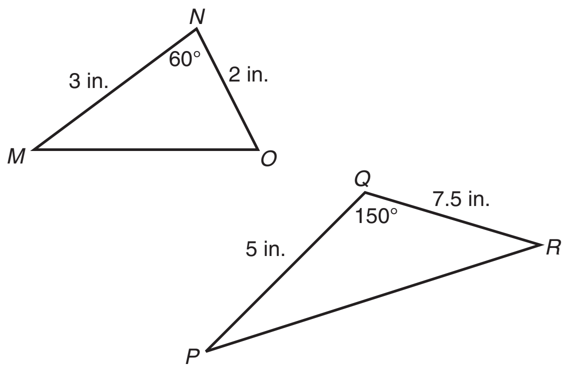
The triangles are similar by the Side-Side-Side Similarity Theorem because all of the corresponding sides are proportional.

$\frac{AB}{KL} = \frac{8}{6} = \frac{4}{3}$

$\frac{BC}{JK} = \frac{12}{9} = \frac{4}{3}$

$\frac{AC}{JL} = \frac{16}{12} = \frac{4}{3}$

15.



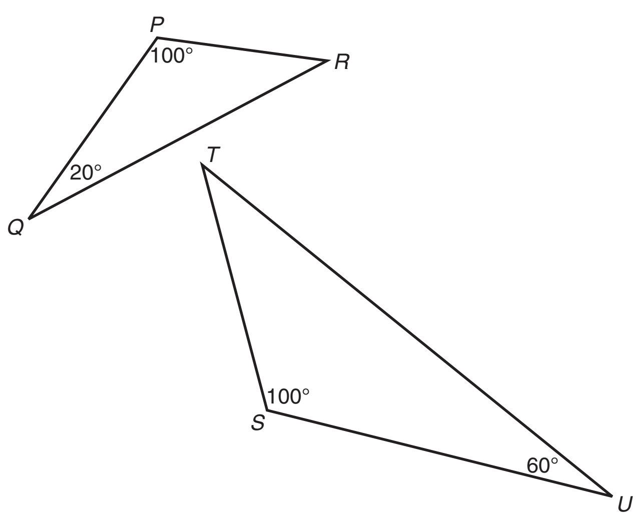
The triangles are not similar. The corresponding sides are proportional, but the included angles are not congruent. So, I cannot use the Side-Angle-Side Similarity Theorem.

$\frac{NO}{PQ} = \frac{2}{5}$

$\frac{MN}{QR} = \frac{3}{7.5} = \frac{2}{5}$

4

16.



The triangles are similar by the Angle-Angle Similarity Theorem. I used the fact that the interior angles of a triangle sum to 180 degrees.

$m\angle P + m\angle Q + m\angle R = 180$

$100 + 20 + m\angle R = 180$

$120 + m\angle R = 180$

$m\angle R = 60$

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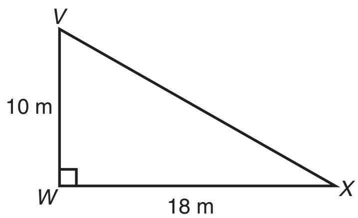
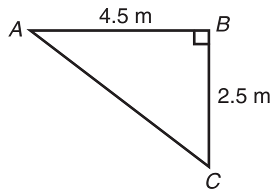
17.



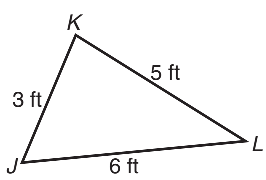


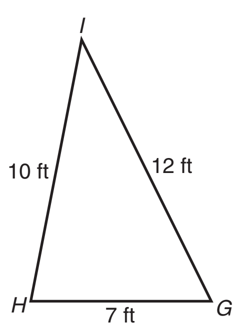
The triangles are similar by the Side-Angle-Side Similarity Theorem. The corresponding sides are proportional and the included angles are congruent.

$\frac{WV}{BC} = \frac{10}{2.5} = \frac{4}{1}$

$\frac{WX}{AB} = \frac{18}{4.5} = \frac{4}{1}$

18.



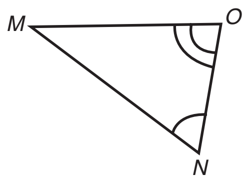


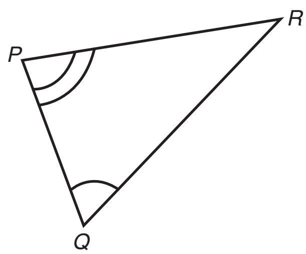
The triangles are not similar because not all of the corresponding sides are proportional. So, I cannot use the Side-Side-Side Similarity Theorem.

$\frac{KJ}{GH} = \frac{3}{7}$

$\frac{KL}{IH} = \frac{5}{10} = \frac{1}{2}$

19.



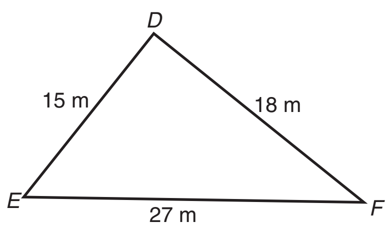


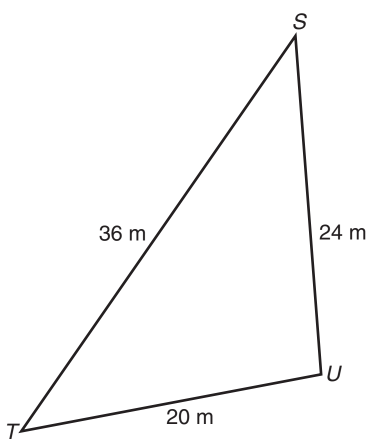
The triangles are similar by the Angle-Angle Similarity Theorem. Two pairs of corresponding angles are congruent.

$\angle N \cong \angle Q$

$\angle O \cong \angle P$

20.





The triangles are similar by the Side-Side-Side Similarity Theorem because all of the corresponding sides are proportional.

$\frac{DE}{TU} = \frac{15}{20} = \frac{3}{4}$

$\frac{DF}{SU} = \frac{18}{24} = \frac{3}{4}$

$\frac{EF}{ST} = \frac{27}{36} = \frac{3}{4}$

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Keep It in Proportion
Theorems About Proportionality

Vocabulary

Match each definition to its corresponding term.

1. Angle Bisector/Proportional Side Theorem
   b

2. Triangle Proportionality Theorem
   a

3. Converse of the Triangle Proportionality Theorem
   c

4. Proportional Segments Theorem
   e

5. Triangle Midsegment Theorem
   d

a. If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

b. A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the sides adjacent to the angle.

c. If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

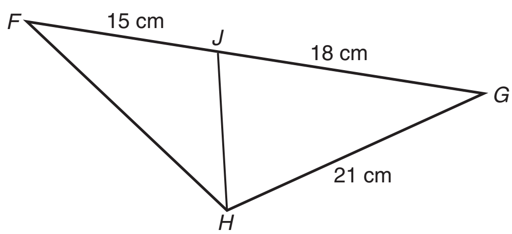
d. The midsegment of a triangle is parallel to the third side of the triangle and half the measure of the third side of the triangle

e. If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Problem Set

Calculate the length of the indicated segment in each figure.

1. $\overline{HJ}$ bisects $\angle H$. Calculate HF.



The length of segment HF is 17.5 centimeters.

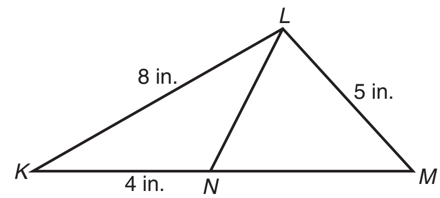
$$\frac{GH}{HF} = \frac{GJ}{JF}$$

$$\frac{21}{HF} = \frac{18}{15}$$

$$18 \cdot HF = 315$$

$$HF = 17.5$$

2. $\overline{LN}$ bisects $\angle L$. Calculate NM.



