Name ____ Date ____________

Time to Get Right
Right Triangle Congruence Theorems

Vocabulary

Choose the diagram that models each right triangle congruence theorem.

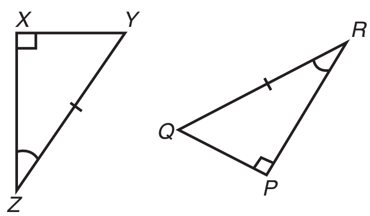
1. Hypotenuse-Leg (HL) Congruence Theorem
   b

2. Leg-Leg (LL) Congruence Theorem
   d

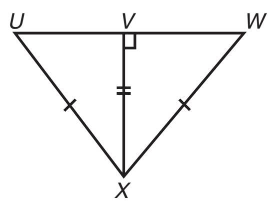
3. Hypotenuse-Angle (HA) Congruence Theorem
   a

4. Leg-Angle (LA) Congruence Theorem
   c

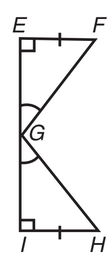
a.



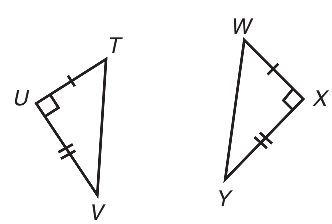
b.



c.



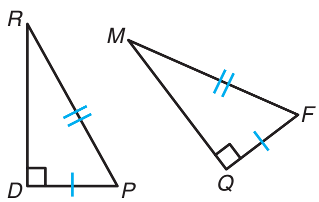
d.



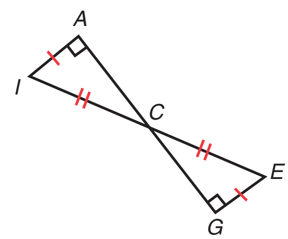
Problem Set

Mark the appropriate sides to make each congruence statement true by the Hypotenuse-Leg Congruence Theorem.

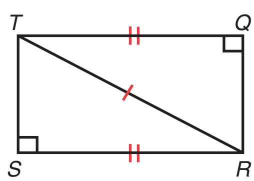
1. $\triangle DPR \cong \triangle QFM$



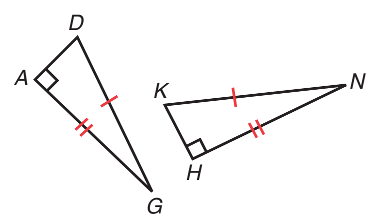
2. $\triangle ACI \cong \triangle GCE$



3. $\triangle QTR \cong \triangle SRT$

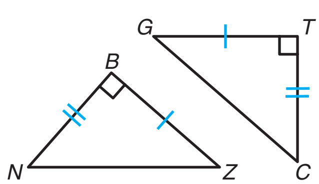


4. $\triangle ADG \cong \triangle HKN$

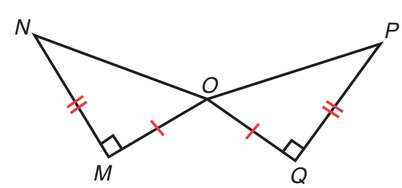


Mark the appropriate sides to make each congruence statement true by the Leg-Leg Congruence Theorem.

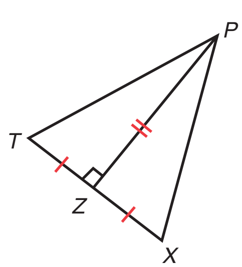
5. $\triangle BZN \cong \triangle TGC$



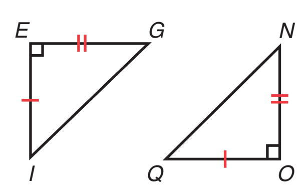
6. $\triangle MNO \cong \triangle QPO$



7. $\triangle PZT \cong \triangle PZX$



8. $\triangle EGI \cong \triangle ONQ$



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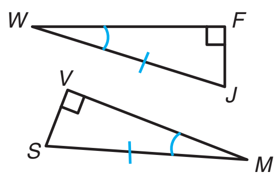
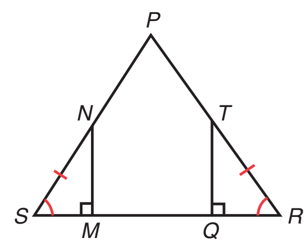
Mark the appropriate sides and angles to make each congruence statement true by the Hypotenuse-Angle Congruence Theorem.

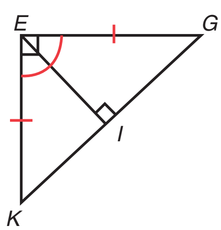
9. $\triangle SVM \cong \triangle JFW$



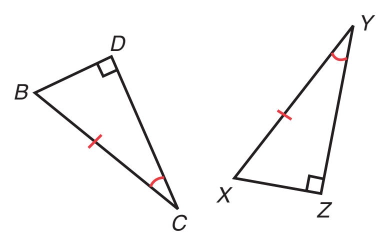
10. $\triangle MSN \cong \triangle QRT$



11. $\triangle IEG \cong \triangle IEK$

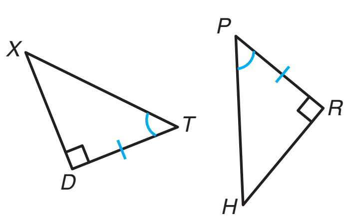


12. $\triangle DCB \cong \triangle ZYX$

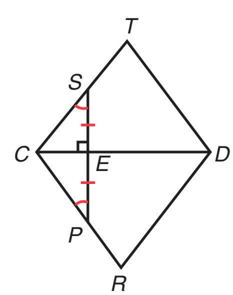


Mark the appropriate sides and angles to make each congruence statement true by the Leg-Angle Congruence Theorem.

