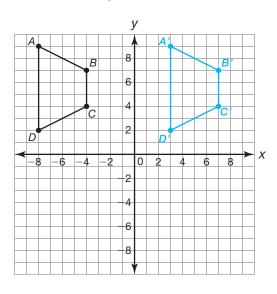
Name -Date _

We Like to Move It! Translating, Rotating, and Reflecting Geometric Figures

Problem Set

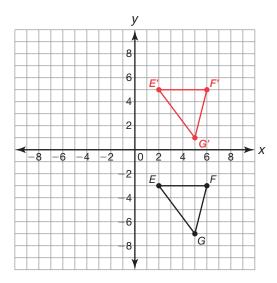
Transform each given geometric figure on the coordinate plane as described.

1. Translate trapezoid *ABCD* 11 units to the right.

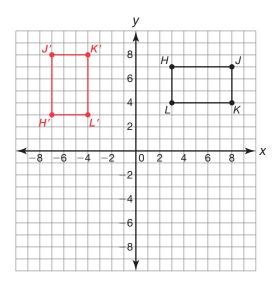


2. Translate triangle *EFG* 8 units up.

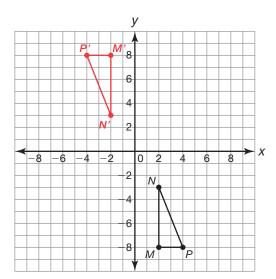
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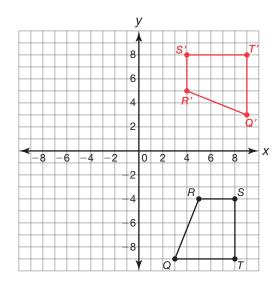
3. Rotate rectangle *HJKL* 90° counterclockwise about the origin.



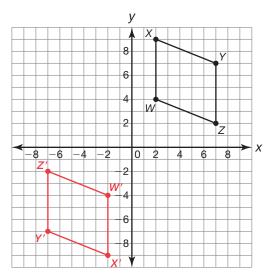
4. Rotate triangle MNP 180° counterclockwise about the origin.



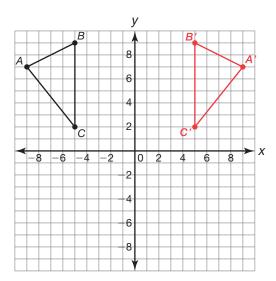
5. Rotate trapezoid *QRST* 90° counterclockwise about the origin.



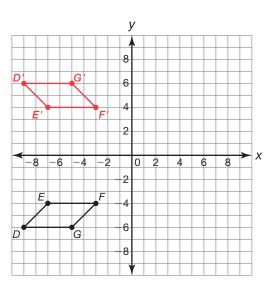
6. Rotate parallelogram *WXYZ* 180° counterclockwise about the origin.



7. Reflect triangle ABC over the y-axis.

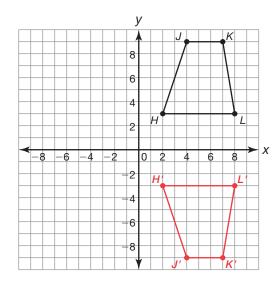


8. Reflect parallelogram *DEFG* over the *x*-axis.

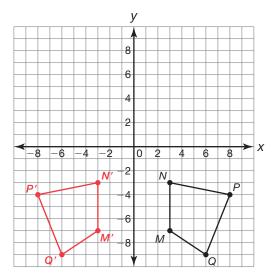


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9. Reflect trapezoid *HJKL* over the *x*-axis.



10. Reflect quadrilateral MNPQ over the *y*-axis.



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Determine the coordinates of each translated image without graphing.

11. The vertices of triangle ABC are A (5, 3), B (2, 8), and C (-4, 5). Translate the triangle 6 units to the left to form triangle A' B' C'.

The vertices of triangle A' B' C' are A' (-1, 3), B' (-4, 8), and <math>C' (-10, 5).

12. The vertices of rectangle DEFG are D(-7, 1), E(-7, 8), F(1, 8), and G(1, 1). Translate the rectangle 10 units down to form rectangle D' E' F' G'.

The vertices of rectangle D' E' F' G' are D' (-7, -9), E' (-7, -2), F' (1, -2), and <math>G' (1, -9).

13. The vertices of parallelogram HJKL are H(2, -6), J(3, -1), K(7, -1), and L(6, -6). Translate the parallelogram 7 units up to form parallelogram H' J' K' L'.

The vertices of parallelogram H' J' K' L' are H' (2, 1), J' (3, 6), K' (7, 6), and <math>L' (6, 1).

14. The vertices of trapezoid MNPQ are M(-6, -5), N(0, -5), P(-1, 2), and Q(-4, 2). Translate the trapezoid 4 units to the right to form trapezoid M' N' P' Q'.

The vertices of trapezoid M' N' P' Q' are M' (-2, -5), N' (4, -5), P' (3, 2), and <math>Q' (0, 2).

15. The vertices of triangle RST are R (0, 3), S (2, 7), and T (3, -1). Translate the triangle 5 units to the left and 3 units up to form triangle R' S' T'.

The vertices of triangle R' S' T' are R' (-5, 6), S' (-3, 10), and T' (-2, 2).

16. The vertices of quadrilateral WXYZ are W(-10, 8), X(-2, -1), Y(0, 0), and Z(3, 7). Translate the quadrilateral 5 units to the right and 8 units down to form quadrilateral W' X' Y' Z'.

The vertices of quadrilateral W' X' Y' Z' are W' (-5, 0), X' (3, -9), Y' (5, -8), and Z' (8, -1).

Determine the coordinates of each rotated image without graphing.

17. The vertices of triangle ABC are A (5, 3), B (2, 8), and C (-4, 5). Rotate the triangle about the origin 90° counterclockwise to form triangle A' B' C'.

The vertices of triangle A' B' C' are A' (-3, 5), B' (-8, 2), and <math>C' (-5, -4).

18. The vertices of rectangle *DEFG* are D(-7, 1), E(-7, 8), F(1, 8), and G(1, 1). Rotate the rectangle about the origin 180° counterclockwise to form rectangle D' E' F' G'.

The vertices of rectangle D' E' F' G' are D' (7, -1), E' (7, -8), F' (-1, -8), and G' (-1, -1).

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19. The vertices of parallelogram HJKL are H (2, -6), J (3, -1), K (7, -1), and L (6, -6). Rotate the parallelogram about the origin 90° counterclockwise to form parallelogram H' J' K' L'.

The vertices of parallelogram H' J' K' L' are H' (6, 2), J' (1, 3), K' (1, 7) and L' (6, 6).

20. The vertices of trapezoid MNPQ are M (-6, -5), N (0, -5), P (-1, 2), and Q (-4, 2). Rotate the trapezoid about the origin 180° counterclockwise to form trapezoid M' N' P' Q'.

The vertices of trapezoid M' N' P' Q' are M' (6, 5), N' (0, 5), P' (1, -2), and Q' (4, -2).

21. The vertices of triangle RST are R (0, 3), S (2, 7), and T (3, -1). Rotate the triangle about the origin 90° counterclockwise to form triangle R' S' T'.

The vertices of triangle R' S' T' are R' (-3, 0), S' (-7, 2), and T' (1, 3).

22. The vertices of quadrilateral WXYZ are W(-10, 8), X(-2, -1), Y(0, 0), and Z(3, 7). Rotate the quadrilateral about the origin 180° counterclockwise to form quadrilateral W'X'Y'Z'.

The vertices of quadrilateral W'X'Y'Z' are W'(10, -8), X'(2, 1), Y'(0, 0), and <math>Z'(-3, -7).

Determine the coordinates of each reflected image without graphing.

23. The vertices of triangle ABC are A (5, 3), B (2, 8), and C (-4, 5). Reflect the triangle over the x-axis to form triangle A' B' C'.

The vertices of triangle A' B' C' are A' (5, -3), B' (2, -8), and C' (-4, -5).

24. The vertices of rectangle *DEFG* are D(-7, 1), E(-7, 8), F(1, 8), and G(1, 1). Reflect the rectangle over the *y*-axis to form rectangle D'E'F'G'.

The vertices of rectangle D' E' F' G' are D' (7, 1), E' (7, 8), F' (-1, 8), and G' (-1, 1).

25. The vertices of parallelogram HJKL are H (2, -6), J (3, -1), K (7, -1), and L (6, -6). Reflect the parallelogram over the x-axis to form parallelogram H' J' K' L'.

The vertices of parallelogram H' J' K' L' are H' (2, 6), J' (3, 1), K' (7, 1), and L' (6, 6).

26. The vertices of trapezoid MNPQ are M (-6, -5), N (0, -5), P (-1, 2), and Q (-4, 2). Reflect the trapezoid over the y-axis to form trapezoid M' N' P' Q'.

The vertices of trapezoid M' N' P' Q' are M' (6, -5), N' (0, -5), P' (1, 2), and <math>Q' (4, 2).

5

27. The vertices of triangle RST are R (0, 3), S (2, 7), and T (3, -1). Reflect the triangle over the x-axis to form triangle R' S' T'.

The vertices of triangle R' S' T' are R' (0, -3), S' (2, -7), and T' (3, 1).

28. The vertices of quadrilateral WXYZ are W(-10, 8), X(-2, -1), Y(0, 0), and Z(3, 7). Reflect the quadrilateral over the y-axis to form quadrilateral W'X'Y'Z'.

The vertices of quadrilateral W' X' Y' Z' are W' (10, 8), X' (2, -1), Y' (0, 0), and Z' (-3, 7).

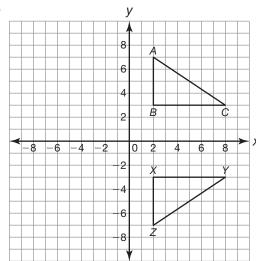
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Hey, Haven't I Seen You Before? **Congruent Triangles**

Problem Set

Identify the transformation used to create $\triangle XYZ$ on each coordinate plane. Identify the congruent angles and the congruent sides. Then, write a triangle congruence statement.

1.



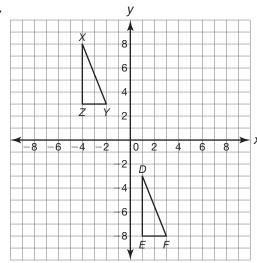
Triangle BCA was reflected over the x-axis to create triangle XYZ.

 $BC \cong \overline{XY}$, $\overline{CA} \cong \overline{YZ}$, and $\overline{BA} \cong \overline{XZ}$; $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle A \cong \angle Z$.

 $\triangle BCA \cong \triangle XYZ$

2.

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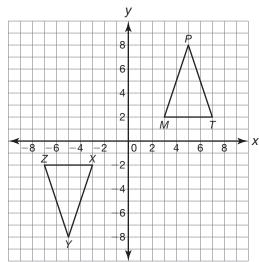


Triangle DFE was translated 5 units to the left and 11 units up to create triangle XYZ.

 $\overline{DF} \cong \overline{XY}, \overline{FE} \cong \overline{YZ}, \text{ and } \overline{DE} \cong \overline{XZ}; \angle D \cong \angle X,$ $\angle F \cong \angle Y$, and $\angle E \cong \angle Z$.

 $\triangle DFE \cong \triangle XYZ$

3.

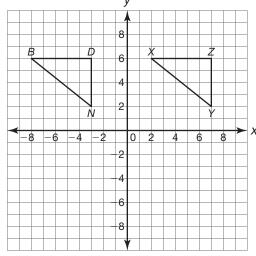


Triangle MPT was rotated 180° counterclockwise or clockwise about the origin to create triangle XYZ.

 $\overline{MP} \cong \overline{XY}, \overline{PT} \cong \overline{YZ}, \text{ and } \overline{MT} \cong \overline{XZ}; \angle M \cong \angle X,$ $\angle P \cong \angle Y$, and $\angle T \cong \angle Z$.

 $\triangle MPT \cong \triangle XYZ$

4.



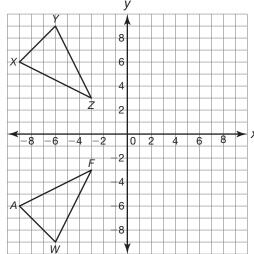
Triangle BND was translated 10 units to the right to create triangle XYZ.