The length of segment NM is 2.5 inches.

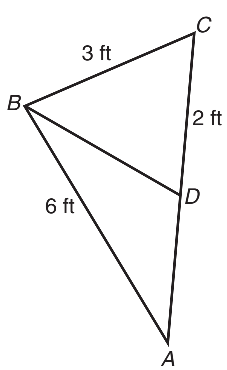
$$\frac{KL}{LM} = \frac{KN}{NM}$$

$$\frac{8}{5} = \frac{4}{NM}$$

$$8 \cdot NM = 20$$

$$NM = 2.5$$

3. $\overline{BD}$ bisects $\angle B$. Calculate AD.



The length of segment AD is 4 feet.

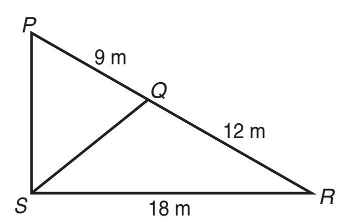
$$\frac{AB}{BC} = \frac{AD}{DC}$$

$$\frac{6}{3} = \frac{AD}{2}$$

$$3 \cdot AD = 12$$

$$AD = 4$$

4. $\overline{SQ}$ bisects $\angle S$. Calculate SP.



The length of segment SP is 13.5 meters.

$$\frac{RS}{SP} = \frac{RQ}{QP}$$

$$\frac{18}{SP} = \frac{12}{9}$$

$$12 \cdot SP = 162$$

$$SP = 13.5$$

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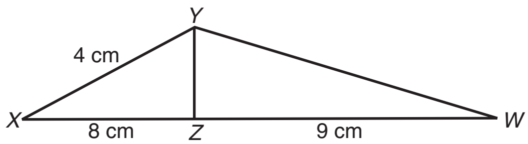
5. $\overline{YZ}$ bisects $\angle Y$. Calculate YW.



The length of segment *YW* is 4.5 centimeters.

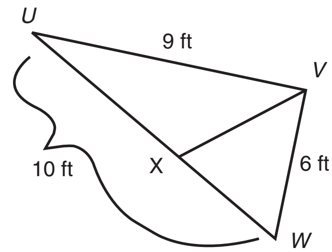
$$\frac{XY}{YW} = \frac{XZ}{ZW}$$

$$\frac{4}{YW} = \frac{8}{9}$$

$$8 \cdot YW = 36$$

$$YW = 4.5$$

6. $\overline{VX}$ bisects $\angle V$. Calculate XW.



The length of segment *XW* is 4 feet.

$$\frac{UV}{VW} = \frac{UX}{XW}$$

$$\frac{9}{6} = \frac{10 - XW}{XW}$$

$$6(10 - XW) = 9 \cdot XW$$

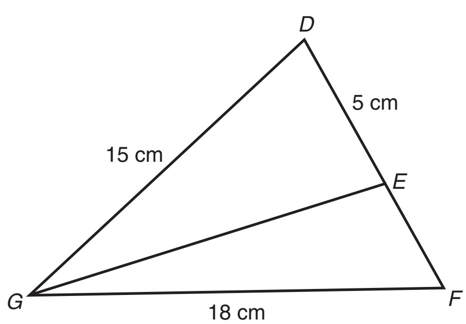
$$60 - 6 \cdot XW = 9 \cdot XW$$

$$60 = 15 \cdot XW$$

$$4 = XW$$

4

7. $\overline{GE}$ bisects $\angle G$. Calculate FD.



The length of segment *FD* is 11 centimeters.

$$\frac{FG}{GD} = \frac{FE}{ED}$$

$$\frac{18}{15} = \frac{FE}{5}$$

$$15 \cdot FE = 90$$

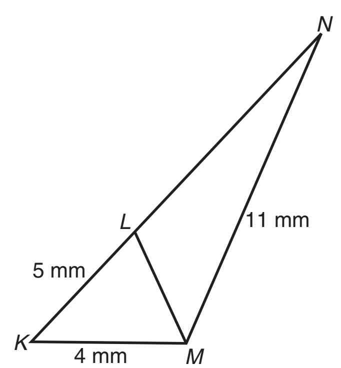
$$FE = 6$$

$$FD = FE + ED$$

$$FD = 6 + 5$$

$$FD = 11$$

8. $\overline{ML}$ bisects $\angle M$. Calculate NL.



The length of segment *NL* is 13.75 millimeters.

$$\frac{NM}{MK} = \frac{NL}{LK}$$

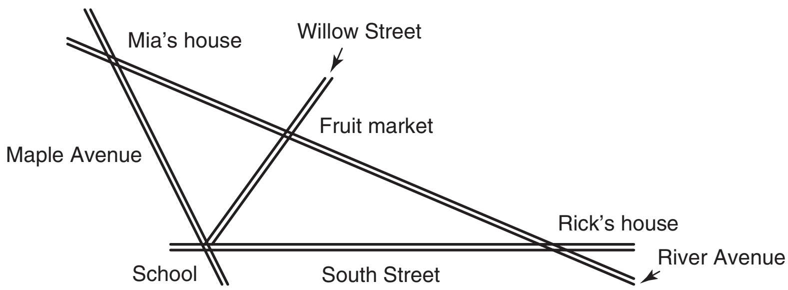
$$\frac{11}{4} = \frac{NL}{5}$$

$$4 \cdot NL = 55$$

$$NL = 13.75$$

Use the given information to answer each question.

9. On the map shown, Willow Street bisects the angle formed by Maple Avenue and South Street. Mia's house is 5 miles from the school and 4 miles from the fruit market. Rick's house is 6 miles from the fruit market. How far is Rick's house from the school?



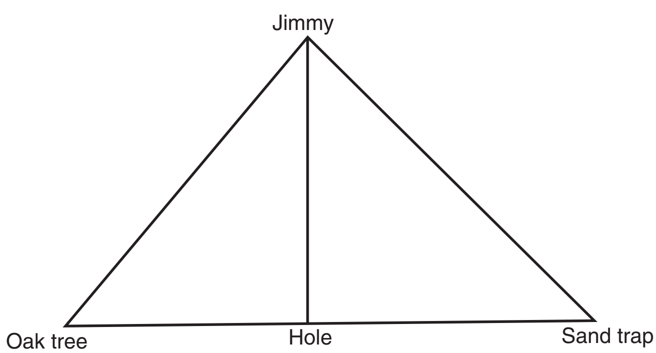
Rick's house is 7.5 miles from the school.

$$\frac{5}{4} = \frac{x}{6}$$

$$4x = 30$$

$$x = 7.5$$

10. Jimmy is hitting a golf ball towards the hole. The line from Jimmy to the hole bisects the angle formed by the lines from Jimmy to the oak tree and from Jimmy to the sand trap. The oak tree is 200 yards from Jimmy, the sand trap is 320 yards from Jimmy, and the hole is 250 yards from the sand trap. How far is the hole from the oak tree?



The hole is 156.25 yards from the oak tree.

$$\frac{200}{x} = \frac{320}{250}$$

$$320x = 50{,}000$$

$$x = 156.25$$

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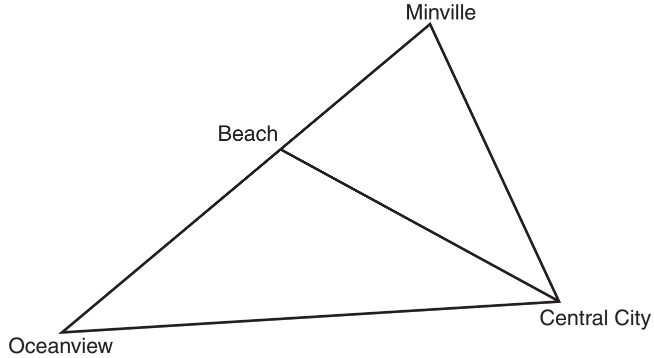
11. The road from Central City on the map shown bisects the angle formed by the roads from Central City to Minville and from Central City to Oceanview. Central City is 12 miles from Oceanview, Minville is 6 miles from the beach, and Oceanview is 8 miles from the beach. How far is Central City from Minville?