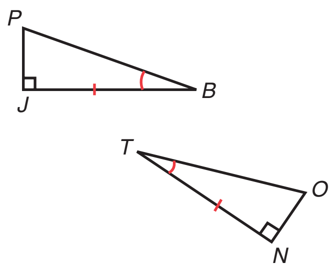
13. $\triangle XTD \cong \triangle HPR$



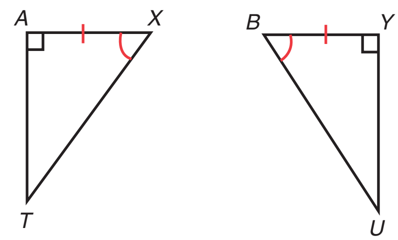
14. $\triangle SEC \cong \triangle PEC$



15. $\triangle PBJ \cong \triangle OTN$



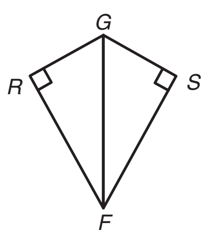
16. $\triangle AXT \cong \triangle YBU$



For each figure, determine if there is enough information to prove that the two triangles are congruent. If so, name the congruence theorem used.

17. Given: $\overline{GF}$ bisects $\angle RGS$, and $\angle R$ and $\angle S$ are right angles.

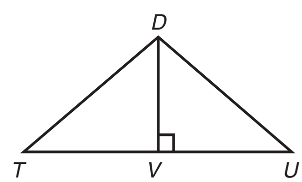
Is $\triangle FRG \cong \triangle FSG$?



Yes. There is enough information to conclude that $\triangle FRG \cong \triangle FSG$ by HA.

18. Given: $\overline{DV} \perp \overline{TU}$

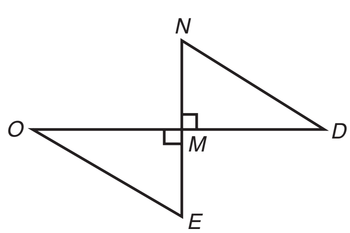
Is $\triangle DVT \cong \triangle DVU$?



No. $\triangle DVT$ might not be congruent to $\triangle DVU$. There is not enough information.

19. Given: $\overline{NM} \cong \overline{EM}$, $\overline{DM} \cong \overline{OM}$, and $\angle NMD$ and $\angle EMO$ are right angles.

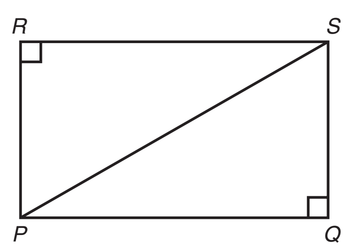
Is $\triangle NMD \cong \triangle EMO$?



Yes. There is enough information to conclude that $\triangle NMD \cong \triangle EMO$ by LL.

20. Given: $\overline{RP} \cong \overline{QS}$, and $\angle R$ and $\angle Q$ are right angles.

Is $\triangle SRP \cong \triangle PQS$?



Yes. There is enough information to conclude that $\triangle SRP \cong \triangle PQS$ by HL.

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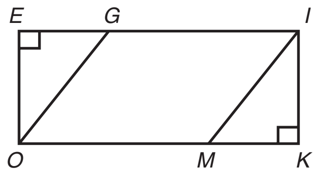
21. Given: $\overline{GO} \cong \overline{MI}$, and $\angle E$ and $\angle K$ are right angles.

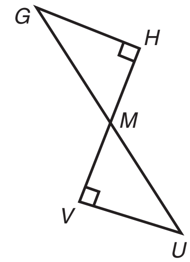
Is $\triangle GEO \cong \triangle MKI$?



No. $\triangle GEO$ might not be congruent to $\triangle MKI$. There is not enough information.

22. Given: $\overline{HM} \cong \overline{VM}$, and $\angle H$ and $\angle V$ are right angles.

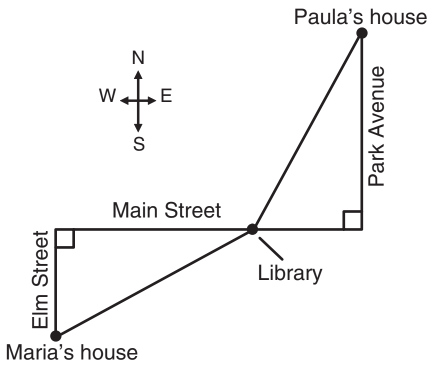
Is $\triangle GHM \cong \triangle UVM$?



Yes. There is enough information to conclude that $\triangle GHM \cong \triangle UVM$ by LA.