 $\overline{BN} \cong \overline{XY}, \overline{ND} \cong \overline{YZ}, \text{ and } \overline{BD} \cong \overline{XZ}; \angle B \cong \angle X,$ $\angle N \cong \angle Y$, and $\angle D \cong \angle Z$.

 $\triangle BND \cong \triangle XYZ$

Name _ Date _

5.

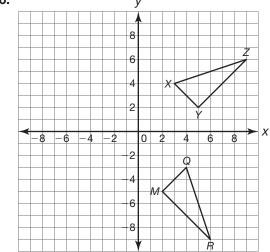


Triangle AWF was reflected over the x-axis to create triangle XYZ.

 $\overline{AW} \cong \overline{XY}$, $\overline{WF} \cong \overline{YZ}$, and $\overline{AF} \cong \overline{XZ}$; $\angle A \cong \angle X$, $\angle W \cong \angle Y$, and $\angle F \cong \angle Z$.

 $\triangle AWF \cong \triangle XYZ$

6.

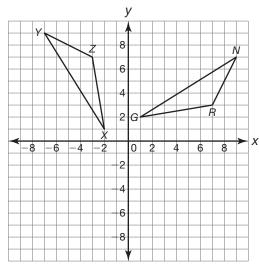


Triangle QMR was rotated 90° counterclockwise about the origin to create triangle XYZ.

 $\overline{QM} \cong \overline{XY}, \overline{MR} \cong \overline{YZ}, \text{ and } \overline{QR} \cong \overline{XZ}; \angle Q \cong \angle X,$ $\angle M \cong \angle Y$, and $\angle R \cong \angle Z$.

 $\triangle QMR \cong \triangle XYZ$

7.

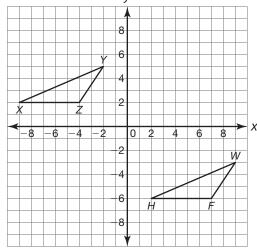


Triangle GNR was rotated 90° counterclockwise about the origin to create triangle XYZ.

 $\overline{GN} \cong \overline{XY}, \overline{NR} \cong \overline{YZ}, \text{ and } \overline{GR} \cong \overline{XZ}; \angle G \cong \angle X,$ $\angle N \cong \angle Y$, and $\angle R \cong \angle Z$.

 $\triangle GNR \cong \triangle XYZ$

8.



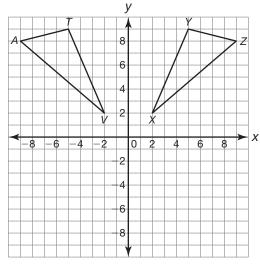
Triangle HWF was translated 11 units to the left and 8 units up to create triangle XYZ.

 $\overline{HW} \cong \overline{XY}, \overline{WF} \cong \overline{YZ}, \text{ and } \overline{HF} \cong \overline{XZ}; \angle H \cong \angle X,$ $\angle W \cong \angle Y$, and $\angle F \cong \angle Z$.

 $\triangle HWF \cong \triangle XYZ$

Date_

9.

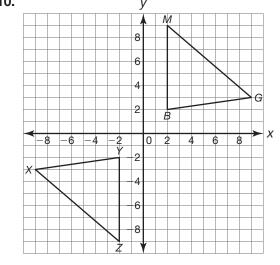


Triangle VTA was reflected over the y-axis to create triangle XYZ.

 $\overline{VT} \cong \overline{XY}$, $\overline{TA} \cong \overline{YZ}$, and $\overline{VA} \cong \overline{XZ}$; $\angle V \cong \angle X$, $\angle T \cong \angle Y$, and $\angle A \cong \angle Z$.

 $\triangle VTA \cong \triangle XYZ$

10.



Triangle GBM was rotated 180° clockwise or counterclockwise about the origin to create triangle XYZ.

 $\overline{GB} \cong \overline{XY}, \overline{BM} \cong \overline{YZ}, \text{ and } \overline{GM} \cong \overline{XZ}; \angle G \cong \angle X,$ $\angle B \cong \angle Y$, and $\angle M \cong \angle Z$.

 $\triangle GBM \cong \triangle XYZ$

11. $\triangle JPM \cong \triangle TRW$

5.2

 $\overline{JP} \cong \overline{TR}, \overline{PM} \cong \overline{RW}, \text{ and } \overline{JM} \cong \overline{TW}; \angle J \cong \angle T, \angle P \cong \angle R, \text{ and } \angle M \cong \angle W.$

12. $\triangle AEU \cong \triangle BCD$

 $\overline{AE} \cong \overline{BC}$, $\overline{EU} \cong \overline{CD}$, and $\overline{AU} \cong \overline{BD}$; $\angle A \cong \angle B$, $\angle E \cong \angle C$, and $\angle U \cong \angle D$.

13. $\triangle LUV \cong \triangle MTH$

 $\overline{LU} \cong \overline{MT}$, $\overline{UV} \cong \overline{TH}$, and $\overline{LV} \cong \overline{HM}$; $\angle L \cong \angle M$, $\angle U \cong \angle T$, and $\angle V \cong \angle H$.

14. $\triangle RWB \cong \triangle VCQ$

 $\overline{RW} \cong \overline{VC}$, $\overline{WB} \cong \overline{CQ}$, and $\overline{RB} \cong \overline{VQ}$; $\angle R \cong \angle V$, $\angle W \cong \angle C$, and $\angle B \cong \angle Q$.

15. $\triangle TOM \cong \triangle BEN$

 $\overline{TO} \cong \overline{BE}$, $\overline{OM} \cong \overline{EN}$, and $\overline{TM} \cong \overline{BN}$; $\angle T \cong \angle B$, $\angle O \cong \angle E$, and $\angle M \cong \angle N$.

16. $\triangle JKL \cong \triangle RST$

 $\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, and $\overline{JL} \cong \overline{RT}$; $\angle J \cong \angle R$, $\angle K \cong \angle S$, and $\angle L \cong \angle T$.

17. $\triangle CAT \cong \triangle SUP$

 $\overline{CA} \cong \overline{SU}, \overline{AT} \cong \overline{UP}, \text{ and } \overline{CT} \cong \overline{SP}; \angle C \cong \angle S, \angle A \cong \angle U, \text{ and } \angle T \cong \angle P.$

18. $\triangle TOP \cong \triangle GUN$

 $\overline{TO} \cong \overline{GU}$, $\overline{OP} \cong \overline{UN}$, and $\overline{TP} \cong \overline{GN}$; $\angle T \cong \angle G$, $\angle O \cong \angle U$, and $\angle P \cong \angle N$.

It's All About the Sides Side-Side Congruence Theorem

Vocabulary

Define the term in your own words.

1. Side-Side-Side (SSS) Congruence Theorem

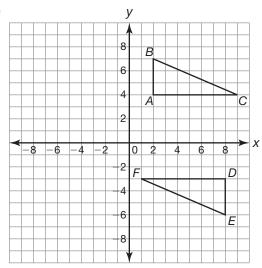
The Side-Side (SSS) Congruence Theorem states that if three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

Problem Set

Determine whether each pair of given triangles are congruent by SSS. Use the Distance Formula and a protractor when necessary.

1.

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$$AB = DE = 3$$

$$AC = DF = 7$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(9-2)^2 + (4-7)^2}$$

$$BC = \sqrt{7^2 + (-3)^2}$$

$$BC = \sqrt{49 + 9}$$

$$BC = \sqrt{58} \approx 7.62$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$EF = \sqrt{(1-8)^2 + (-3-(-6))^2}$$

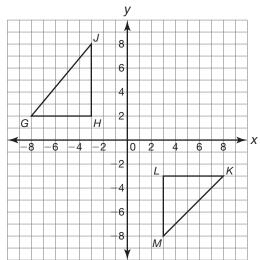
$$EF = \sqrt{(-7)^2 + 3^2}$$

$$EF = \sqrt{49 + 9}$$

$$EF = \sqrt{58} \approx 7.62$$

$$BC = EF$$

2.



$$GH = KL = 5$$

$$HJ = 6$$
, $LM = 5$, $HJ \neq LM$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$GJ = \sqrt{(-3 - (-8))^2 + (8 - 2)^2}$$

$$GJ = \sqrt{5^2 + 6^2}$$

$$GJ = \sqrt{25 + 36}$$

$$GJ = \sqrt{61} \approx 7.81$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KM = \sqrt{(3-8)^2 + (-8-(-3))^2}$$

$$KM = \sqrt{(-5)^2 + (-5)^2}$$

$$KM = \sqrt{25 + 25}$$

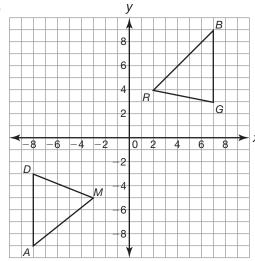
$$KM = \sqrt{50} \approx 7.07$$

The triangles are not congruent.



3.

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$$AD = BG = 6$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DM = \sqrt{(-3 - (-8))^2 + (-5 - (-3))^2}$$

$$DM = \sqrt{5^2 + (-2)^2}$$

$$DM = \sqrt{25 + 4}$$

$$DM = \sqrt{29} \approx 5.39$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$GR = \sqrt{(2-7)^2 + (4-3)^2}$$

$$GR = \sqrt{(-5)^2 + 1^2}$$

$$GR = \sqrt{25 + 1}$$

$$GR = \sqrt{26} \approx 5.10$$

$$DM \neq GR$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AM = \sqrt{(-3 - (-8))^2 + (-5 - (-9))^2}$$

$$AM = \sqrt{(-3 - (-8))^2 + (-5 - (-9))^2}$$

$$AM = \sqrt{5^2 + 4^2}$$

$$AM = \sqrt{25 + 16}$$

$$AM = \sqrt{41} \approx 6.40$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BR = \sqrt{(2-7)^2 + (4-9)^2}$$

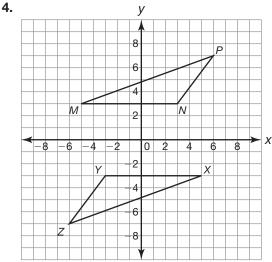
$$BR = \sqrt{(-5)^2 + (-5)^2}$$

$$BR = \sqrt{25 + 25}$$

$$BR = \sqrt{50} \approx 7.07$$

$$AM \neq BR$$

The triangles are not congruent.



$$MN = XY = 8$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(6-3)^2 + (7-3)^2}$$

$$NP = \sqrt{3^2 + 4^2}$$

$$NP = \sqrt{9 + 16}$$

$$NP = \sqrt{25} = 5$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$YZ = \sqrt{(-6 - (-3))^2 + (-7 - (-3))^2}$$

$$YZ = \sqrt{(-3)^2 + (-4)^2}$$

$$YZ = \sqrt{9 + 16}$$

$$YZ = \sqrt{25} = 5$$

$$NP = YZ$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MP = \sqrt{(6 - (-5))^2 + (7 - 3))^2}$$

$$MP = \sqrt{11^2 + 4^2}$$

$$MP = \sqrt{121 + 16}$$

$$MP = \sqrt{137} \approx 11.70$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$XZ = \sqrt{(-6-5)^2 + (-7-(-3))^2}$$

$$XZ = \sqrt{(-11)^2 + (-4)^2}$$

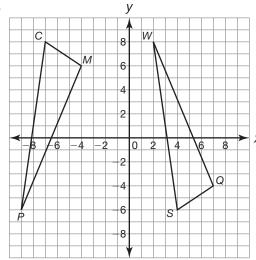
$$XZ = \sqrt{121 + 16}$$

$$XZ = \sqrt{137} \approx 11.70$$

$$MP = XZ$$

Date .