Central City is 9 miles from Minville.

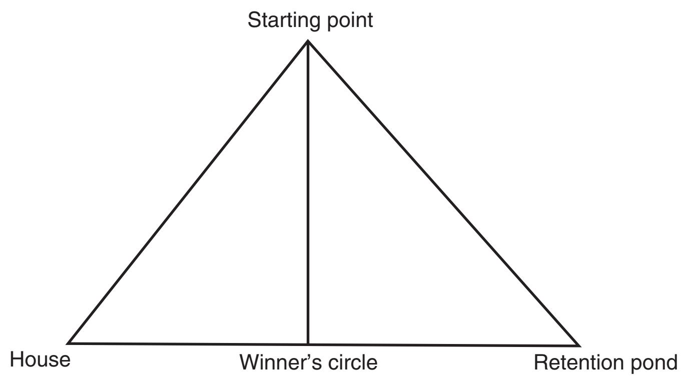
$$\frac{12}{8} = \frac{x}{6}$$

$8x = 72$

$x = 9$



12. Luigi is racing a remote control car from the starting point to the winner's circle.That path bisects the angle formed by the lines from the starting point to the house and from the starting point to the retention pond. The house and the retention pond are each 500 feet from the starting point. The house is 720 feet from the retention pond. How far is the winner's circle from the retention pond?



The winner's circle is 360 feet from the retention pond.

$$\frac{500}{x} = \frac{500}{720 - x}$$

$500x = 500(720 - x)$

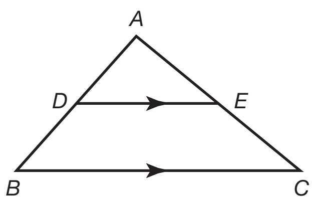
$500x = 360{,}000 - 500x$

$1000x = 360{,}000$

$x = 360$

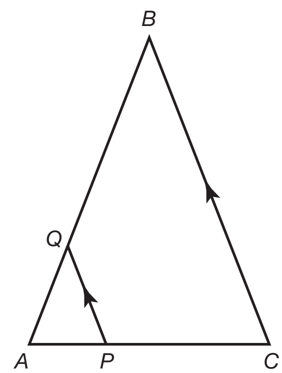
Use the diagram and given information to write a statement that can be justified using the Proportional Segments Theorem, Triangle Proportionality Theorem, or its Converse. State the theorem used.

13.



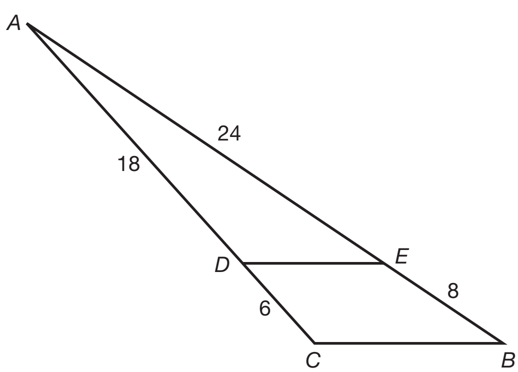
$\frac{AD}{DB} = \frac{AE}{EC}$, Triangle Proportionality Theorem

14.



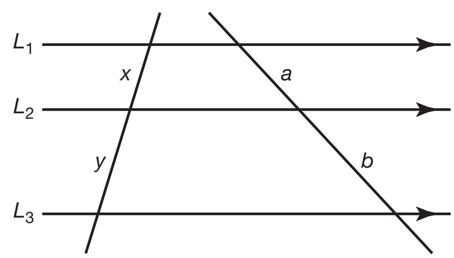
$\frac{AQ}{QB} = \frac{AP}{PC}$, Triangle Proportionality Theorem

15.



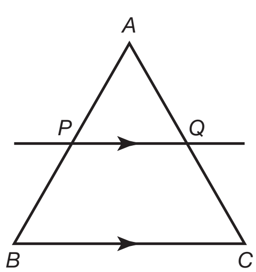
$\overline{DE} \parallel \overline{CB}$, Converse of Triangle Proportionality Theorem

16.



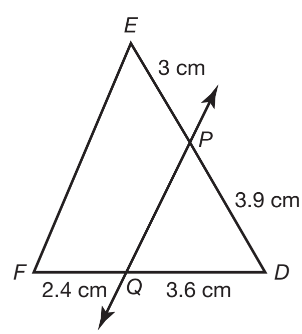
$\frac{x}{y} = \frac{a}{b}$, Proportional Segments Theorem

17.



$\frac{BP}{PA} = \frac{CQ}{QA}$, Triangle Proportionality Theorem

18.



$\overline{EF} \parallel \overline{PQ}$, Converse of Triangle Proportionality Theorem

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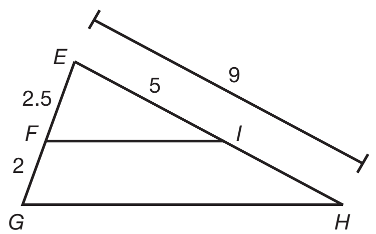
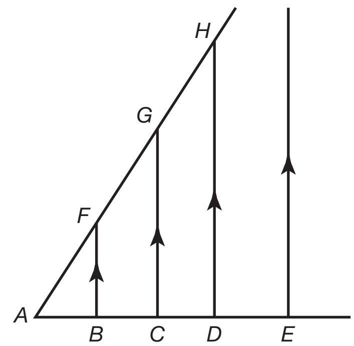
19.



$\overline{GH} \parallel \overline{FI}$, Converse of Triangle Proportionality Theorem

20.

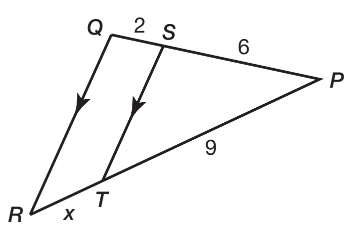


$\frac{BC}{CD} = \frac{FG}{GH}$, $\frac{AF}{FG} = \frac{AB}{BC}$,

Proportional Segments Theorem

Use the Triangle Proportionality Theorem and the Proportional Segments Theorem to determine the missing value.

21.

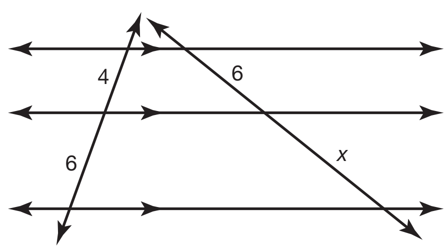


$\frac{x}{9} = \frac{2}{6}$

$6x = 18$

$x = 3$

22.

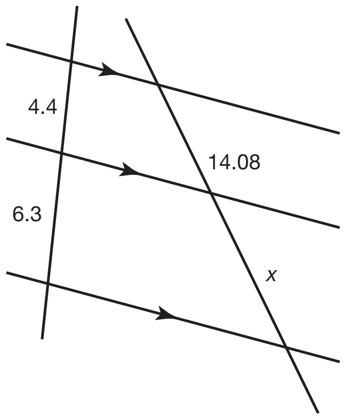


$\frac{4}{6} = \frac{6}{x}$

$4x = 36$

$x = 9$

23.