Use the given information to answer each question.

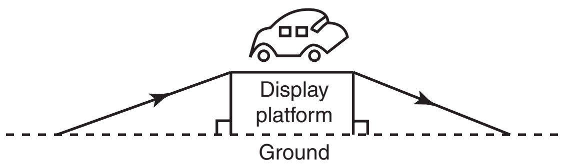
23. Two friends are meeting at the library. Maria leaves her house and walks north on Elm Street and then east on Main Street to reach the library. Paula leaves her house and walks south on Park Avenue and then west on Main Street to reach the library. Maria walks the same distance on Elm Street as Paula walks on Main Street, and she walks the same distance on Main Street as Paula walks on Park Avenue. Is there enough information to determine whether Maria's walking distance is the same as Paula's walking distance?



Yes. Maria's walking distance to the library is equal to Paula's walking distance. The triangles formed are right triangles. The corresponding legs of the triangles are congruent. So, by the Leg-Leg Congruence Theorem, the triangles are congruent. If the triangles are congruent, the hypotenuses are congruent.

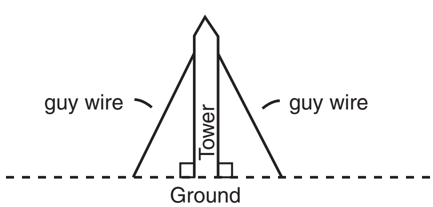
6

24. An auto dealership displays one of their cars by driving it up a ramp onto a display platform. Later they will drive the car off the platform using a ramp on the opposite side. Both ramps form a right triangle with the ground and the platform. Is there enough information to determine whether the two ramps have the same length? Explain.



No. There is not enough information to determine whether the two ramps have the same length. The triangles formed by the ramps, the vertical sides of the platform, and the ground are right triangles. But, the lengths of the legs, the lengths of the hypotenuses, and the measures of the acute angles are unknown.

25. A radio station erected a new transmission antenna to provide its listeners with better reception. The antenna was built perpendicular to the ground, and to keep the antenna from swaying in the wind two guy wires were attached from it to the ground on opposite sides of the antenna. Is there enough information to determine if the guy wires have the same length? Explain.



No. There is not enough information to determine whether the guy wires have the same length. The triangles formed by the antenna, the guy wires, and the ground are right triangles. But, the lengths of the legs, the lengths of the hypotenuses, and the measures of the acute angles are unknown.

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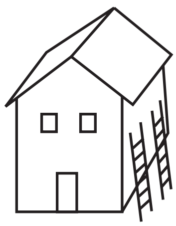
26. Two ladders resting on level ground are leaning against the side of a house. The bottom of each ladder is exactly 2.5 feet directly out from the base of the house. The point at which each ladder rests against the house is 10 feet directly above the base of the house. Is there enough information to determine whether the two ladders have the same length? Explain.

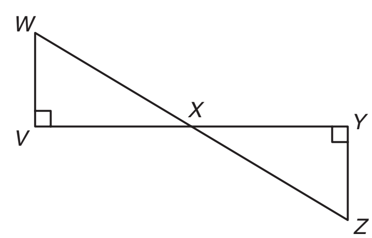


Yes. The triangles formed by the ladders, the ground, and the side of the house are right triangles. Each leg of one triangle is congruent to the corresponding leg of the other triangle, making the two triangles congruent by LL. The ladders form the hypotenuses of the triangles. Since the triangles are congruent, the hypotenuses are congruent. Therefore, the ladders have the same length.

Create a two-column proof to prove each statement.

27. Given: $\overline{WZ}$ bisects $\overline{VY}$, $\overline{WV} \perp \overline{VY}$, and $\overline{YZ} \perp \overline{VY}$

Prove: $\triangle WVX \cong \triangle ZYX$

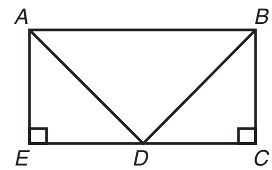


Statements	Reasons
1. $\overline{WV} \perp \overline{VY}$ and $\overline{YZ} \perp \overline{VY}$	1. Given
2. $\angle WVX$ and $\angle ZYX$ are right angles.	2. Definition of perpendicular angles
3. $\triangle WVX$ and $\triangle ZYX$ are right triangles.	3. Definition of right triangles
4. $\overline{WZ}$ bisects $\overline{VY}$.	4. Given
5. $\overline{VX} \cong \overline{YX}$	5. Definition of segment bisector
6. $\angle WXV \cong \angle ZXY$	6. Vertical Angle Theorem
7. $\triangle WVX \cong \triangle ZYX$	7. LA Congruence Theorem

6

28. Given: Point D is the midpoint of $\overline{EC}$, $\triangle ADB$ is an isosceles triangle with base $\overline{AB}$, and $\angle E$ and $\angle C$ are right angles.