5.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$CP = \sqrt{(-9 - (-7))^2 + (-6 - 8)^2}$$

$$CP = \sqrt{(-2)^2 + (-14)^2}$$

$$CP = \sqrt{4 + 196}$$

$$CP = \sqrt{200} \approx 14.14$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$SW = \sqrt{(2-4)^2 + (8-(-6))^2}$$

$$SW = \sqrt{(-2)^2 + (14)^2}$$

$$SW = \sqrt{4 + 196}$$

$$SW = \sqrt{200} \approx 14.14$$

$$CP = SW$$

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$CM = \sqrt{(-4 - (-7))^2 + (6 - 8))^2}$$

$$CM = \sqrt{3^2 + (-2)^2}$$

$$CM = \sqrt{9 + 4}$$

$$CM = \sqrt{13} \approx 3.61$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$SQ = \sqrt{(7-4)^2 + (-4-(-6))^2}$$

$$SQ = \sqrt{3^2 + 2^2}$$

$$SQ = \sqrt{9+4}$$

$$SQ = \sqrt{13} \approx 3.61$$

$$CM = SQ$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MP = \sqrt{(-9 - (-4))^2 + (-6 - 6)^2}$$

$$MP = \sqrt{(-5)^2 + (-12)^2}$$

$$MP = \sqrt{25 + 144}$$

$$MP = \sqrt{169} = 13$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

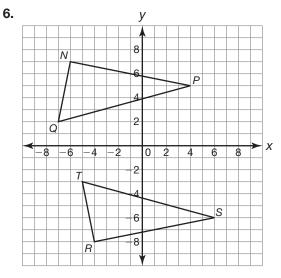
$$QW = \sqrt{(2-7)^2 + (8-(-4))^2}$$

$$QW = \sqrt{(-5)^2 + 12^2}$$

$$QW = \sqrt{25 + 144}$$

$$QW = \sqrt{169} = 13$$

$$MP = QW$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NQ = \sqrt{(-7 - (-6))^2 + (2 - 7)^2}$$

$$NQ = \sqrt{(-1)^2 + (-5)^2}$$

$$NQ = \sqrt{1 + 25}$$

$$NQ = \sqrt{26} \approx 5.10$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$RT = \sqrt{(-5 - (-4))^2 + (-3 - (-8))^2}$$

$$RT = \sqrt{(-1)^2 + 5^2}$$

$$RT = \sqrt{1 + 25}$$

$$RT = \sqrt{26} \approx 5.10$$

$$NQ = RT$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(4 - (-6))^2 + (5 - 7))^2}$$

$$NP = \sqrt{10^2 + (-2)^2}$$

$$NP = \sqrt{100 + 4}$$

$$NP = \sqrt{104} \approx 10.20$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$RS = \sqrt{(6 - (-4))^2 + (-6 - (-8))^2}$$

$$RS = \sqrt{10^2 + 2^2}$$

$$RS = \sqrt{100 + 4}$$

$$RS = \sqrt{104} \approx 10.20$$

$$NP = RS$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(-7-4)^2 + (2-5)^2}$$

$$PQ = \sqrt{(-11)^2 + (-3)^2}$$

$$PQ = \sqrt{121 + 9}$$

$$PQ = \sqrt{130} \approx 14.40$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$ST = \sqrt{(-5-6)^2 + (-3-(-6))^2}$$

$$ST = \sqrt{(-11)^2 + 3^2}$$

$$ST = \sqrt{121 + 9}$$

$$ST = \sqrt{130} \approx 14.40$$

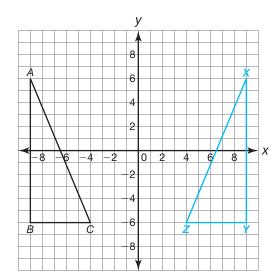
$$PQ = ST$$

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Name _ Date .

Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by SSS. Use the Distance Formula and a protractor when necessary.

7. Reflect $\triangle ABC$ over the *y*-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by SSS.



$$AB = XY = 12$$

$$BC = YZ = 5$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(-4 - (-9))^2 + (-6 - 6)^2}$$

$$AC = \sqrt{5^2 + (-12)^2}$$

$$AC = \sqrt{25 + 144}$$

$$AC = \sqrt{169} = 13$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$XZ = \sqrt{(4-9)^2 + (-6-6)^2}$$

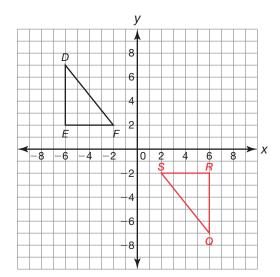
$$XZ = \sqrt{(-5)^2 + (-12)^2}$$

$$XZ = \sqrt{25 + 144}$$

$$XZ = \sqrt{169} = 13$$

$$AC = XZ$$

8. Rotate $\triangle DEF$ 180° clockwise about the origin to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by SSS.



$$DE = QR = 5$$

$$EF = RS = 4$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DF = \sqrt{(-2 - (-6))^2 + (2 - 7)^2}$$

$$DF = \sqrt{4^2 + (-5)^2}$$

$$DF = \sqrt{16 + 25}$$

$$DF = \sqrt{41} \approx 6.4$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$QS = \sqrt{(2-6)^2 + (-2-(-7))^2}$$

$$QS = \sqrt{(-4)^2 + 5^2}$$

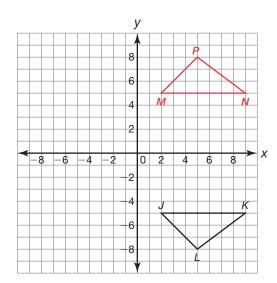
$$QS = \sqrt{16 + 25}$$

$$QS = \sqrt{41} \approx 6.4$$

$$DF = QS$$

Name _ Date _

9. Reflect $\triangle JKL$ over the *x*-axis to form $\triangle MNP$. Verify that $\triangle JKL \cong \triangle MNP$ by SSS.



$$JK = MN = 7$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KL = \sqrt{(5-9)^2 + (-8-(-5))^2}$$

$$KL = \sqrt{(-4)^2 + (-3)^2}$$

$$KL = \sqrt{16 + 9}$$

$$KL = \sqrt{25} = 5$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(5-9)^2 + (8-5)^2}$$

$$NP = \sqrt{(-4)^2 + 3^2}$$

$$NP = \sqrt{16 + 9}$$

$$NP = \sqrt{25} = 5$$

$$KL = NP$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JL = \sqrt{(5-2)^2 + (-8-(-5))^2}$$

$$JL = \sqrt{3^2 + (-3)^2}$$

$$JL = \sqrt{9+9}$$

$$JL = \sqrt{18} \approx 4.24$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MP = \sqrt{(5-2)^2 + (8-5)^2}$$

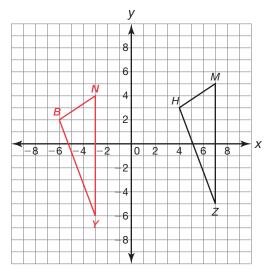
$$MP = \sqrt{3^2 + 3^2}$$

$$MP = \sqrt{9 + 9}$$

$$MP = \sqrt{18} \approx 4.24$$

$$JL = MP$$

10. Translate $\triangle HMZ$ 10 units to the left and 1 unit down to form $\triangle BNY$. Verify that $\triangle HMZ \cong \triangle BNY$ by SSS.



$$MZ = NY = 10$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$HM = \sqrt{(7-4)^2 + (5-3)^2}$$

$$HM = \sqrt{3^2 + 2^2}$$

$$HM = \sqrt{9 + 4}$$

$$HM = \sqrt{13} \approx 3.61$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BN = \sqrt{(-3 - (-6))^2 + (4 - 2)^2}$$

$$BN = \sqrt{3^2 + 2^2}$$

$$BN = \sqrt{9 + 4}$$

$$BN = \sqrt{13} \approx 3.61$$

$$HM = BN$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$HZ = \sqrt{(7-4)^2 + (-5-3)^2}$$

$$HZ = \sqrt{3^2 + (-8)^2}$$

$$HZ = \sqrt{9 + 64}$$

$$HZ = \sqrt{73} \approx 8.54$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BY = \sqrt{(-3 - (-6))^2 + (-6 - 2)^2}$$

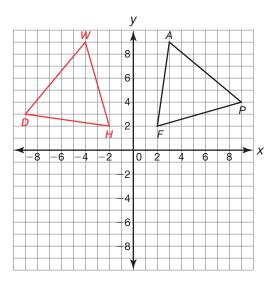
$$BY = \sqrt{3^2 + (-8)^2}$$

$$BY = \sqrt{9 + 64}$$

$$BY = \sqrt{73} \approx 8.54$$

$$HZ = BY$$

11. Rotate $\triangle AFP$ 90° counterclockwise about the origin to form $\triangle DHW$. Verify that $\triangle AFP \cong \triangle DHW$ by SSS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AP = \sqrt{(9-3)^2 + (4-9)^2}$$

$$AP = \sqrt{6^2 + (-5)^2}$$

$$AP = \sqrt{36 + 25}$$

$$AP = \sqrt{61} \approx 7.81$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DW = \sqrt{(-4 - (-9))^2 + (9 - 3)^2}$$

$$DW = \sqrt{5^2 + 6^2}$$

$$DW = \sqrt{25 + 36}$$

$$DW = \sqrt{61} \approx 7.81$$

$$AP = DW$$

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AF = \sqrt{(2-3)^2 + (2-9)^2}$$

$$AF = \sqrt{(-1)^2 + (-7)^2}$$

$$AF = \sqrt{1 + 49}$$

$$AF = \sqrt{50} \approx 7.07$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DH = \sqrt{(-2 - (-9))^2 + (2 - 3)^2}$$

$$DH = \sqrt{7^2 + (-1)^2}$$

$$DH = \sqrt{49 + 1}$$

$$DH = \sqrt{50} \approx 7.07$$

$$AF = DH$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$FP = \sqrt{(9-2)^2 + (4-2)^2}$$

$$FP = \sqrt{7^2 + 2^2}$$

$$FP = \sqrt{49 + 4}$$

$$FP = \sqrt{53} \approx 7.28$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

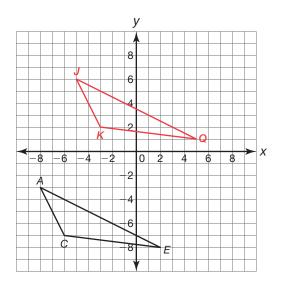
$$HW = \sqrt{(-4 - (-2))^2 + (9 - 2)^2}$$

$$HW = \sqrt{(-2)^2 + 7^2}$$

$$HW = \sqrt{4 + 49}$$

$$HW = \sqrt{53} \approx 7.28$$

$$FP = HW$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AE = \sqrt{(2 - (-8))^2 + (-8 - (-3))^2}$$

$$AE = \sqrt{10^2 + (-5)^2}$$

$$AE = \sqrt{100 + 25}$$

$$AE = \sqrt{125} \approx 11.18$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JQ = \sqrt{(5 - (-5))^2 + (1 - 6)^2}$$

$$JQ = \sqrt{10^2 + (-5)^2}$$

$$JQ = \sqrt{100 + 25}$$

$$JQ = \sqrt{125} \approx 11.18$$

$$AE = JQ$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(-6 - (-8))^2 + (-7 - (-3))^2}$$

$$AC = \sqrt{2^2 + (-4)^2}$$

$$AC = \sqrt{4 + 16}$$

$$AC = \sqrt{20} \approx 4.47$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JK = \sqrt{(-3 - (-5))^2 + (2 - 6)^2}$$

$$JK = \sqrt{2^2 + (-4)^2}$$

$$JK = \sqrt{4 + 16}$$

$$JK = \sqrt{20} \approx 4.47$$

$$AC = JK$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$CE = \sqrt{(2 - (-6))^2 + (-8 - (-7))^2}$$

$$CE = \sqrt{8^2 + (-1)^2}$$

$$CE = \sqrt{64 + 1}$$

$$CE = \sqrt{65} \approx 8.06$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KQ = \sqrt{(5 - (-3))^2 + (1 - 2)^2}$$

$$KQ = \sqrt{8^2 + (-1)^2}$$

$$KQ = \sqrt{64 + 1}$$

$$KQ = \sqrt{65} \approx 8.06$$

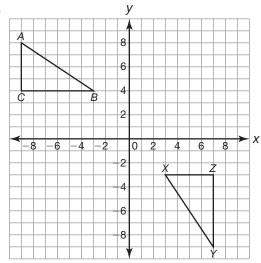
$$CE = KQ$$

Make Sure the Angle Is Included **Side-Angle-Side Congruence Theorem**

Vocabulary

Describe how to prove the given triangles are congruent. Use the Side-Angle-Side Congruence Theorem in your answer.

1.

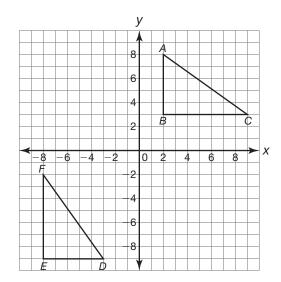


The Side-Angle-Side Congruence Theorem states that if two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent. In the triangles shown, AC = XZ = 4, and BC = YZ = 6. The included angle ($\angle C$) between \overline{AC} and \overline{BC} is congruent to the included angle ($\angle Z$) between \overline{XZ} and \overline{YZ} , because $m \angle C = m \angle Z = 90^{\circ}$. Therefore, $\triangle ABC \cong \triangle XYZ$ according to the SAS Congruence Theorem.