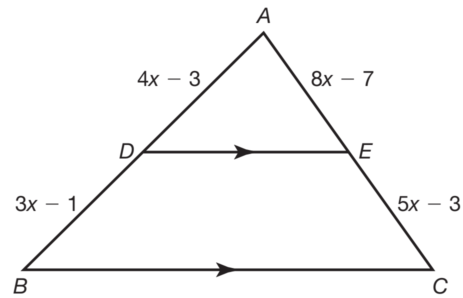


$\frac{4.4}{6.3} = \frac{14.08}{x}$

$4.4x = 88.704$

$x = 20.16$

24.



$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$

$(3x - 1)(8x - 7) = (5x - 3)(4x - 3)$

$24x^2 - 29x + 7 = 20x^2 - 27x + 9$

$4x^2 - 2x - 2 = 0$

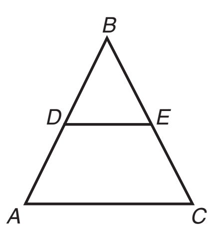
$(4x + 2)(x - 1) = 0$

$x = -\frac{1}{2}, 1$

$x \neq -\frac{1}{2}$, because you get a negative distance when substituted back into the equation so $x = 1$.

Use the diagram and given information to write two statements that can be justified using the Triangle Midsegment Theorem.

25.



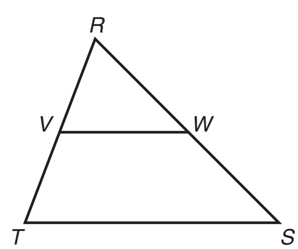
Given: ABC is a triangle

D is the midpoint of $\overline{AB}$

E is the midpoint of BC

$\overline{DE} \parallel \overline{AC}$, $DE = \frac{1}{2}AC$

26.



Given: RST is a triangle

V is the midpoint of $\overline{RT}$

W is the midpoint of $\overline{RS}$

$\overline{VW} \parallel \overline{ST}$, $VW = \frac{1}{2}ST$

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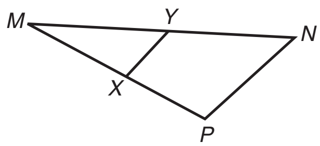
27.



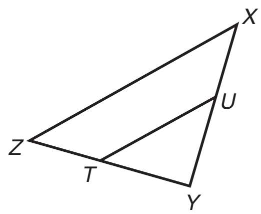
Given: MNP is a triangle

X is the midpoint of $\overline{MP}$

Y is the midpoint of $\overline{MN}$

$\overline{XY} \parallel \overline{NP}$, $XY = \frac{1}{2}NP$

28.



Given: XYZ is a triangle

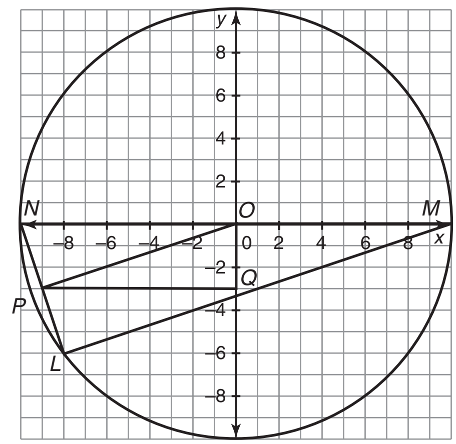
T is the midpoint of $\overline{YZ}$

U is the midpoint of $\overline{XY}$

$\overline{TU} \parallel \overline{XZ}$, $TU = \frac{1}{2}XZ$

Given each diagram, compare the measures described. Simplify your answers, but do not evaluate any radicals.

29. The sides of triangle LMN have midpoints $O(0, 0)$, $P(-9, -3)$, and $Q(0, -3)$. Compare the length of $\overline{OP}$ to the length of $\overline{LM}$.



Segment LM is two times the length of segment OP.

$OP = \sqrt{(-9 - 0)^2 + (-3 - 0)^2}$

$= \sqrt{(-9)^2 + (-3)^2} = \sqrt{81 + 9}$

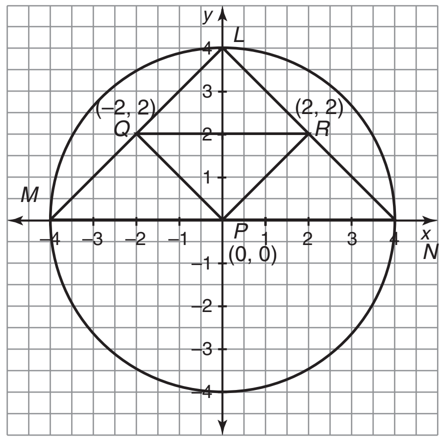
$= \sqrt{90} = 3\sqrt{10}$

$LM = \sqrt{(10 - (-8))^2 + (0 - (-6))^2}$

$= \sqrt{18^2 + 6^2} = \sqrt{324 + 36}$

$= \sqrt{360} = 6\sqrt{10}$

30. The sides of triangle LMN have midpoint $P(0, 0)$, $Q(-2, 2)$, and $R(2, 2)$. Compare the length of $\overline{QP}$ to the length of $\overline{LN}$.



Segment LN is two times the length of segment QP.

$$QP = \sqrt{(-2-0)^2 + (2-0)^2}$$
$$= \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4}$$
$$= \sqrt{8} = 2\sqrt{2}$$
$$LN = \sqrt{(4-0)^2 + (0-4)^2}$$
$$= \sqrt{4^2 + (-4)^2} = \sqrt{16+16}$$
$$= \sqrt{32} = 4\sqrt{2}$$

4

Name ____________________ Date ____________

Geometric Mean
More Similar Triangles

Vocabulary

Write the term from the box that best completes each statement.

Right Triangle/Altitude Similarity Theorem	geometric mean
Right Triangle Altitude/Hypotenuse Theorem	Right Triangle Altitude/Leg Theorem

1. The Right Triangle/Altitude Similarity Theorem states that if an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

2. The Right Triangle Altitude/Leg Theorem states that if the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the measure of the hypotenuse and the measure of the segment of the hypotenuse adjacent to the leg.

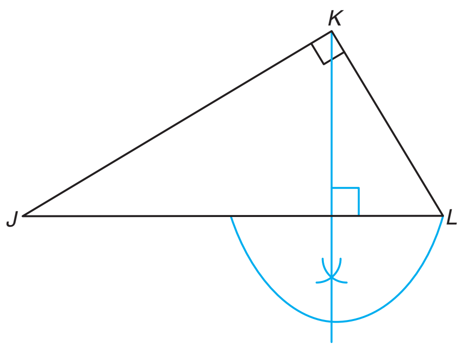
3. The geometric mean of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

4. The Right Triangle Altitude/Hypotenuse Theorem states that the measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

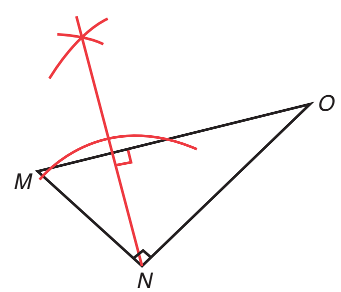
Problem Set

Construct an altitude to the hypotenuse of each right triangle.

1.

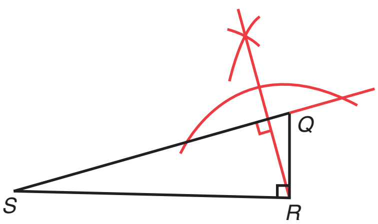


2.

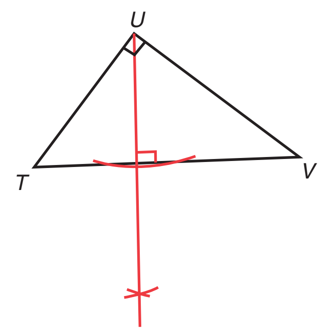


4

3.