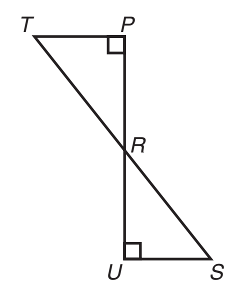
Prove: $\triangle AED \cong \triangle BCD$



Statements	Reasons
1. $\angle E$ and $\angle C$ are right angles.	1. Given
2. $\triangle AED$ and $\triangle BCD$ are right triangles.	2. Definition of right triangles
3. Point D is the midpoint of $\overline{EC}$.	3. Given
4. $\overline{ED} \cong \overline{CD}$	4. Definition of midpoint
5. $\triangle ADB$ is an isosceles triangle with base $\overline{AB}$.	5. Given
6. $\overline{AD} \cong \overline{BD}$	6. Definition of isosceles triangle
7. $\triangle AED \cong \triangle BCD$	7. HL Congruence Theorem

29. Given: $\overline{SU} \perp \overline{UP}$, $\overline{TP} \perp \overline{UP}$, and $\overline{UR} \cong \overline{PR}$

Prove: $\triangle SUR \cong \triangle TPR$



Statements	Reasons
1. $\overline{SU} \perp \overline{UP}$ and $\overline{TP} \perp \overline{UP}$	1. Given
2. $\angle U$ and $\angle P$ are right angles.	2. Definition of perpendicular lines
3. $\triangle SUR$ and $\triangle TPR$ are right triangles.	3. Definition of right triangles
4. $\overline{UR} \cong \overline{PR}$	4. Given
5. $\angle PRT$ and $\angle URS$ are vertical angles.	5. Definition of vertical angles
6. $\angle PRT \cong \angle URS$	6. Vertical Angle Theorem
7. $\triangle SUR \cong \triangle TPR$	7. LA Congruence Theorem

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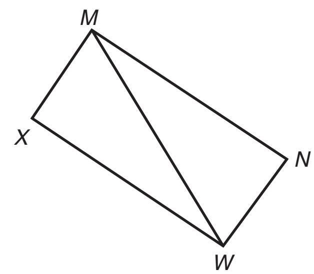
30. Given: Rectangle *MNWX* and $\angle NMW \cong \angle XWM$

Prove: $\triangle MNW \cong \triangle WXM$



Statements	Reasons
1. *MNWX* is a rectangle	1. Given
2. $\angle N$ and $\angle X$ are right angles.	2. Definition of rectangle
3. $\triangle MNW$ and $\triangle WXM$ are right triangles.	3. Definition of right triangles
4. $MW = WM$	4. Reflexive Property of Equality
5. $\overline{MW} \cong \overline{WM}$	5. Definition of congruent segments
6. $\angle NMW \cong \angle XWM$	6. Given
7. $\triangle MNW \cong \triangle WXM$	7. HA Congruence Theorem

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CPCTC
Corresponding Parts of Congruent Triangles are Congruent

Vocabulary

Provide an example to illustrate each term.

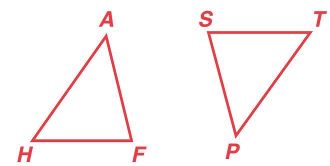
1. Corresponding parts of congruent triangles are congruent (CPCTC)

   Examples will vary.

   Given: $\triangle AFH \cong \triangle PST$

   The corresponding congruent parts are:

   $\angle A \cong \angle P$, $\angle F \cong \angle S$, $\angle H \cong \angle T$,
   $\overline{AF} \cong \overline{PS}$, $\overline{FH} \cong \overline{ST}$, $\overline{HA} \cong \overline{TP}$

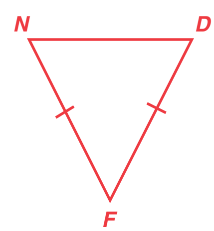


2. Isosceles Triangle Base Angle Theorem

   Examples will vary.

   Given: $\triangle FND$ with $\overline{FN} \cong \overline{FD}$

   The Isosceles Triangle Base Angle Theorem states that $\angle N \cong \angle D$.

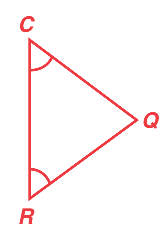


3. Isosceles Triangle Base Angle Converse Theorem

   Examples will vary.

   Given: $\triangle RCQ$ with $\angle R \cong \angle C$

   The Isosceles Triangle Base Angle Converse Theorem states that $\overline{QC} \cong \overline{QR}$.

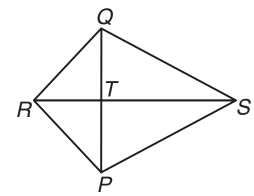


Problem Set

Create a two-column proof to prove each statement.