Problem Set

Determine whether each pair of given triangles are congruent by SAS. Use the Distance Formula and a protractor when necessary.

1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by SAS.

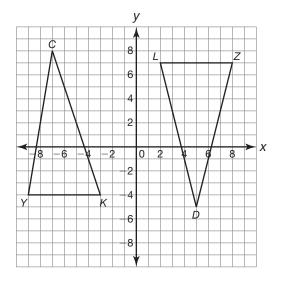


$$AB = DE = 5$$

 $BC = EF = 7$
 $m\angle B = m\angle E = 90^{\circ}$

The triangles are congruent by the SAS Congruence Theorem.

2. Determine whether $\triangle CKY$ is congruent to $\triangle DLZ$ by SAS.



$$m \angle Y = 81^{\circ}, m \angle Z = 76^{\circ}$$

$$m \angle Y \neq m \angle Z$$

The triangles are not congruent.

$$KY = LZ = 6$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$CY = \sqrt{(-9 - (-7))^2 + (-4 - 8)^2}$$

$$CY = \sqrt{(-2)^2 + (-12)^2}$$

$$CY = \sqrt{4 + 144}$$

$$CY = \sqrt{148} \approx 12.17$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DZ = \sqrt{(8-5)^2 + (7-(-5))^2}$$

$$DZ = \sqrt{3^2 + (12)^2}$$

$$DZ = \sqrt{9 + 144}$$

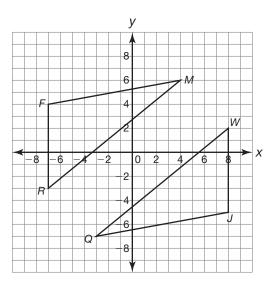
$$DZ = \sqrt{153} \approx 12.37$$

$$CY \neq DZ$$

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3. Determine whether $\triangle FMR$ is congruent to $\triangle JQW$ by SAS.



$$m \angle F = m \angle J = 100^{\circ}$$

The triangles are congruent by the SAS Congruence Theorem.

$$FR = JW = 7$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$FM = \sqrt{(4 - (-7))^2 + (6 - 4)^2}$$

$$FM = \sqrt{11^2 + 2^2}$$

$$FM = \sqrt{121 + 4}$$

$$FM = \sqrt{125} \approx 11.18$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JQ = \sqrt{(-3-8)^2 + (-7-(-5))^2}$$

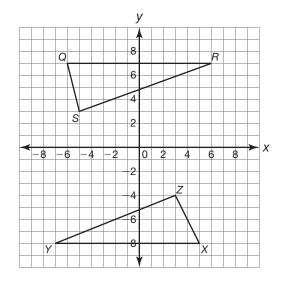
$$JQ = \sqrt{(-11)^2 + (-2)^2}$$

$$JQ = \sqrt{121 + 4}$$

$$JQ = \sqrt{125} \approx 11.18$$

$$FM = JQ$$

4. Determine whether $\triangle QRS$ is congruent to $\triangle XYZ$ by SAS.



$$m \angle R = 20^{\circ}, m \angle Y = 22^{\circ}$$

$$m \angle R \neq m \angle Y$$

The triangles are not congruent.

$$QR = XY = 12$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$RS = \sqrt{(-5-6)^2 + (3-7)^2}$$

$$RS = \sqrt{(-11)^2 + (-4)^2}$$

$$RS = \sqrt{121 + 16}$$

$$RS = \sqrt{137} \approx 11.70$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$YZ = \sqrt{(3 - (-7))^2 + (-4 - (-8))^2}$$

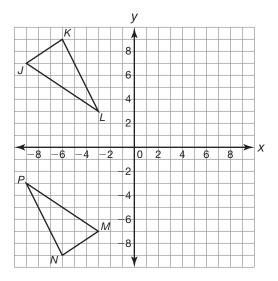
$$YZ = \sqrt{10^2 + 4^2}$$

$$YZ = \sqrt{100 + 16}$$

$$YZ = \sqrt{116} \approx 10.77$$

$$RS \neq YZ$$

5. Determine whether $\triangle JKL$ is congruent to $\triangle MNP$ by SAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JK = \sqrt{(-6 - (-9))^2 + (9 - 7)^2}$$

$$JK = \sqrt{3^2 + 2^2}$$

$$JK = \sqrt{9 + 4}$$

$$JK = \sqrt{13}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MN = \sqrt{(-6 - (-3))^2 + (-9 - (-7))^2}$$

$$MN = \sqrt{(-3)^2 + (-2)^2}$$

$$MN = \sqrt{9 + 4}$$

$$MN = \sqrt{13}$$

$$JK = MN$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KL = \sqrt{(-3 - (-6))^2 + (3 - 9)^2}$$

$$KL = \sqrt{3^2 + (-6)^2}$$

$$KL = \sqrt{9 + 36}$$

$$KL = \sqrt{45}$$

$$KL \approx 6.71$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(-9 - (-6))^2 + (-3 - (-9))^2}$$

$$NP = \sqrt{(-3)^2 + 6^2}$$

$$NP = \sqrt{9 + 36}$$

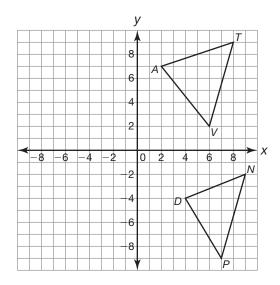
$$NP = \sqrt{45}$$

$$KL = NP$$

$$m \angle K = m \angle N = 83^{\circ}$$

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6. Determine whether $\triangle ATV$ is congruent to $\triangle DNP$ by SAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AT = \sqrt{(8-2)^2 + (9-7)^2}$$

$$AT = \sqrt{6^2 + 2^2}$$

$$AT = \sqrt{36 + 4}$$

$$AT = \sqrt{40}$$

$$AT \approx 6.32$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DN = \sqrt{(9-4)^2 + (-2-(-4))^2}$$

$$DN = \sqrt{5^2 + 2^2}$$

$$DN = \sqrt{25 + 4}$$

$$DN = \sqrt{29}$$

$$AT \neq DN$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$TV = \sqrt{(6-8)^2 + (2-9)^2}$$

$$TV = \sqrt{(-2)^2 + (-7)^2}$$

$$TV = \sqrt{4 + 49}$$

$$TV = \sqrt{53}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(7-9)^2 + (-9-(-2))^2}$$

$$NP = \sqrt{(-2)^2 + (-7)^2}$$

$$NP = \sqrt{4 + 49}$$

$$NP = \sqrt{53}$$

$$TV = NP$$

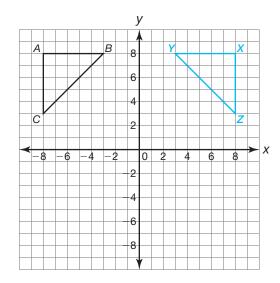
$$m \angle T = 56^{\circ}, m \angle N = 52^{\circ}$$

$$m \angle T \neq m \angle N$$

The triangles are not congruent.

Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by SAS. Use the Distance Formula and a protractor when necessary.

7. Reflect $\triangle ABC$ over the *y*-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by SAS.

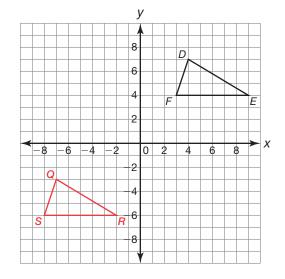


$$AB = XY = 5$$
$$AC = XZ = 5$$

$$m \angle A = m \angle X = 90^{\circ}$$

The triangles are congruent by the SAS Congruence Theorem.

8. Translate $\triangle DEF$ 11 units to the left and 10 units down to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by SAS.



$$EF = RS = 6$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DF = \sqrt{(3-4)^2 + (4-7)^2}$$

$$DF = \sqrt{(-1)^2 + (-3)^2}$$

$$DF = \sqrt{1 + 9}$$

$$DF = \sqrt{10} \approx 3.16$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$QS = \sqrt{(-8 - (-7))^2 + (-6 - (-3))^2}$$

$$QS = \sqrt{(-1)^2 + (-3)^2}$$

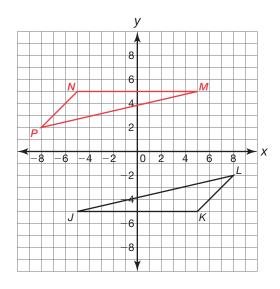
$$QS = \sqrt{1+9}$$

$$QS = \sqrt{10} \approx 3.16$$

$$DF = QS$$

$$m \angle F = m \angle S = 72^{\circ}$$

9. Rotate $\triangle JKL$ 180° counterclockwise about the origin to form $\triangle MNP$. Verify that $\triangle JKL \cong \triangle MNP$ by SAS.



$$m \angle K = m \angle N = 135^{\circ}$$

The triangles are congruent by the SAS Congruence Theorem.

$$JK = MN = 10$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KL = \sqrt{(8 - 5)^2 + (-2 - (-5))^2}$$

$$KL = \sqrt{3^2 + 3^2}$$

$$KL = \sqrt{9+9}$$

$$KL = \sqrt{18} \approx 4.24$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(-8 - (-5))^2 + (2 - 5)^2}$$

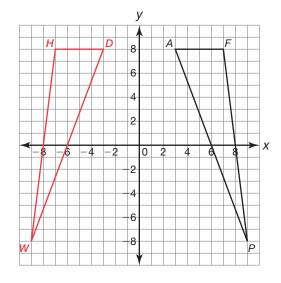
$$NP = \sqrt{(-3)^2 + (-3)^2}$$

$$NP = \sqrt{9+9}$$

$$NP = \sqrt{18} \approx 4.24$$

$$KL = NP$$

10. Reflect $\triangle AFP$ over the *y*-axis to form $\triangle DHW$. Verify that $\triangle AFP \cong \triangle DHW$ by SAS.



$$m \angle F = m \angle H \approx 97^{\circ}$$

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$$AF = DH = 4$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$FP = \sqrt{(9-7)^2 + (-8-8)^2}$$

$$FP = \sqrt{2^2 + (-16)^2}$$

$$FP = \sqrt{4 + 256}$$

$$FP = \sqrt{260} \approx 16.12$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$HW = \sqrt{(-9 - (-7))^2 + (-8 - 8)^2}$$

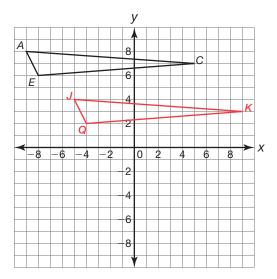
$$HW = \sqrt{(-2)^2 + (-16)^2}$$

$$HW = \sqrt{4 + 256}$$

$$HW = \sqrt{260} \approx 16.12$$

$$FP = HW$$

11. Translate $\triangle ACE$ 4 units to the right and 4 units down to form $\triangle JKQ$. Verify that $\triangle ACE \cong \triangle JKQ$ by SAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(5 - (-9))^2 + (7 - 8)^2}$$

$$AC = \sqrt{14^2 + (-1)^2}$$

$$AC = \sqrt{196 + 1}$$

$$AC = \sqrt{197}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JK = \sqrt{(9 - (-5))^2 + (3 - 4)^2}$$

$$JK = \sqrt{14^2 + (-1)^2}$$

$$JK = \sqrt{196 + 1}$$

$$JK = \sqrt{197}$$

$$AC = JK$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AE = \sqrt{(-8 - (-9))^2 + (6 - 8)^2}$$

$$AE = \sqrt{1^2 + (-2)^2}$$

$$AE = \sqrt{1 + 4}$$

$$AE = \sqrt{5}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JQ = \sqrt{(-4 - (-5))^2 + (2 - 4)^2}$$

$$JQ = \sqrt{1^2 + (-2)^2}$$

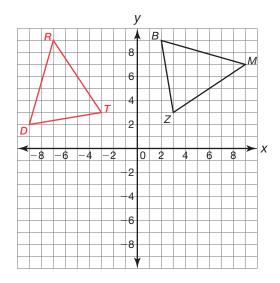
$$JQ = \sqrt{1+4}$$

$$JQ = \sqrt{5}$$

$$AE = JQ$$

$$m \angle A = m \angle J = 59^{\circ}$$

12. Rotate $\triangle BMZ$ 90° counterclockwise about the origin to form $\triangle DRT$. Verify that $\triangle BMZ \cong \triangle DRT$ by SAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BM = \sqrt{(9-2)^2 + (7-9)^2}$$

$$BM = \sqrt{7^2 + (-2)^2}$$

$$BM = \sqrt{49 + 4}$$

$$BM = \sqrt{53}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DR = \sqrt{(-7 - (-9))^2 + (9 - 2)^2}$$

$$DR = \sqrt{2^2 + 7^2}$$

$$DR = \sqrt{4 + 49}$$

$$DR = \sqrt{53}$$

$$BM = DR$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MZ = \sqrt{(3-9)^2 + (3-7)^2}$$

$$MZ = \sqrt{(-6)^2 + (-4)^2}$$

$$MZ = \sqrt{36 + 16}$$

$$MZ = \sqrt{52}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$RT = \sqrt{(-3 - (-7))^2 + (3 - 9)^2}$$

$$RT = \sqrt{4^2 + (-6)^2}$$

$$RT = \sqrt{16 + 36}$$

$$RT = \sqrt{52}$$

$$MZ = RT$$

$$m \angle M = m \angle R = 50^{\circ}$$

Determine the angle measure or side measure that is needed in order to prove that each set of triangles are congruent by SAS.