4.



Use each similarity statement to write the corresponding sides of the triangles as proportions.

5. $\triangle CGJ \sim \triangle MKP$

$\frac{CG}{MK} = \frac{GJ}{KP} = \frac{CJ}{MP}$

6. $\triangle XZC \sim \triangle YMN$

$\frac{XZ}{YM} = \frac{ZC}{MN} = \frac{XC}{YN}$

7. $\triangle ADF \sim \triangle GLM$

$\frac{AD}{GL} = \frac{DF}{LM} = \frac{AF}{GM}$

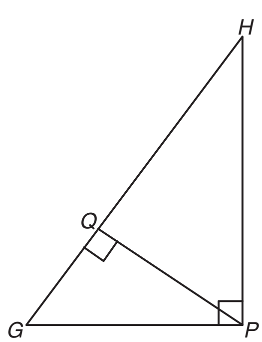
8. $\triangle WNY \sim \triangle CQR$

$\frac{WN}{CQ} = \frac{NY}{QR} = \frac{WY}{CR}$

4

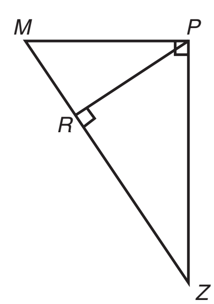
Use the Right Triangle/Altitude Similarity Theorem to write three similarity statements involving the triangles in each diagram.

9.



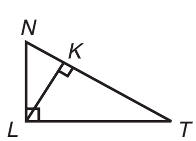
$\triangle HPG \sim \triangle PQG$, $\triangle HPG \sim \triangle HQP$, $\triangle PQG \sim \triangle HQP$

10.



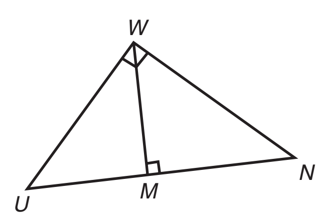
$\triangle MPZ \sim \triangle MRP$, $\triangle MPZ \sim \triangle PRZ$, $\triangle MRP \sim \triangle PRZ$

11.



$\triangle NLT \sim \triangle NKL$, $\triangle NLT \sim \triangle LKT$, $\triangle NKL \sim \triangle LKT$

12.



$\triangle UWN \sim \triangle UMW$, $\triangle UWN \sim \triangle WMN$, $\triangle UMW \sim \triangle WMN$

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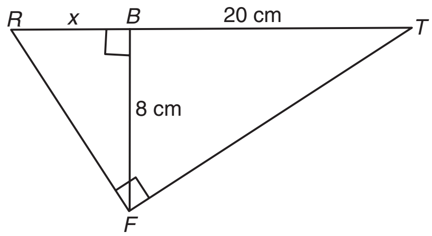
Solve for x.

13.



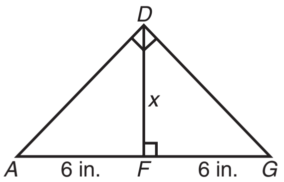
$$\frac{FB}{TB} = \frac{RB}{FB}$$

$$\frac{8}{20} = \frac{x}{8}$$

$$20x = 64$$

$$x = 3.2 \text{ cm}$$

14.



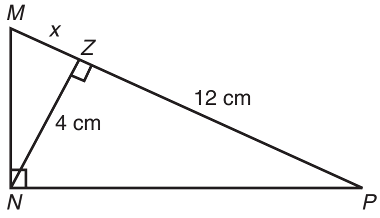
$$\frac{AF}{DF} = \frac{DF}{FG}$$

$$\frac{6}{x} = \frac{x}{6}$$

$$x^2 = 36$$

$$x = 6 \text{ in.}$$

15.



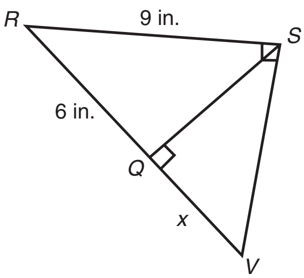
$$\frac{MZ}{NZ} = \frac{NZ}{ZP}$$

$$\frac{x}{4} = \frac{4}{12}$$

$$12x = 16$$

$$x = 1.\overline{3} \text{ cm}$$

16.



$$\frac{RV}{RS} = \frac{RS}{RQ}$$

$$\frac{6 + x}{9} = \frac{9}{6}$$

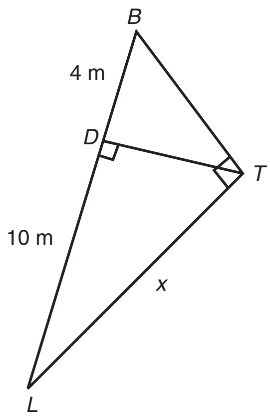
$$6(6 + x) = 81$$

$$36 + 6x = 81$$

$$6x = 45$$

$$x = 7.5 \text{ in.}$$

17.



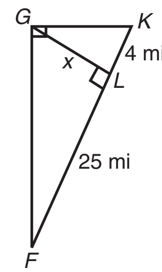
$$\frac{BL}{LT} = \frac{LT}{DL}$$

$$\frac{14}{x} = \frac{x}{10}$$

$$x^2 = 140$$

$$x = \sqrt{140} = 2\sqrt{35} \text{ m}$$

18.



$$\frac{KL}{GL} = \frac{GL}{LF}$$

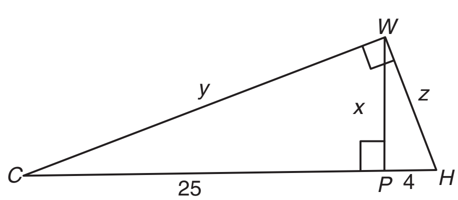
$$\frac{4}{x} = \frac{x}{25}$$

$$x^2 = 100$$

$$x = 10 \text{ mi}$$

Solve for x, y, and z.

19.



$$\frac{CP}{WP} = \frac{WP}{PH}$$

$$\frac{25}{x} = \frac{x}{4}$$

$$x^2 = 100$$

$$x = 10$$

$$x^2 + 25^2 = y^2$$

$$100 + 625 = y^2$$

$$725 = y^2$$

$$5\sqrt{29} = y$$

$$4^2 + x^2 = z^2$$

$$16 + 100 = z^2$$

$$116 = z^2$$

$$2\sqrt{29} = z$$

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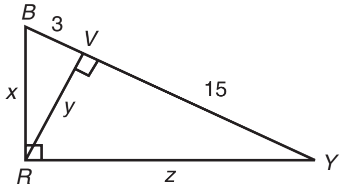
20.



$\frac{BY}{BR} = \frac{BR}{BV}$

$\frac{18}{x} = \frac{x}{3}$

$x^2 = 54$

$x = \sqrt{54} = 3\sqrt{6}$

$\frac{BV}{RV} = \frac{RV}{VY}$

$\frac{3}{y} = \frac{y}{15}$

$y^2 = 45$

$y = \sqrt{45} = 3\sqrt{5}$

$\frac{BY}{RY} = \frac{RY}{VY}$

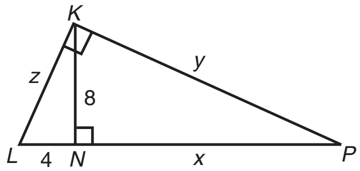
$\frac{18}{z} = \frac{z}{15}$

$\frac{18}{z} = \frac{z}{15}$

$z^2 = 270$

$z = \sqrt{270} = 3\sqrt{30}$

21.