1. Given: $\overline{RS}$ is the $\perp$ bisector of $\overline{PQ}$.

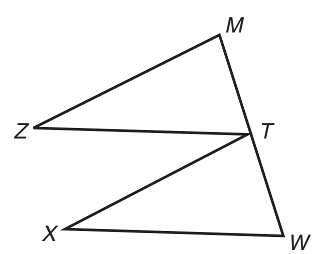
Prove: $\angle SPT \cong \angle SQT$



Statements	Reasons
1. $\overline{RS}$ is the $\perp$ bisector of $\overline{PQ}$.	1. Given
2. $\overline{RS} \perp \overline{PQ}$	2. Definition of perpendicular bisector
3. $\angle PTS$ and $\angle QTS$ are right angles.	3. Definition of perpendicular lines
4. $\triangle PTS$ and $\triangle QTS$ are right triangles.	4. Definition of right triangles
5. $\overline{RS}$ bisects $\overline{PQ}$	5. Definition of perpendicular bisector
6. $\overline{PT} \cong \overline{QT}$	6. Definition of bisect
7. $\overline{TS} \cong \overline{TS}$	7. Reflexive Property of $\cong$
8. $\triangle PTS \cong \triangle QTS$	8. Leg-Leg Congruence Theorem
9. $\angle SPT \cong \angle SQT$	9. CPCTC

2. Given: $\overline{TZ} \cong \overline{WX}$, $\overline{TM} \cong \overline{WT}$, and $\overline{TZ} \parallel \overline{WX}$

Prove: $\overline{MZ} \cong \overline{TX}$



Statements	Reasons
1. $\overline{TZ} \cong \overline{WX}$	1. Given
2. $\overline{TM} \cong \overline{WT}$	2. Given
3. $\overline{TZ} \parallel \overline{WX}$	3. Given
4. $\angle MTZ$ and $\angle TWX$ are corresponding angles.	4. Definition of corresponding angles
5. $\angle MTZ \cong \angle TWX$	5. Corresponding Angles Postulate
6. $\triangle MTZ \cong \triangle TWX$	6. SAS Congruence Theorem
7. $\overline{MZ} \cong \overline{TX}$	7. CPCTC

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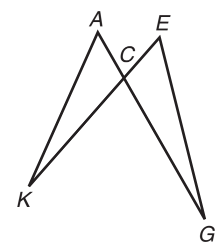
3. Given: $\overline{AG}$ and $\overline{EK}$ intersect at C, $\overline{AC} \cong \overline{EC}$, $\overline{CK} \cong \overline{CG}$

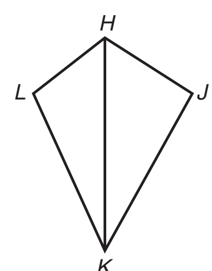
Prove: $\angle K \cong \angle G$



Statements	Reasons
1. $\overline{AG}$ and $\overline{EK}$ intersect at C	1. Given
2. $\overline{AC} \cong \overline{EC}$	2. Given
3. $\overline{CK} \cong \overline{CG}$	3. Given
4. $\angle ACK \cong \angle ECG$	4. Vertical Angles Theorem
5. $\triangle ACK \cong \triangle ECG$	5. SAS Congruence Theorem
6. $\angle K \cong \angle G$	6. CPCTC

4. Given: $\angle JHK \cong \angle LHK$, $\angle JKH \cong \angle LKH$

Prove: $\overline{JK} \cong \overline{LK}$



Statements	Reasons
1. $\angle JHK \cong \angle LHK$	1. Given
2. $\angle JKH \cong \angle LKH$	2. Given
3. $\overline{HK} \cong \overline{HK}$	3. Reflexive Property of $\cong$
4. $\triangle HJK \cong \triangle HLK$	4. ASA Congruence Theorem
5. $\overline{JK} \cong \overline{LK}$	5. CPCTC

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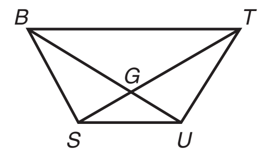
5. Given: $\triangle UGT \cong \triangle SGB$

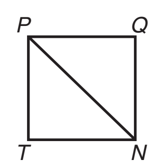
Prove: $\angle TUS \cong \angle BSU$



Statements	Reasons
1. $\triangle UGT \cong \triangle SGB$	1. Given
2. $\overline{TU} \cong \overline{BS}$	2. CPCTC
3. $\overline{SG} \cong \overline{UG}$	3. CPCTC
4. $\overline{GT} \cong \overline{GB}$	4. CPCTC
5. $SG = UG$	5. Definition of congruent segments
6. $GT = GB$	6. Definition of congruent segments
7. $SG + GT = UG + GB$	7. Addition Property of Equality
8. $SG + GT = ST$	8. Segment Addition Postulate
9. $UG + GB = UB$	9. Segment Addition Postulate
10. $ST = UB$	10. Substitution Property
11. $\overline{ST} \cong \overline{UB}$	11. Definition of congruent segments
12. $\angle STU \cong \angle UBS$	12. CPCTC
13. $\triangle STU \cong \triangle UBS$	13. SAS Congruence Theorem
14. $\angle TUS \cong \angle BSU$	14. CPCTC

6. Given: $\angle TPN \cong \angle TNP$, $\overline{TP} \cong \overline{QP}$

Prove: $\overline{TN} \cong \overline{QP}$



Statements	Reasons
1. $\angle TPN \cong \angle TNP$	1. Given
2. $\overline{TN} \cong \overline{TP}$	2. Base Angle Converse Theorem
3. $\overline{TP} \cong \overline{QP}$	3. Given
4. $\overline{TN} \cong \overline{QP}$	4. Transitive Property of $\cong$