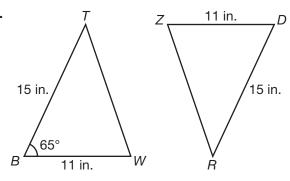
13. In
$$\triangle ART$$
, $AR=12$, $RT=8$, and $m \angle R=70^\circ$. In $\triangle BSW$, $BS=12$, and $m \angle S=70^\circ$. $SW=8$

14. In
$$\triangle CDE$$
, $CD = 7$, and $DE = 11$. In $\triangle FGH$, $FG = 7$, $GH = 11$, and $m \angle G = 45^{\circ}$. $m \angle D = 45^{\circ}$

15. In
$$\triangle JKL$$
, $JK = 2$, $KL = 3$, and $m \angle K = 60^\circ$. In $\triangle MNP$, $NP = 3$, and $m \angle N = 60^\circ$. $MN = 2$

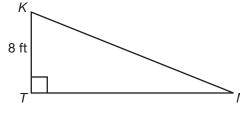
16. In
$$\triangle QRS$$
, $QS = 6$, $RS = 4$, and $m \angle S = 20^{\circ}$. In $\triangle TUV$, $TV = 6$, and $UV = 4$. $m \angle V = 20^{\circ}$

17.

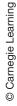


$$m \angle D = 65^{\circ}$$

18.



$$MT = 20 \text{ ft}$$



8 ft

20 ft

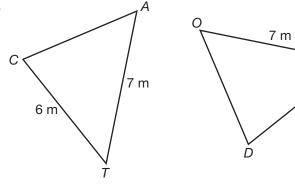
50°

6 m

Name _

Date_

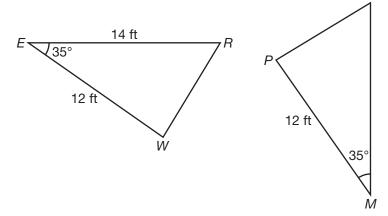
19.



 $m \angle T = 50^{\circ}$

20.

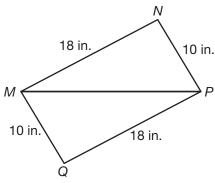
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MV = 14 ft

or SAS. Write the congruence statements to justify your reasoning.

21.
$$\land MNP \stackrel{?}{\cong} \land PQM$$



The triangles are congruent by SSS.

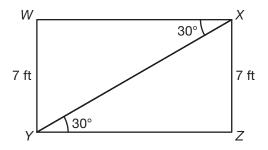
$$\overline{MN} \cong \overline{PQ}$$

$$\overline{NP} \cong \overline{QM}$$

$$\overline{MP} \cong \overline{PM}$$

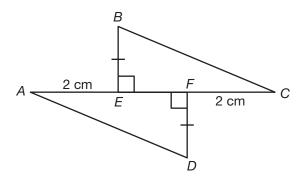
22. $\triangle WXY \stackrel{?}{\cong} \triangle ZYX$

Determine whether there is enough information to prove that each pair of triangles are congruent by SSS



There is not enough information to determine whether the triangles are congruent by SSS or SAS. SAS does not apply because the congruent angles in the figure are not included angles.

23. $\land BCE \stackrel{?}{\cong} \land DAF$

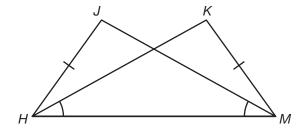


The triangles are congruent by SAS.

$$\overline{BE} \cong \overline{DF}$$

$$\overline{CE} \cong \overline{AF}$$

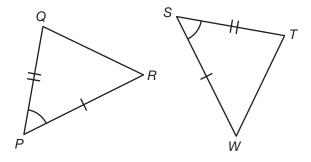
24. △*HJM* ≈ △*MKH*



There is not enough information to determine whether the triangles are congruent by SSS or SAS. SAS does not apply because the congruent angles in the figure are not included angles.

Name _ Date _

25.
$$\triangle PQR \stackrel{?}{\cong} \triangle STW$$



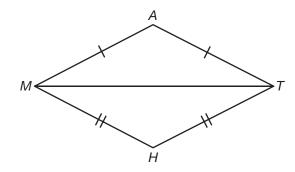
The triangles are congruent by SAS.

$$\overline{PQ} \cong \overline{ST}$$

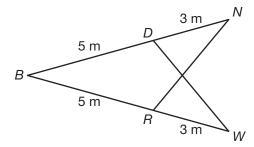
∠QPR ≅ ∠TSW

$$\overline{PR} \cong \overline{SW}$$

26. △*MAT*
$$\stackrel{?}{\cong}$$
 △*MHT*



There is not enough information to determine whether the triangles are congruent by SSS or SAS.



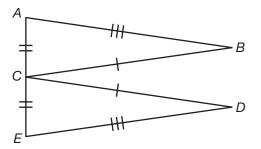
The triangles are congruent by SAS.

$$\overline{BD} \cong \overline{BR}$$

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$$\angle DBW \cong \angle RBN$$

$$\overline{BW} \cong \overline{BN}$$



The triangles are congruent by SSS.

$$\overline{AB} \cong \overline{ED}$$

$$\overline{BC} \cong \overline{DC}$$

$$\overline{AC} \cong \overline{EC}$$

N	5.5

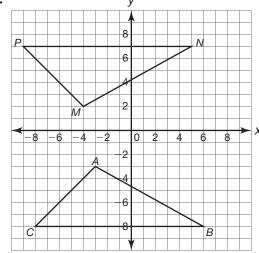
Name _ Date.

Angle to the Left of Me, Angle to the Right of Me **Angle-Side-Angle Congruence Theorem**

Vocabulary

Describe how to prove the given triangles are congruent. Use the Angle-Side-Angle Congruence Theorem in your answer.

1.

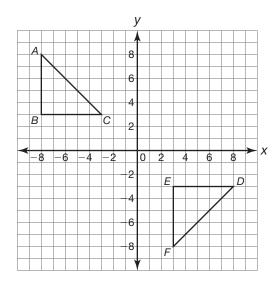


The Angle-Side-Angle Congruence Theorem states that if two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent. In the triangles shown, $m \angle C = m \angle P = 45^{\circ}$, and $m \angle B = m \angle N = 29^{\circ}$. The included side (\overline{BC}) between $\angle B$ and $\angle C$ is congruent to the included side (\overline{NP}) between $\angle N$ and $\angle P$, because BC = NP = 14. Therefore, $\triangle ABC \cong \triangle MNP$ according to the ASA Congruence Theorem.

Problem Set

Determine whether each pair of given triangles are congruent by ASA. Use the Distance Formula and a protractor when necessary.

1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by ASA.

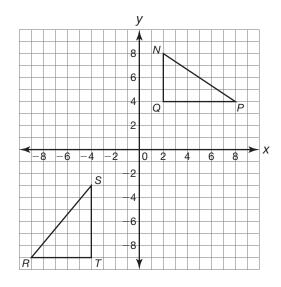


$$m \angle B = m \angle E = 90^{\circ}$$

 $m \angle C = m \angle F = 45^{\circ}$
 $BC = EF = 5$

The triangles are congruent by the ASA Congruence Theorem.

2. Determine whether $\triangle NPQ$ is congruent to $\triangle RST$ by ASA.



$$m\angle Q = m\angle T = 90^{\circ}$$

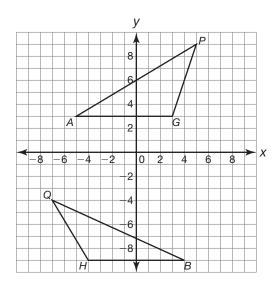
 $m\angle P = 34^{\circ}, m\angle S = 40^{\circ},$
 $m\angle P \neq m\angle S$
 $PQ = ST = 6$

The triangles are not congruent.

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Date _____

3. Determine whether $\triangle AGP$ is congruent to $\triangle BHQ$ by ASA.



$$m \angle A = 31^{\circ}, m \angle B = 24^{\circ}$$

$$m \angle A \neq m \angle B$$

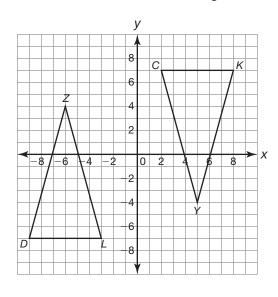
$$m \angle G = 108^{\circ}, m \angle H = 121^{\circ}$$

$$m \angle G \neq m \angle H$$

$$AG = BH = 8$$

The triangles are not congruent.

4. Determine whether $\triangle CKY$ is congruent to $\triangle DLZ$ by ASA.

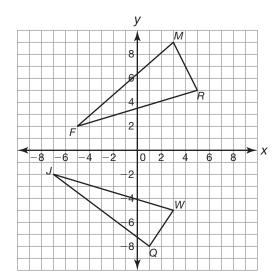


$$m \angle C = m \angle D = 75^{\circ}$$

$$m \angle K = m \angle L = 75^{\circ}$$

$$CK = DL = 6$$

5. Determine whether $\triangle FMR$ is congruent to $\triangle JQW$ by ASA.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$FM = \sqrt{(3 - (-5))^2 + (9 - 2)^2}$$

$$FM = \sqrt{8^2 + 7^2}$$

$$FM = \sqrt{64 + 49}$$

$$FM = \sqrt{113} \approx 10.63$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JQ = \sqrt{(1 - (-7))^2 + (-8 - (-2))^2}$$

$$JQ = \sqrt{8^2 + (-6)^2}$$

$$JQ = \sqrt{64 + 36}$$

$$JQ = \sqrt{100} = 10$$

$$FM \neq JQ$$

$$m \angle F = 24^{\circ}, m \angle J = 20^{\circ}$$

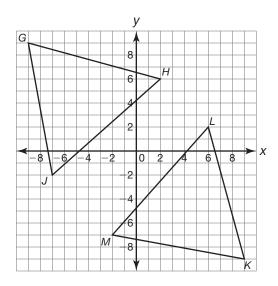
$$m \angle F \neq m \angle J$$

$$m \angle M = 75^{\circ}, m \angle Q = 87^{\circ}$$

$$m \angle M \neq m \angle Q$$

The triangles are not congruent.

6. Determine whether $\triangle GHJ$ is congruent to $\triangle KLM$ by ASA.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$HJ = \sqrt{(-7-2)^2 + (-2-6)^2}$$

$$HJ = \sqrt{(-9)^2 + (-8)^2}$$

$$HJ = \sqrt{81 + 64}$$

$$HJ = \sqrt{145} \approx 12.04$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$LM = \sqrt{(-2-6)^2 + (-7-2)^2}$$

$$LM = \sqrt{(-8)^2 + (-9)^2}$$

$$LM = \sqrt{64 + 81}$$

$$LM = \sqrt{145} \approx 12.04$$

$$HJ = LM$$

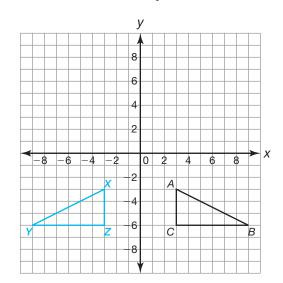
$$m \angle H = m \angle L = 57^{\circ}$$

$$m \angle J = m \angle M = 59^{\circ}$$

Date _

Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by ASA. Use the Distance Formula and a protractor when necessary.