$\frac{LN}{KN} = \frac{KN}{NP}$

$\frac{4}{8} = \frac{8}{x}$

$4x = 64$

$x = 16$

$\frac{LP}{KP} = \frac{KP}{NP}$

$\frac{20}{y} = \frac{y}{16}$

$y^2 = 320$

$y = \sqrt{320} = 8\sqrt{5}$

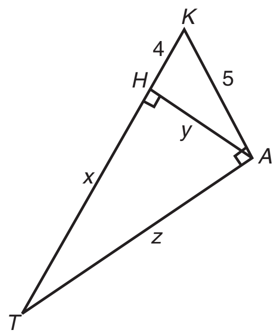
$\frac{LP}{LK} = \frac{LK}{LN}$

$\frac{20}{z} = \frac{z}{4}$

$z^2 = 80$

$z = \sqrt{80} = 4\sqrt{5}$

22.



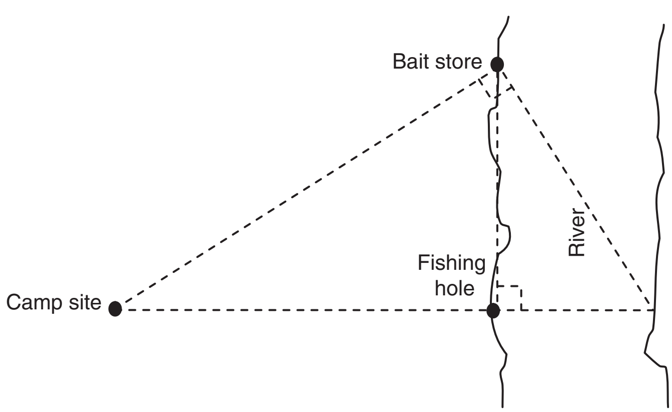
$$\frac{TK}{KA} = \frac{KA}{HK}$$
$$\frac{4 + x}{5} = \frac{5}{4}$$
$$4(4 + x) = 25$$
$$16 + 4x = 25$$
$$4x = 9$$
$$x = 2.25$$

$$\frac{TH}{HA} = \frac{HA}{HK}$$
$$\frac{2.25}{y} = \frac{y}{4}$$
$$y^2 = 9$$
$$y = 3$$

$$\frac{TK}{TA} = \frac{TA}{TH}$$
$$\frac{6.25}{z} = \frac{z}{2.25}$$
$$z^2 = 14.0625$$
$$z = \sqrt{14.0625} = 3.75$$

Use the given information to answer each question.

23. You are on a fishing trip with your friends. The diagram shows the location of the river, fishing hole, camp site, and bait store. The camp site is located 200 feet from the fishing hole. The bait store is located 110 feet from the fishing hole. How wide is the river?



The river is 60.5 feet wide.

$$\frac{200}{110} = \frac{110}{x}$$
$$200x = 12{,}100$$
$$x = 60.5$$

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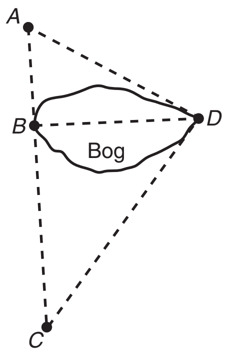
24. You are standing at point D in the diagram looking across a bog at point B. Point B is 84 yards from point A and 189 yards from point C. How wide across is the bog?



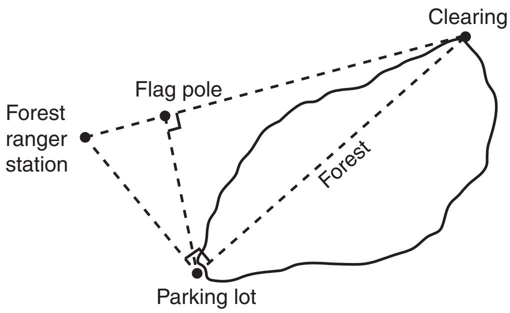
The bog is 126 yards wide.

$$\frac{84}{x} = \frac{x}{189}$$

$$x^2 = 15{,}876$$

$$x = \sqrt{15{,}876} = 126$$

25. Marsha wants to walk from the parking lot through the forest to the clearing, as shown in the diagram. She knows that the forest ranger station is 154 feet from the flag pole and the flag pole is 350 feet from the clearing. How far is the parking lot from the clearing?



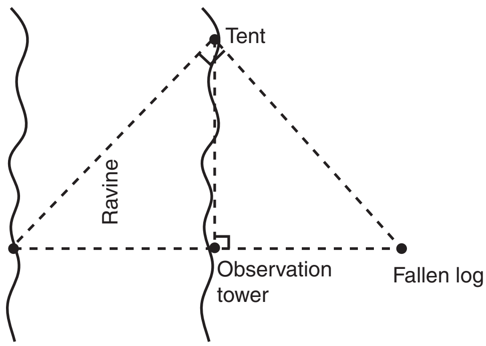
The parking lot is 420 feet from the clearing.

$$\frac{504}{x} = \frac{x}{350}$$

$$x^2 = 176{,}400$$

$$x = \sqrt{176{,}400} = 420$$

26. Andre is camping with his uncle at one edge of a ravine. The diagram shows the location of their tent. The tent is 1.2 miles from the fallen log and the fallen log is 0.75 miles from the observation tower. How wide is the ravine?



The ravine is 1.17 miles wide.

$$\frac{x + 0.75}{1.2} = \frac{1.2}{0.75}$$

$$0.75(x + 0.75) = 1.44$$

$$0.75x + 0.5625 = 1.44$$

$$0.75x = 0.8775$$

$$x = 1.17$$

4

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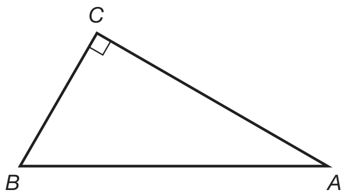
Proving the Pythagorean Theorem
Proving the Pythagorean Theorem and the Converse of the Pythagorean Theorem

Problem Set

Write the Given and Prove statements that should be used to prove the indicated theorem using each diagram.

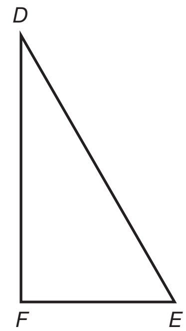
1. Prove the Pythagorean Theorem.



Given: $\triangle ABC$ with right angle C

Prove: $AC^2 + BC^2 = AB^2$

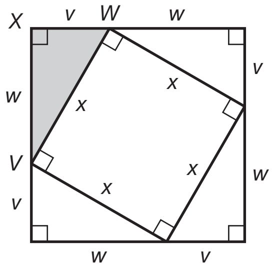
2. Prove the Converse of the Pythagorean Theorem.



Given: $\triangle DEF$ with $DF^2 + EF^2 = DE^2$

Prove: Angle F is a right angle.

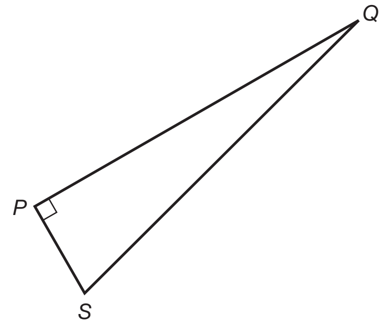
3. Prove the Pythagorean Theorem.



Given: $\triangle VWX$ with right angle X

Prove: $v^2 + w^2 = x^2$

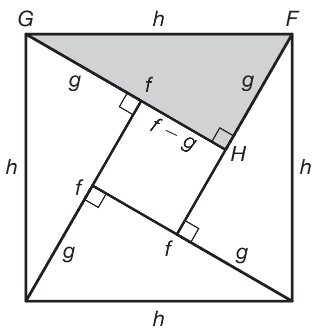
4. Prove the Pythagorean Theorem.