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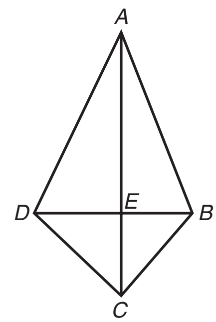
7. Given: $\overline{AC} \perp \overline{DB}$, $\overline{AC}$ bisects $\overline{DB}$

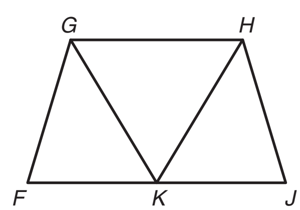
Prove: $\overline{AD} \cong \overline{AB}$



Statements	Reasons
1. $\overline{AC} \perp \overline{DB}$	1. Given
2. $\angle DEA$ is a right angle.	2. Definition of perpendicular lines
3. $\angle BEA$ is a right angle.	3. Definition of perpendicular lines
4. $\triangle DEA$ is a right triangle.	4. Definition of right triangle
5. $\triangle BEA$ is a right triangle.	5. Definition of right triangle
6. $\overline{AC}$ bisects $\overline{DB}$	6. Given
7. $\overline{DE} \cong \overline{BE}$	7. Definition of bisect
8. $\overline{AE} \cong \overline{AE}$	8. Reflexive Property of $\cong$
9. $\triangle DEA \cong \triangle BEA$	9. Leg-Leg Congruence Theorem
10. $\overline{AD} \cong \overline{AB}$	10. CPCTC

8. Given: $\angle KGH \cong \angle KHG$, $\overline{FG} \cong \overline{JH}$, $\overline{FK} \cong \overline{JK}$

Prove: $\angle F \cong \angle J$



Statements	Reasons
1. $\angle KGH \cong \angle KHG$	1. Given
2. $\overline{GK} \cong \overline{HK}$	2. Base Angle Converse Theorem
3. $\overline{FG} \cong \overline{JH}$	3. Given
4. $\overline{FK} \cong \overline{JK}$	4. Given
5. $\triangle FGK \cong \triangle JHK$	5. SSS Congruence Theorem
6. $\angle F \cong \angle J$	6. CPCTC

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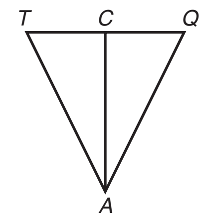
9. Given: $\overline{AT} \cong \overline{AQ}$, $\overline{AC}$ bisects $\angle TAQ$

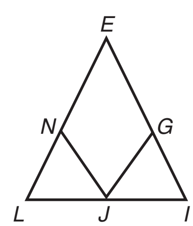
Prove: $\overline{AC}$ bisects $\overline{TQ}$



Statements	Reasons
1. $\overline{AT} \cong \overline{AQ}$	1. Given
2. $\angle T \cong \angle Q$	2. Base Angle Theorem
3. $\overline{AC}$ *bisects* $\angle TAQ$	3. Given
4. $\angle TAC \cong \angle QAC$	4. Definition of bisect
5. $\triangle TAC \cong \triangle QAC$	5. ASA Congruence Theorem
6. $\overline{TC} \cong \overline{QC}$	6. CPCTC
7. $\overline{AC}$ bisects $\overline{TQ}$	7. Definition of bisect

10. Given: $\overline{EL} \cong \overline{EI}$, $\angle LNJ \cong IGJ$, J is the midpoint of $\overline{LI}$

Prove: $\overline{NJ} \cong \overline{GJ}$



Statements	Reasons
1. $\overline{EL} \cong \overline{EI}$	1. Given
2. $\angle ELJ \cong \angle EIJ$	2. Base Angle Theorem
3. J is the midpoint of $\overline{LI}$.	3. Given
4. $\overline{LJ} \cong \overline{IJ}$	4. Definition of midpoint
5. $\angle LNJ \cong \angle IGJ$	5. Given
6. $\triangle LNJ \cong \triangle IGJ$	6. AAS Congruence Theorem
7. $\overline{NJ} \cong \overline{GJ}$	7. CPCTC

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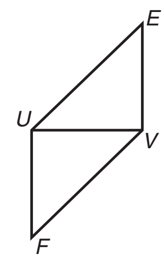
11. Given: $\angle E \cong \angle EUV$, $\angle F \cong \angle FVU$

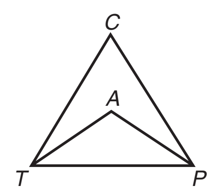
Prove: $\overline{UF} \cong \overline{VE}$



Statements	Reasons
1. $\angle E \cong \angle EUV$	1. Given
2. $\overline{VU} \cong \overline{VE}$	2. Base Angle Converse Theorem
3. $\angle F \cong \angle FVU$	3. Given
4. $\overline{UF} \cong \overline{VU}$	4. Base Angle Converse Theorem
5. $\overline{UF} \cong \overline{VE}$	5. Transitive Property of $\cong$