7. Reflect $\triangle ABC$ over the *y*-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by SAS.



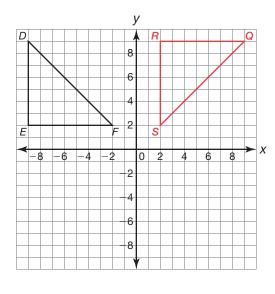
$$m \angle C = m \angle Z = 90^{\circ}$$

$$m \angle A = m \angle X = 63^{\circ}$$

$$AC = XZ = 3$$

The triangles are congruent by the ASA Congruence Theorem.

8. Rotate $\triangle DEF$ 90° clockwise about the origin to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by SAS.



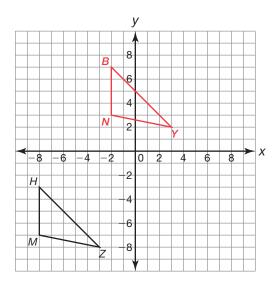
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$$m \angle E = m \angle R = 90^{\circ}$$

$$m \angle F = m \angle S = 45^{\circ}$$

$$EF = RS = 7$$

9. Translate $\triangle HMZ$ 6 units to the right and 10 units up to form $\triangle BNY$. Verify that $\triangle HMZ \cong \triangle BNY$ by ASA.

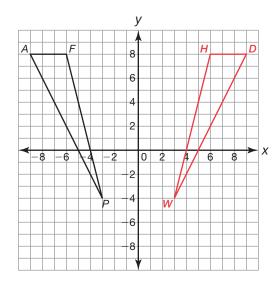


$$m \angle H = m \angle B = 45^{\circ}$$

 $m \angle M = m \angle N = 101^{\circ}$
 $HM = BN = 4$

The triangles are congruent by the ASA Congruence Theorem.

10. Reflect $\triangle AFP$ over the *y*-axis to form $\triangle DHW$. Verify that $\triangle AFP \cong \triangle DHW$ by ASA.

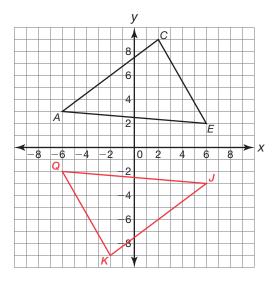


$$m \angle A = m \angle D = 63^{\circ}$$

 $m \angle F = m \angle H = 104^{\circ}$

$$AF = DH = 3$$

11. Rotate $\triangle ACE$ 180° counterclockwise about the origin to form $\triangle JKQ$. Verify that $\triangle ACE \cong \triangle JKQ$ by SAS.



$$m \angle A = m \angle J = 42^{\circ}$$

$$m \angle C = m \angle K = 83^{\circ}$$

The triangles are congruent by the ASA Congruence Theorem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(2 - (-6))^2 + (9 - 3)^2}$$

$$AC = \sqrt{8^2 + 6^2}$$

$$AC = \sqrt{64 + 36}$$

$$AC = \sqrt{100} = 10$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JK = \sqrt{(-2-6)^2 + (-9-(-3))^2}$$

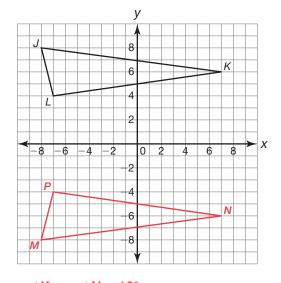
$$JK = \sqrt{(-8)^2 + (-6)^2}$$

$$JK = \sqrt{64 + 36}$$

$$JK = \sqrt{100} = 10$$

$$AC = JK$$

12. Reflect $\triangle JKL$ over the *x*-axis to form $\triangle MNP$. Verify that $\triangle JKL \cong \triangle MNP$ by ASA.



$$m \angle K = m \angle N = 16^{\circ}$$

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$$m \angle L = m \angle P = 96^{\circ}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$KL = \sqrt{(-7-7)^2 + (4-6)^2}$$

$$KL = \sqrt{(-14)^2 + (-2)^2}$$

$$KL = \sqrt{196 + 4}$$

$$KL = \sqrt{200} \approx 14.14$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(-7-7)^2 + (-4-(-6))^2}$$

$$NP = \sqrt{(-14)^2 + 2^2}$$

$$NP = \sqrt{196 + 4}$$

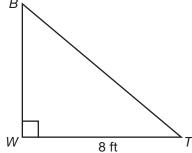
$$NP = \sqrt{200} \approx 14.14$$

$$KL = NP$$

Determine the angle measure or side measure that is needed in order to prove that each set of triangles are congruent by ASA.

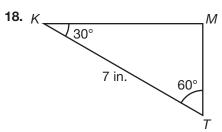
- **13.** In $\triangle ADZ$, $m \angle A = 20^{\circ}$, AD = 9, and $m \angle D = 70^{\circ}$. In $\triangle BEN$, BE = 9, and $m \angle E = 70^{\circ}$. $m \angle B = 20^{\circ}$
- **14.** In $\triangle CUP$, $m \angle U = 45^{\circ}$, and $m \angle P = 55^{\circ}$. In $\triangle HAT$, AT = 14, $m \angle A = 45^{\circ}$, and $m \angle T = 55^{\circ}$. UP = 14
- **15.** In $\triangle HOW$, $m \angle H = 10^{\circ}$, HW = 3, and $m \angle W = 60^{\circ}$. In $\triangle FAR$, FR = 3, and $m \angle F = 10^{\circ}$. $m \angle R = 60^{\circ}$
- **16.** In $\triangle DRY$, $m \angle D = 100^{\circ}$, DR = 25, and $m \angle R = 30^{\circ}$. In $\triangle WET$, $m \angle W = 100^{\circ}$, and $m \angle E = 30^{\circ}$. WE = 25

17. B



40° 8 ft

 $m \angle T = 40^{\circ}$

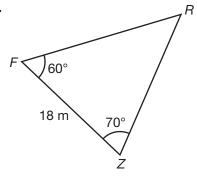


230° ل 60°

LN = 7 in.

Date _

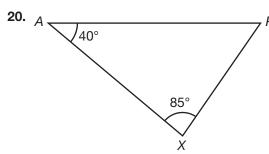
19.

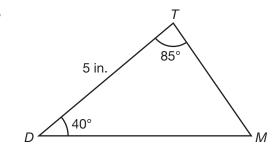


Ģ 18 m

m∠*G* = 60°

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AX = 5 in.

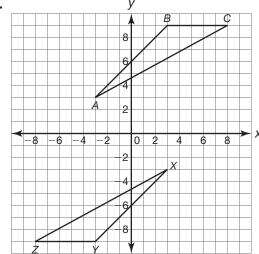
Name	Date	

Sides Not Included Angle-Angle-Side Congruence Theorem

Vocabulary

Describe how to prove the given triangles are congruent. Use the Angle-Angle-Side Congruence Theorem in your answer.

1.

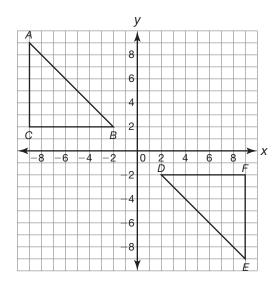


The Angle-Angle-Side Congruence Theorem states that if two angles and a non-included side of one triangle are congruent to the corresponding angles and the corresponding non-included side of a second triangle, then the triangles are congruent. In the triangles shown, $m \angle A = m \angle X = 16^{\circ}$, and $m \angle B = m \angle Y = 135^{\circ}$. The non-included side (\overline{BC}) of $\angle A$ and $\angle B$ is congruent to the corresponding non-included side (\overline{YZ}) of $\angle X$ and $\angle Y$, because BC = YZ = 5. Therefore, $\triangle ABC \cong \triangle XYZ$ according to the AAS Congruence Theorem.

Problem Set

Determine whether each set of given triangles are congruent by AAS. Use the Distance Formula and a protractor when necessary.

1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by AAS.



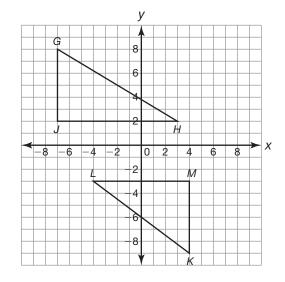
$$m \angle A = m \angle D = 45^{\circ}$$

$$m \angle B = m \angle E = 45^{\circ}$$

$$BC = EF = 7$$

The triangles are congruent by the AAS Congruence Theorem.

2. Determine whether $\triangle GHJ$ is congruent to $\triangle KLM$ by AAS.



$$m \angle H = 31^{\circ}, m \angle L = 37^{\circ}$$

$$m \angle H \neq m \angle L$$

$$m \angle J = m \angle M = 90^{\circ}$$

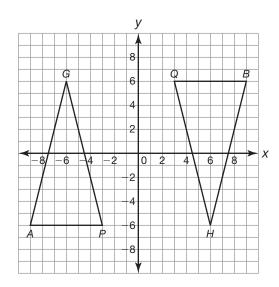
$$GJ = KM = 6$$

The triangles are not congruent.

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Date _____

3. Determine whether $\triangle AGP$ is congruent to $\triangle BHQ$ by AAS.



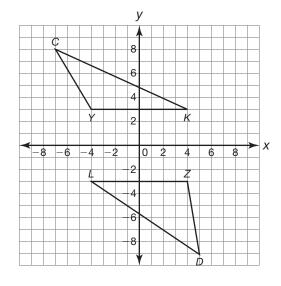
$$m \angle G = m \angle H = 28^{\circ}$$

$$m \angle A = m \angle B = 76^{\circ}$$

$$AP = BQ = 6$$

The triangles are congruent by the AAS Congruence Theorem.

4. Determine whether $\triangle CKY$ is congruent to $\triangle DLZ$ by AAS.



$$m \angle C = 35^{\circ}, m \angle D = 47^{\circ}$$

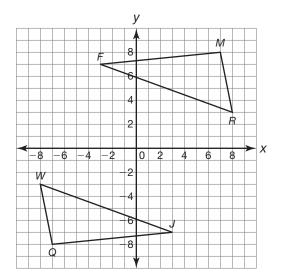
$$m \angle C \neq m \angle D$$

$$m \angle K = 24^{\circ}, m \angle L = 34^{\circ}$$

$$m \angle K \neq m \angle L$$

$$KY = LZ = 8$$

The triangles are not congruent.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$FM = \sqrt{(7 - (-3))^2 + (8 - 7)^2}$$

$$FM = \sqrt{10^2 + 1^2}$$

$$FM = \sqrt{100 + 1}$$

$$FM = \sqrt{101} \approx 10.05$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JQ = \sqrt{(-7-3)^2 + (-8-(-7))^2}$$

$$JQ = \sqrt{(-10)^2 + (-1)^2}$$

$$JQ = \sqrt{100 + 1}$$

$$JQ = \sqrt{101} \approx 10.05$$

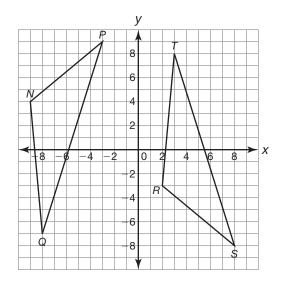
$$FM = JQ$$

$$m \angle F = m \angle J = 26^{\circ}$$

$$m \angle R = m \angle W = 59^{\circ}$$

The triangles are congruent by the AAS Congruence Theorem.

6. Determine whether $\triangle NPQ$ is congruent to $\triangle RST$ by AAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$NP = \sqrt{(-3 - (-9))^2 + (9 - 4)^2}$$

$$NP = \sqrt{6^2 + 5^2}$$

$$NP = \sqrt{36 + 25}$$

$$NP = \sqrt{61} \approx 7.81$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$RS = \sqrt{(8-2)^2 + (-8-(-3))^2}$$

$$RS = \sqrt{6^2 + (-5)^2}$$

$$RS = \sqrt{36 + 25}$$

$$RS = \sqrt{61} \approx 7.81$$

$$NP = RS$$

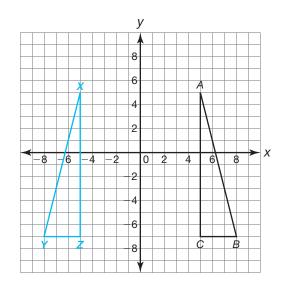
$$m \angle N = m \angle R = 125^{\circ}$$

$$m \angle Q = m \angle T = 23^{\circ}$$

Name __ Date _

Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by AAS. Use the Distance Formula and a protractor when necessary.