Given: $\triangle QSP$ with right angle P

Prove: $QP^2 + SP^2 = QS^2$

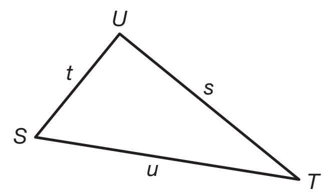
5. Prove the Pythagorean Theorem.



Given: $\triangle FGH$ with right angle H

Prove: $f^2 + g^2 = h^2$

6. Prove the Converse of the Pythagorean Theorem.

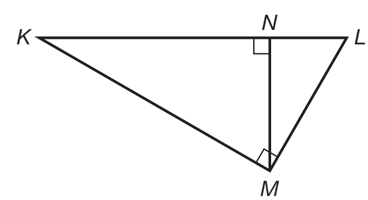


Given: $\triangle STU$ with $s^2 + t^2 = u^2$

Prove: Angle U is a right angle.

Prove each theorem for the diagram that is given. Use the method indicated. In some cases, the proof has been started for you.

7. Prove the Pythagorean Theorem using similar triangles.



Draw an altitude to the hypotenuse at point N.

According to the Right Triangle/Similarity Theorem, $\triangle KLM \sim \triangle LMN \sim \triangle MKN$.

According to the definition of similar triangles, the sides of similar triangles are proportional.

$$\frac{KL}{KM} = \frac{KM}{KN}$$
$$KM^2 = KL \times KN$$

$$\frac{KL}{ML} = \frac{ML}{NL}$$
$$ML^2 = KL \times NL$$

$$KM^2 + ML^2 = KL \times KN + KL \times NL$$
$$KM^2 + ML^2 = KL(KN + NL)$$
$$KM^2 + ML^2 = KL(KL)$$
$$KM^2 + ML^2 = KL^2$$

4

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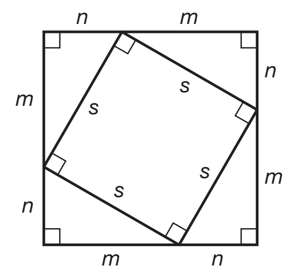
8. Prove the Pythagorean Theorem using algebraic reasoning.



The area of the larger square:

$(n + m)^2 = n^2 + 2nm + m^2$

The area of the four right triangles:

$4\left(\frac{1}{2}nm\right) = 2nm$

The area of the smaller square:

s^2

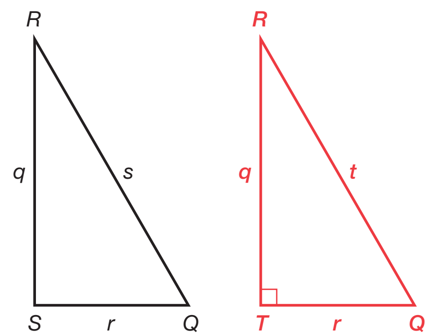
The area of the four right triangles plus the area of the smaller square equals the area of the larger square:

$n^2 + 2nm + m^2 = s^2 + 2nm$

$n^2 + m^2 = s^2$

9. Prove the Converse of the Pythagorean Theorem using algebraic reasoning and the SSS Theorem.

Construct a right triangle that has legs the same lengths as the original triangle, q and r, opposite angles Q and R. Label the hypotenuse of this triangle t, opposite the right angle T.



By the Pythagorean Theorem,

$t^2 = q^2 + r^2$

$t = \sqrt{q^2 + r^2}$

We are given the relationship for the original triangle:

$s^2 = q^2 + r^2$

$s = \sqrt{q^2 + r^2}$

By the transitive property, and since all distances are positive, then, $s = t$.

By the SSS Theorem, the triangles are congruent, $\triangle QRS \cong \triangle QRT$.

All corresponding angles of congruent triangles are congruent, so

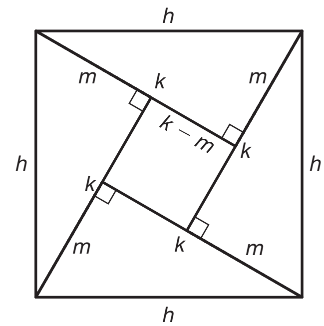
$\angle S = \text{T}$

$\angle S = 90°$

By definition, a triangle with a 90 degree angle is a right triangle.

4

10. Prove the Pythagorean Theorem using similarity.



The area of the larger square:

h^2

The area of the four right triangles:

$4\left(\frac{1}{2}km\right) = 2km$

The area of the smaller square:

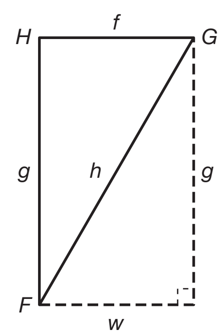
$(k - m)^2 = k^2 + m^2 - 2km$

The area of the four right triangles plus the area of the smaller square equals the area of the larger square:

$k^2 + m^2 - 2km + 2km = h^2$

$k^2 + m^2 = h^2$

11. Prove the Converse of the Pythagorean Theorem using algebraic reasoning.



Construct a right triangle that shares side h with the original triangle and has one leg length g, as in the original triangle, and the other leg length w.

By the Pythagorean Theorem, $h^2 = w^2 + g^2$.

We are given the relationship for the original triangle:

$h^2 = f^2 + g^2$

By the transitive property, and since all distances are positive, then,

$w^2 + g^2 = f^2 + g^2$

$w^2 = g^2$

$w = g$

By the SSS Theorem, the triangles are congruent.

All corresponding angles of congruent triangles are congruent, so $\angle H = 90°$

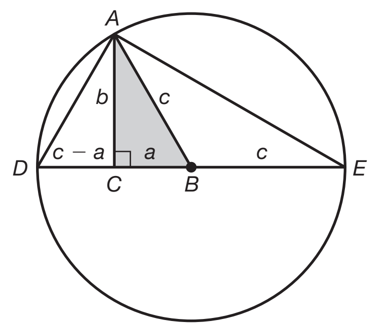
By definition, a triangle with a 90 degree angle is a right triangle.

Name ______________________________ Date ____________

12. Prove the Pythagorean Theorem using similar triangles.

Given: $\triangle ABC$ with right angle C

Place side a along the diameter of a circle of radius c so that B is at the center of the circle.



According to Right Triangle Altitude/Hypotenuse Theorem, $\triangle ADC \sim \triangle AEC$.

According to the definition of similar triangles, the sides of similar triangles are proportional.

$$\frac{c + a}{b} = \frac{b}{c - a}$$

$$(c + a)(c - a) = b^2$$

$$c^2 - a^2 = b^2$$

$$c^2 = a^2 + b^2$$

4

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Indirect Measurement
Application of Similar Triangles

Vocabulary

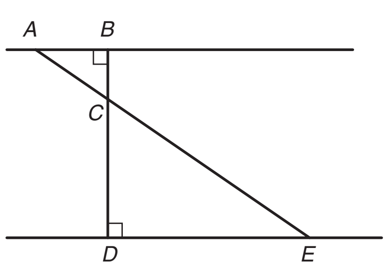
Provide an example of the term.

1. indirect measurement

Indirect measurement is a technique that uses proportions to calculate measurements. It is useful when measuring something directly is very difficult. Knowledge of similar triangles can be very helpful.

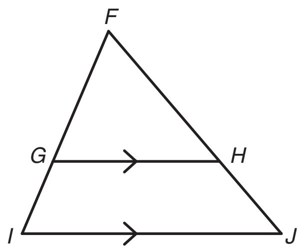
Problem Set

Explain how you know that each pair of triangles are similar.

1. 