12. Given: $\overline{CT} \cong \overline{CP}$, $\overline{AT} \cong \overline{AP}$

Prove: $m\angle CTA = m\angle CPA$

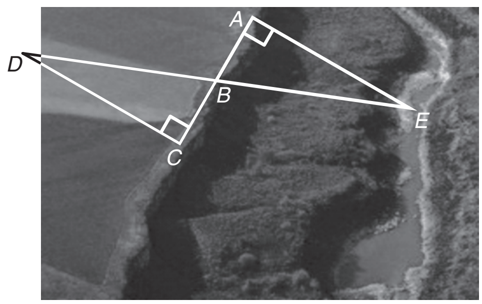


Statements	Reasons
1. $\overline{CT} \cong \overline{CP}$	1. Given
2. $\angle CTP \cong \angle CPT$	2. Base Angle Theorem
3. $m\angle CTP = m\angle CPT$	3. Definition of congruent angles
4. $\overline{AT} \cong \overline{AP}$	4. Given
5. $\angle ATP \cong \angle APT$	5. Base Angle Theorem
6. $m\angle ATP = m\angle APT$	6. Definition of congruent angles
7. $m\angle CTP = m\angle CTA + m\angle ATP$	7. Angle Addition Postulate
8. $m\angle CPT = m\angle CPA + m\angle APT$	8. Angle Addition Postulate
9. $m\angle CTA + m\angle ATP = m\angle CPA + m\angle APT$	9. Substitution Property
10. $m\angle CTA = m\angle CPA$	10. Subtraction Property of Equality

6

Use the given information to answer each question.

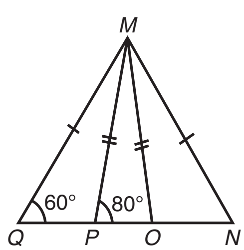
13. Samantha is hiking through the forest and she comes upon a canyon. She wants to know how wide the canyon is. She measures the distance between points *A* and *B* to be 35 feet. Then, she measures the distance between points *B* and *C* to be 35 feet. Finally, she measures the distance between points *C* and *D* to be 80 feet. How wide is the canyon? Explain.



The canyon is 80 feet wide.

The triangles are congruent by the Leg-Angle Congruence Theorem. Corresponding parts of congruent triangles are congruent, so $\overline{CD} \cong \overline{AE}$.

14. Explain why $m\angle NMO = 20°$.



Using $\triangle QMN$ and the Base Angle Theorem, $m\angle MNO = 60°$. Using $\triangle PMO$ and the Base Angle Theorem, $m\angle POM = 80°$. Since $\angle POM$ and $\angle MON$ are supplementary, $m\angle MON = 100°$. Since the sum of the measures of the angles in a triangle is 180°, $m\angle NMO = 20°$.

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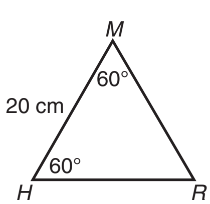
15. Calculate *MR* given that the perimeter of $\triangle HMR$ is 60 centimeters.



$MR = 20$ cm. Using the Base Angle Converse Theorem, $MR = HR$. Solve the perimeter equation $x + x + 20 = 60$, where $x = MR$ and $x = HR$. So, $x = 20$.

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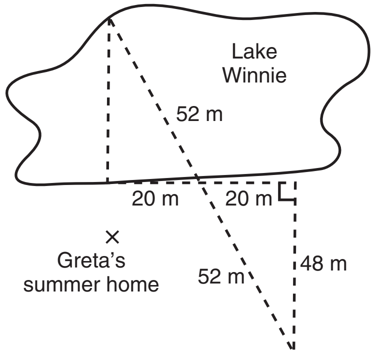
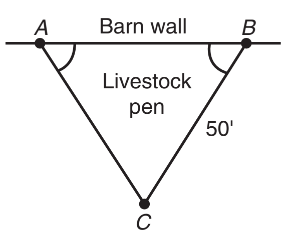
16. Greta has a summer home on Lake Winnie. Using the diagram, how wide is Lake Winnie?



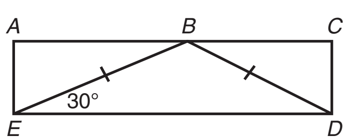
Lake Winnie is 80 feet wide. The triangles are congruent by the Hypotenuse-Leg Congruence Theorem and corresponding parts of congruent triangles are congruent, so the width of Lake Winnie is equal to the length of the 48 meter leg of the triangle that is displayed below the lake.

17. Jill is building a livestock pen in the shape of a triangle. She is using one side of a barn for one of the sides of her pen and has already placed posts in the ground at points *A*, *B*, and *C*, as shown in the diagram. If she places fence posts every 10 feet, how many more posts does she need? Note: There will be no other posts placed along the barn wall.



Eight posts are needed to complete the fence. Using the Base Angle Converse Theorem, I know the length of side *AC* is equal to the length of side *BC*. She will need four more posts for side *AC* and four more posts for side *BC*.

18. Given rectangle *ACDE*, calculate the measure of $\angle CDB$.



The measure of $m\angle CDB = 60°$. Using the Base Angle Theorem, $m\angle BDE = 30°$. Since *ACDE* is a rectangle, $m\angle CDE = 90°$. So $m\angle CDB = m\angle CDE - m\angle BDE = 90° - 30° = 60°$.

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Congruence Theorems in Action
Isosceles Triangle Theorems

Vocabulary

Choose the term from the box that best completes each sentence.