7. Reflect $\triangle ABC$ over the *y*-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by AAS.



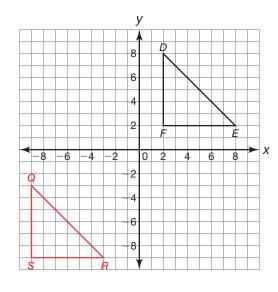
$$m \angle B = m \angle Y = 76^{\circ}$$

$$m \angle C = m \angle Z = 90^{\circ}$$

$$AC = XZ = 12$$

The triangles are congruent by the AAS Congruence Theorem.

8. Translate $\triangle DEF$ 11 units to the left and 11 units down to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by AAS.



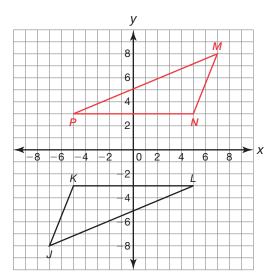
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$$m \angle D = m \angle Q = 45^{\circ}$$

$$m \angle E = m \angle R = 45^{\circ}$$

$$EF = RS = 6$$

9. Rotate $\triangle JKL$ 180° counterclockwise about the origin to form $\triangle MNP$. Verify that $\triangle JKL \cong \triangle MNP$ by AAS.



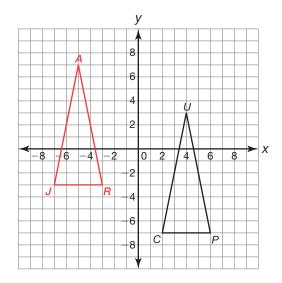
$$m \angle J = m \angle M = 46^{\circ}$$

$$m \angle K = m \angle N = 112^{\circ}$$

$$KL = NP = 10$$

The triangles are congruent by the AAS Congruence Theorem.

10. Translate $\triangle CUP$ 9 units to the left and 4 units up to form $\triangle JAR$. Verify that $\triangle CUP \cong \triangle JAR$ by AAS.



$$m \angle U = m \angle A = 23^{\circ}$$

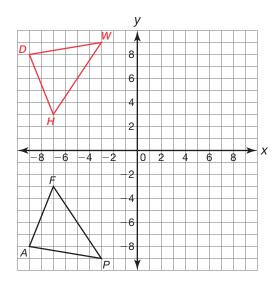
$$m \angle P = m \angle R = 79^{\circ}$$

$$CP = JR = 4$$

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Date ___

11. Reflect $\triangle AFP$ over the *x*-axis to form $\triangle DHW$. Verify that $\triangle AFP \cong \triangle DHW$ by AAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AP = \sqrt{(-3 - (-9))^2 + (-9 - (-8))^2}$$

$$AP = \sqrt{6^2 + (-1)^2}$$

$$AP = \sqrt{36 + 1}$$

$$AP = \sqrt{37} \approx 6.08$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DW = \sqrt{(-3 - (-9))^2 + (9 - 8)^2}$$

$$DW = \sqrt{6^2 + 1^2}$$

$$DW = \sqrt{36 + 1}$$

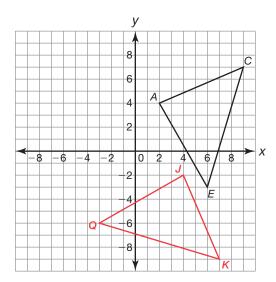
$$DW = \sqrt{37} \approx 6.08$$

$$AP = DW$$

$$m \angle A = m \angle D = 78^{\circ}$$

$$m \angle F = m \angle H = 55^{\circ}$$

12. Rotate $\triangle ACE$ 270° counterclockwise about the origin to form $\triangle JKQ$. Verify that $\triangle ACE \cong \triangle JKQ$ by AAS.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(9-2)^2 + (7-4)^2}$$

$$AC = \sqrt{7^2 + 3^2}$$

$$AC = \sqrt{49 + 9}$$

$$AC = \sqrt{58} \approx 7.62$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$JK = \sqrt{(7-4)^2 + (-9-(-2))^2}$$

$$JK = \sqrt{3^2 + (-7)^2}$$

$$JK = \sqrt{9 + 49}$$

$$JK = \sqrt{58} \approx 7.62$$

$$AC = JK$$

$$m \angle A = m \angle J = 83^{\circ}$$

$$m \angle E = m \angle Q = 46^{\circ}$$

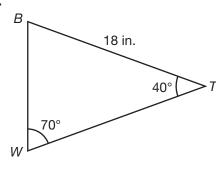
The triangles are congruent by the AAS Congruence Theorem.

Determine the angle measure or side measure that is needed in order to prove that each set of triangles are congruent by AAS.

- **13.** In $\triangle ANT$, $m \angle A = 30^\circ$, $m \angle N = 60^\circ$, and NT = 5. In $\triangle BUG$, $m \angle U = 60^\circ$, and UG = 5. $m \angle B = 30^{\circ}$
- **14.** In $\triangle BCD$, $m \angle B = 25^{\circ}$, and $m \angle D = 105^{\circ}$. In $\triangle RST$, RS = 12, $m \angle R = 25^{\circ}$, and $m \angle T = 105^{\circ}$. BC = 12
- **15.** In $\triangle EMZ$, $m \angle E = 40^{\circ}$, EZ = 7, and $m \angle M = 70^{\circ}$. In $\triangle DGP$, DP = 7, and $m \angle D = 40^{\circ}$. $m \angle G = 70^{\circ}$
- **16.** In $\triangle BMX$, $m \angle M = 90^{\circ}$, BM = 16, and $m \angle X = 15^{\circ}$. In $\triangle CNY$, $m \angle N = 90^{\circ}$, and $m \angle Y = 15^{\circ}$. CN = 16

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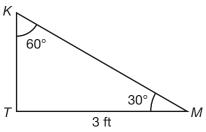
17.



18 in. 40°

 $m \angle Z = 70^{\circ}$

18.

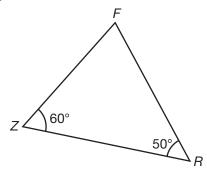


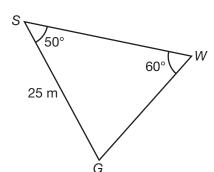
30° 60°

XN = 3 ft

19.

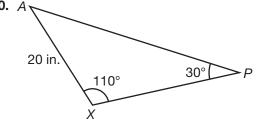
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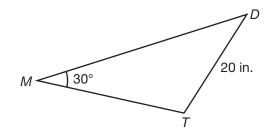


FR = 25 m

20. A

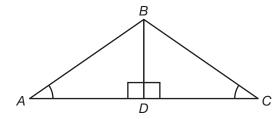


 $m \angle T = 110^{\circ}$



Determine whether there is enough information to prove that each pair of triangles are congruent by ASA or AAS. Write the congruence statements to justify your reasoning.

21. △*ABD* $\stackrel{?}{\cong}$ △*CBD*



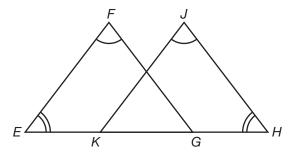
The triangles are congruent by AAS.

∠BAD ≅ ∠BCD

∠ADB ≅ ∠CDB

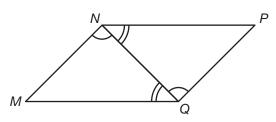
 $\overline{BD} \cong \overline{BD}$

22. $\triangle EFG \stackrel{?}{\cong} \triangle HJK$



There is not enough information to determine whether the triangles are congruent by ASA or AAS.

23. △*MNQ* [?] △*PQN*



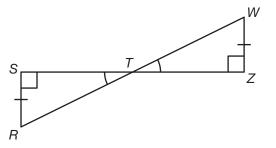
The triangles are congruent by ASA.

 $\angle MNQ \cong \angle PQN$

 $\overline{NQ} \cong \overline{QN}$

 $\angle MQN \cong \angle PNQ$

24. △*RST* $\stackrel{?}{\cong}$ △*WZT*



The triangles are congruent by AAS.

 $\angle RTS \cong \angle WTZ$

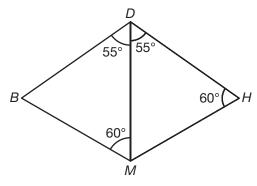
 $\angle RST \cong \angle WZT$

 $\overline{RS} \cong \overline{WZ}$

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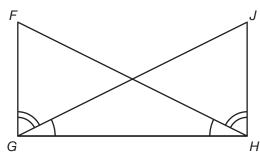
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25. △*BDM* $\stackrel{?}{\cong}$ △*MDH*



There is not enough information to determine whether the triangles are congruent by ASA or AAS.

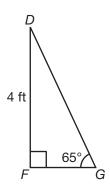
26. △*FGH* $\stackrel{?}{\cong}$ △*JHG*



The triangles are congruent by ASA.

$$\overline{GH} \cong \overline{HG}$$

27. $\triangle DFG \stackrel{?}{\cong} \triangle JMT$



65° 4 ft

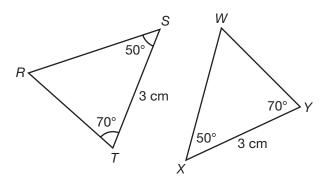
The triangles are congruent by AAS.

$$\angle DGF \cong \angle JTM$$

$$\angle DFG \cong \angle JMT$$

$$\overline{DF} \cong \overline{JM}$$

28. $\triangle RST \stackrel{?}{=} \triangle WXY$



The triangles are congruent by ASA.

$$\angle RST \cong \angle WXY$$

$$\overline{ST} \cong \overline{XY}$$

$$\angle RTS \cong \angle WYX$$

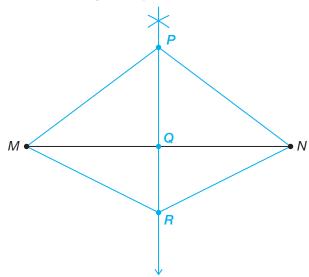
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Any Other Theorems You Forgot to Mention? **Using Congruent Triangles**

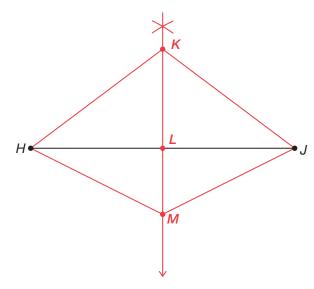
Problem Set

Construct a perpendicular bisector to each line segment. Connect points on the bisector on either side of the line segment to form the new line segment indicated.

1. \overline{MN} bisected by \overline{PR} at point Q

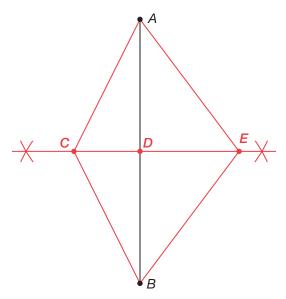


2. \overline{HJ} bisected by \overline{KM} at point L

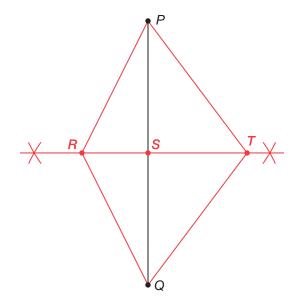


3. \overline{AB} bisected by \overline{CE} at point D

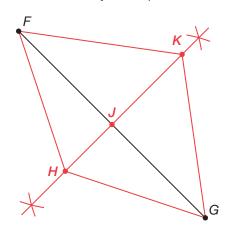
5.7



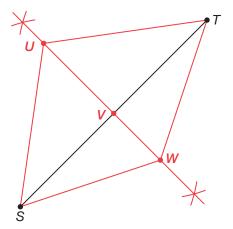
4. \overline{PQ} bisected by \overline{RT} at point S



5. \overline{FG} bisected by \overline{HK} at point J



6. \overline{ST} bisected by \overline{UW} at point V

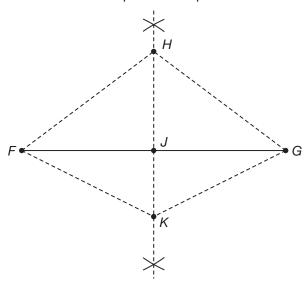


Name _ Date .