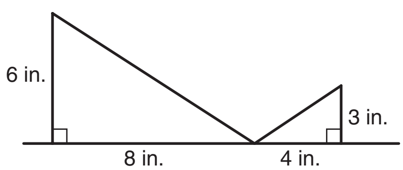
The angles where the vertices of the triangle intersect are vertical angles, so angles $\angle ACB$ and $\angle ECD$ are congruent. The angles formed by BD intersecting the two parallel lines are right angles, so they are also congruent. So by the Angle-Angle Similarity Theorem, the triangles formed are similar.

2.



Angles *FGH* and *FIJ* are corresponding angles, so those angles are congruent. Similarly, *FHG* and *FJI* are also corresponding angles, so those angles are also congruent. So by the Angle-Angle Similarity Theorem, the triangles formed are similar.

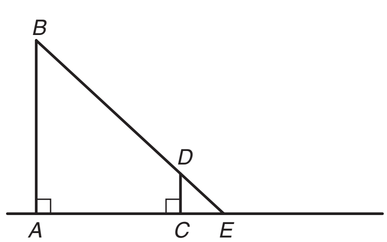
3.



The known corresponding sides of the triangles are proportional: $\frac{6}{3} = \frac{2}{1}$ and $\frac{8}{4} = \frac{2}{1}$.

The angle between the known sides is a right angle for both triangles, so those angles are congruent. Therefore, by the Side-Angle-Side Similarity Postulate, the triangles are similar.

4.



The angles formed by the horizontal line intersecting the two vertical lines are right angles, so those angles are congruent. Both triangles share angle *B* and angle *E*. So by the Angle-Angle Similarity Theorem, the triangles formed are similar.

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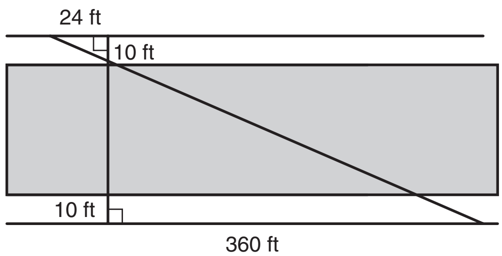
Use indirect measurement to calculate the missing distance.

5. Elly and Jeff are on opposite sides of a canyon that runs east to west, according to the graphic. They want to know how wide the canyon is. Each person stands 10 feet from the edge. Then, Elly walks 24 feet west, and Jeff walks 360 feet east.



What is the width of the canyon?

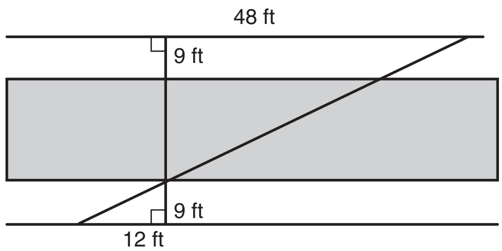
The distance across the canyon is 140 feet.

$$\frac{10 + x}{10} = \frac{360}{24}$$

$10 + x = 150$

$x = 140$

6. Zoe and Ramon are hiking on a glacier. They become separated by a crevasse running east to west. Each person stands 9 feet from the edge. Then, Zoe walks 48 feet east, and Ramon walks 12 feet west.



What is the width of the crevasse?

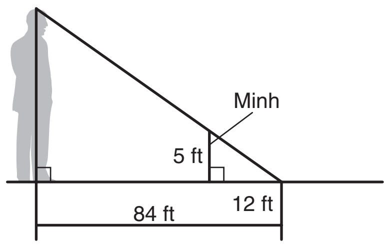
The distance from one side of the crevasse to the other is 27 feet.

$$\frac{x + 9}{9} = \frac{48}{12}$$

$x + 9 = 36$

$x = 27$

7. Minh wanted to measure the height of a statue. She lined herself up with the statue's shadow so that the tip of her shadow met the tip of the statue's shadow. She marked the spot where she was standing. Then, she measured the distance from where she was standing to the tip of the shadow, and from the statue to the tip of the shadow.



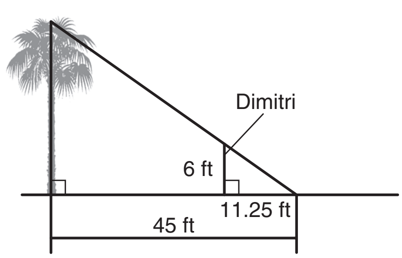
What is the height of the statue?

The height of the statue is 35 feet.

$\frac{x}{5} = \frac{84}{12}$

$x = 35$

8. Dimitri wants to measure the height of a palm tree. He lines himself up with the palm tree's shadow so that the tip of his shadow meets the tip of the palm tree's shadow. Then, he asks a friend to measure the distance from where he was standing to the tip of his shadow and the distance from the palm tree to the tip of its shadow.



What is the height of the palm tree?

The palm tree is 24 feet tall.

$\frac{x}{6} = \frac{45}{11.25}$

$x = 24$

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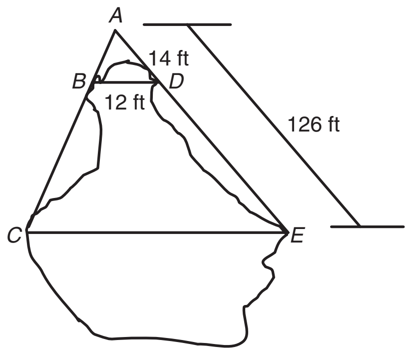
9. Andre is making a map of a state park. He finds a small bog, and he wants to measure the distance across the widest part. He first marks the points *A*, *C*, and *E*. Andre measures the distances shown on the image. Andre also marks point *B* along *AC* and point *D* along *AE*, such that *BD* is parallel to *CE*.



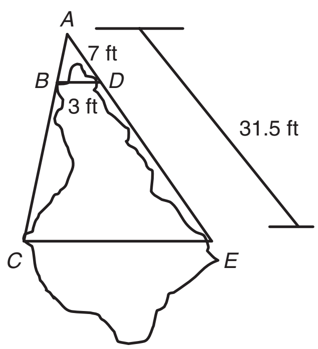
What is the width of the bog at the widest point?

The bog is 108 feet across at the widest point.

$$\frac{x}{12} = \frac{126}{14}$$

$$x = 108$$

10. Shira finds a tidal pool while walking on the beach. She wants to know the maximum width of the tidal pool. Using indirect measurement, she begins by marking the points *A*, *C*, and *E*. Shira measures the distances shown on the image. Next, Shira marks point *B* along *AC* and point *D* along *AE*, such that *BD* is parallel to *CE*.



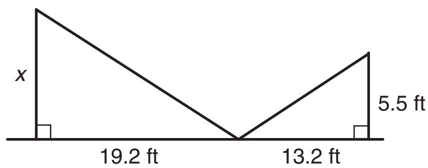
What is the distance across the tidal pool at its widest point?

The tidal pool is 13.5 feet across at the widest point.

$$\frac{x}{3} = \frac{31.5}{7}$$

$$x = 13.5$$

11. Keisha is visiting a museum. She wants to know the height of one of the sculptures. She places a small mirror on the ground between herself and the sculpture, then she backs up until she can see the top of the sculpture in the mirror.



What is the height of the sculpture?

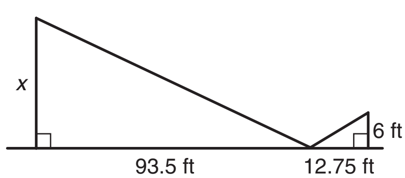
The sculpture is 8 feet in height.

$$\frac{x}{5.5} = \frac{19.2}{13.2}$$

$$x = 8$$

4

12. Micah wants to know the height of his school. He places a small mirror on the ground between himself and the school, then he backs up until he can see the highest point of the school in the mirror.



What is the height of Micah's school?

Micah's school is 44 feet in height.

$$\frac{x}{6} = \frac{93.5}{12.75}$$

$$x = 44$$