Use a triangle congruence theorem to complete each proof. Some of the statements and reasons are provided for you.

7. Given: \overline{HK} is a perpendicular bisector of \overline{FJ} at point J

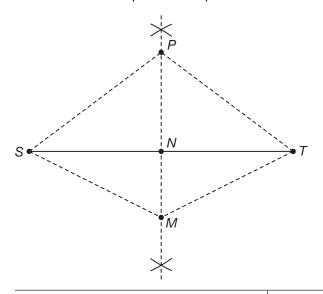
Prove: Point *H* is equidistant to points *F* and *G*



- 1. FG ⊥ HK, HK bisects FG
- 2. $\angle FJH$ and $\angle GJH$ are right angles.
- 3. $\angle FJH \cong \angle GJH$
- 4. $\overline{FJ} \cong \overline{GJ}$
- 5. $\overline{HJ} \cong \overline{HJ}$
- 6. $\triangle FJH \cong \triangle GJH$
- 7. *FH* ≅ *GH*
- 8. Point H is equidistant to points F and G

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- **6. SAS Congruence Theorem**
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

8. Given: \overline{PM} is a perpendicular bisector of \overline{ST} at point N Prove: Point M is equidistant to points S and T

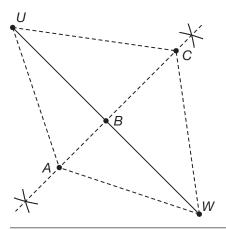


- 1. $\overline{ST} \perp \overline{PM}, \overline{PM}$ bisects \overline{ST}
- 2. \angle SNM and \angle TNM are right angles.
- 3. ∠SNM ≅ ∠TNM
- 4. $\overline{SN} \cong \overline{TN}$
- 5. $\overline{MN} \cong \overline{MN}$
- 6. $\triangle SNM \cong \triangle TNM$
- 7. $\overline{SM} \cong \overline{TM}$
- 8. Point M is equidistant to points S and T

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- 6. SAS Congruence Theorem
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

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9. Given: \overline{CA} is a perpendicular bisector of \overline{UW} at point B Prove: Point C is equidistant to points U and W

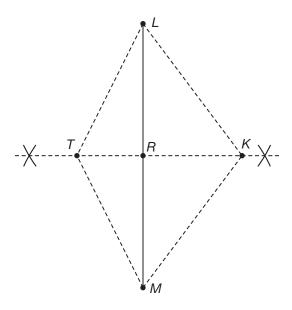


- 1. $\overline{UW} \perp \overline{CA}$, \overline{CA} bisects \overline{UW}
- 2. ∠UBC and ∠WBC are right angles.
- 3. ∠UBC ≅ ∠WBC
- 4. $\overline{UB} \cong \overline{WB}$
- 5. $\overline{CB} \cong \overline{CB}$
- 6. $\triangle UBC \cong \triangle WBC$
- 7. $\overline{UC} \cong \overline{WC}$
- 8. Point C is equidistant to points U and W

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- 6. SAS Congruence Theorem
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

10. Given: \overline{TK} is a perpendicular bisector of \overline{LM} at point R

Prove: Point T is equidistant to points L and M

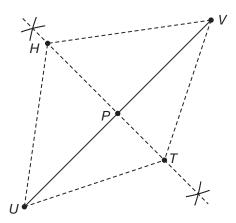


- 1. $\overline{LM} \perp \overline{TK}, \overline{TK}$ bisects \overline{LM}
- 2. $\angle LRT$ and $\angle MRT$ are right angles.
- 3. $\angle LRT \cong \angle MRT$
- 4. $\overline{LR} \cong \overline{MR}$
- 5. $\overline{TR} \cong \overline{TR}$
- 6. $\triangle LRT \cong \triangle MRT$
- 7. $\overline{LR} \cong \overline{MT}$
- 8. Point *T* is equidistant to points L and M

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- 6. SAS Congruence Theorem
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

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11. Given: \overline{HT} is a perpendicular bisector of \overline{UV} at point PProve: Point H is equidistant to points U and V



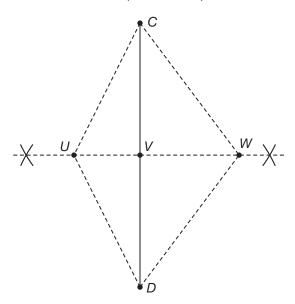
- 1. $\overline{UV} \perp \overline{HT}, \overline{HT}$ bisects \overline{UV}
- **2.** $\angle UPH$ and $\angle VPH$ are right angles.
- 3. ∠UPH ≅ ∠VPH
- 4. $\overline{UP} \cong \overline{VP}$
- 5. $\overline{HP} \cong \overline{HP}$
- 6. $\triangle UPH \cong \triangle VPH$
- 7. $\overline{UH} \cong \overline{VH}$
- 8. Point H is equidistant to points U and V

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- 6. SAS Congruence Theorem
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

Pro

12. Given: \overline{UW} is a perpendicular bisector of \overline{CD} at point V

Prove: Point *U* is equidistant to points *C* and *D*



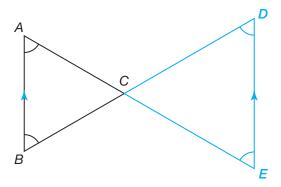
- 1. CD \(\perp \) UW, UW bisects CD
- 2. $\angle CVU$ and $\angle DVU$ are right angles.
- 3. $\angle CVU \cong \angle DVU$
- 4. $\overline{CV} \cong \overline{DV}$
- 5. $\overline{UV} \cong \overline{UV}$
- 6. $\triangle CVU \cong \triangle DVU$
- 7. $\overline{CU} \cong \overline{DU}$
- 8. Point *U* is equidistant to points *C* and *D*

- 1. Definition of perpendicular bisector
- 2. Definition of perpendicular lines
- 3. All right angles are congruent.
- 4. Definition of bisect
- 5. Reflexive Property
- 6. SAS Congruence Theorem
- 7. Corresponding sides of congruent triangles are congruent
- 8. Definition of equidistant

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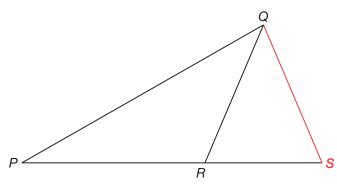
Complete each diagram to provide a counterexample that proves the indicated theorem does not work for congruent triangles. Explain your reasoning. A hint is provided in each case.

13. Angle-Angle-Angle (Hint: Use vertical angles.)



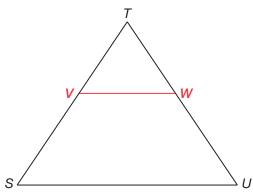
Extend \overline{AC} and \overline{BC} , and connect points D and E so that \overline{AB} is parallel to \overline{DE} . Since vertical angles are congruent, all three corresponding angles of the two triangles are congruent. The side lengths, however, are different, so $\triangle ABC$ is not congruent to $\triangle DEC$.

14. Side-Side-Angle (Hint: Draw a triangle that shares $\angle P$ with the given triangle.)



Angle P and \overline{PQ} are the same for both triangles. We can draw QS the same length as \overline{QR} . Then $\triangle PQR$ and $\triangle PQS$ have side-side-angle congruency, but since $\angle QRS$ is not congruent to $\angle QSR$, the triangles are not congruent.

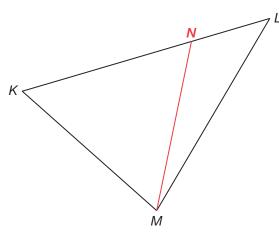
15. Angle-Angle (Hint: Draw a triangle that shares $\angle T$ with the given triangle.)



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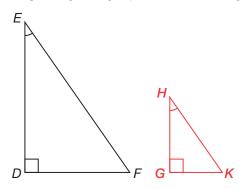
Draw \overline{VW} parallel to \overline{SU} . Since alternative interior angles are congruent, $\angle V \cong \angle S$ and $\angle W \cong \angle U$. All three corresponding angles of the two triangles are congruent. The side lengths, however, are different, so $\triangle STU$ is not congruent to $\triangle VTW$.

16. Side-Side-Angle (Hint: Draw a triangle that shares $\angle L$ with the given triangle.)



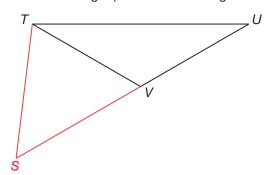
Angle L and \overline{LM} are the same for both triangles. We can draw \overline{MN} the same length as \overline{KM} . Then $\triangle LMK$ and $\triangle LMN$ have side-side-angle congruency, but since ∠LMK is not congruent to ∠LMN, the triangles are not congruent.

17. Angle-Angle (Hint: Draw a triangle that has an angle with the same measure as $\angle E$.)



Draw $\triangle GHK$ such that $\angle G = 90^{\circ}$ and $\angle H \cong \angle E$. Since right angles are congruent, $\angle D \cong \angle G$. Since the two triangles are two corresponding angles, we also know that $\angle F \cong \angle K$. All three corresponding angles of the two triangles are congruent. The side lengths, however, are different, so $\triangle DEF$ is not congruent to $\triangle GHK$.

18. Side-Side-Angle (Hint: Draw a triangle that shares $\angle U$ with the given triangle.)



Angle U and \overline{TU} are the same for both triangles. We can draw \overline{ST} the same length as \overline{TV} . Then $\triangle STU$ and $\triangle VTU$ have side-side-angle congruency, but since ∠STU is not congruent to ∠VTU, the triangles are not congruent.

Date.

State the congruence theorem that proves the triangles in each diagram are congruent. If not enough information is given, name an example of information that could be given that you could use to prove congruency. Explain your reasoning.

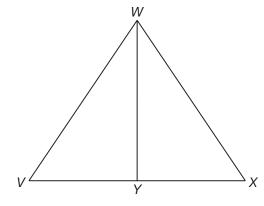
19. Given: $\overline{VW} \cong \overline{XW}$

Prove: $\triangle VYW \cong \triangle XYW$

Not enough information is given.

If $\angle VWY \cong \angle XWY$ is given, then $\triangle VYW \cong \triangle XYW$

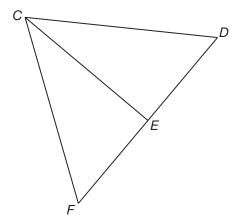
by the SAS Triangle Congruence Theorem.



20. Given: \overline{CE} is a perpendicular bisector of \overline{FD}

Prove: $\triangle FEC \cong \triangle DEC$

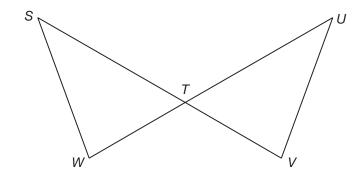
 $\overline{CE} \cong \overline{DE}$ by the Reflexive Property. $\triangle FEC \cong \triangle DEC$ using the SAS Triangle Congruence Theorem.



21. Given: $\angle WST \cong \angle VUT$, $\overline{ST} \cong \overline{UT}$

Prove: $\triangle WST \cong \triangle VUT$

 $\angle STW \cong \angle UTV$ because they are vertical angles. $\triangle WST \cong \triangle VUT$ using the ASA Triangle Congruence Theorem.



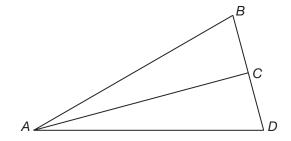
22. Given: $\triangle ABD$ is isosceles with $\overline{AB} \cong \overline{AD}$

Prove: $\triangle ABC \cong \triangle ADC$

Not enough information is given.

If $\angle BAC \cong \angle DAC$ is given, then $\triangle ABC \cong \triangle ADC$

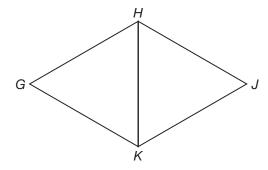
by the SAS Triangle Congruence Theorem.



23. Given: $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$

Prove: $\triangle GHK \cong \triangle JKH$

 $\overline{HK} \cong \overline{HK}$ by the Reflexive Property. $\triangle GHK \cong \triangle JKH$ using the SSS Triangle Congruence Theorem.



24. Given: $\overline{PT} \cong \overline{SR}$, $\angle PQT \cong \angle RQS$

Prove: $\triangle TPQ \cong \triangle SRQ$

Since right angles are congruent,

 $\angle PTQ \cong \angle RSQ$. $\triangle GHK \cong \triangle JKH$ using the AAS

Triangle Congruence Theorem.