Isosceles Triangle Altitude to Congruent Sides Theorem	Isosceles Triangle Base Theorem
Isosceles Triangle Vertex Angle Theorem	vertex angle
Isosceles Triangle Perpendicular Bisector Theorem	Isosceles Triangle Angle Bisector to Congruent Sides Theorem

1. A(n) <u>vertex angle</u> is the angle formed by the two congruent legs in an isosceles triangle.

2. In an isosceles triangle, the altitudes to the congruent sides are congruent, as stated in the <u>Isosceles Triangle Altitude to Congruent Sides Theorem</u>.

3. In an isosceles triangle, the angle bisectors to the congruent sides are congruent, as stated in the <u>Isosceles Triangle Angle Bisector to Congruent Sides Theorem</u>.

4. The <u>Isosceles Triangle Perpendicular Bisector Theorem</u> states that the altitude from the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

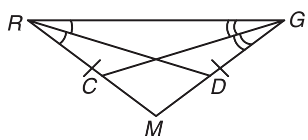
5. The <u>Isosceles Triangle Base Theorem</u> states that the altitude to the base of an isosceles triangle bisects the base.

6. The altitude to the base of an isosceles triangle bisects the vertex angle, as stated in the <u>Isosceles Triangle Vertex Angle Theorem</u>.

Problem Set

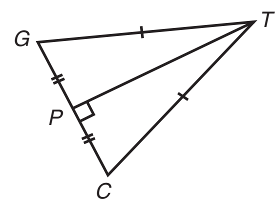
Write the theorem that justifies the truth of each statement.

1. In isosceles $\triangle MRG$, $\overline{RD} \cong \overline{GC}$.



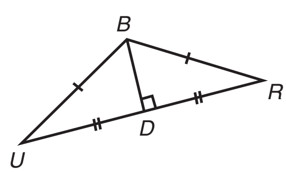
Isosceles Triangle Angle Bisector to Congruent Sides Theorem

2. In isosceles $\triangle TGC$ with altitude $\overline{TP}$, $\overline{TP} \perp \overline{GC}$, and $\overline{GP} \cong \overline{CP}$.



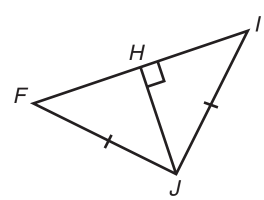
Isosceles Triangle Perpendicular Bisector Theorem

3. In isosceles $\triangle BRU$ with altitude $\overline{BD}$, $\overline{UD} \cong \overline{RD}$.



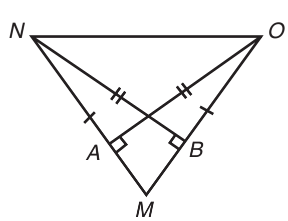
Isosceles Triangle Base Theorem

4. In isosceles $\triangle JFI$ with altitude $\overline{JH}$, $\angle HJF \cong \angle HJI$.



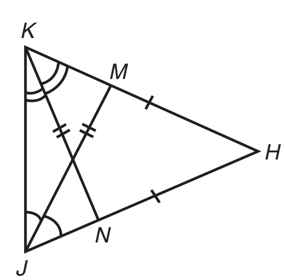
Isosceles Triangle Vertex Angle Theorem

5. In isosceles $\triangle MNO$, $\overline{OA} \cong NB$.



Isosceles Triangle Altitude to Congruent Sides Theorem

6. In isosceles $\triangle HJK$, $\overline{KN}$ bisects $\angle HKJ$, $\overline{JM}$ bisects $\angle HJK$, and $\overline{MJ} \cong \overline{NK}$.



Isosceles Triangle Angle Bisector to Congruent Sides Theorem

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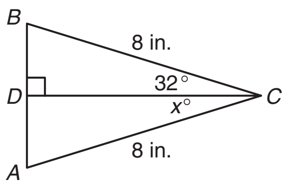
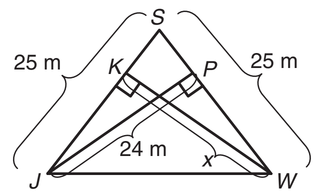
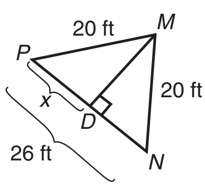
Determine the value of *x* in each isosceles triangle.

7. 

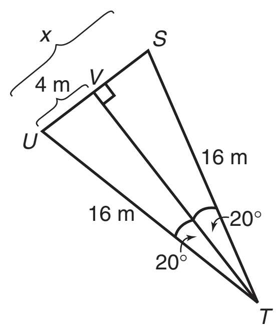
$x = 32°$

8. 

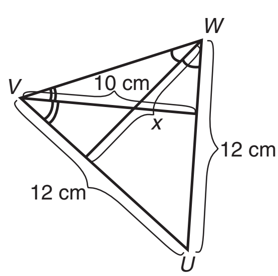
$x = 24$ m

9. 

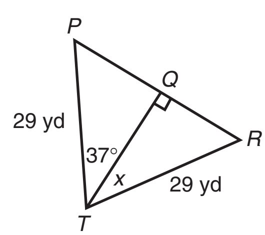
$x = 13$ ft

10. 

$x = 8$ m

11. 

$x = 10$ cm

12. 

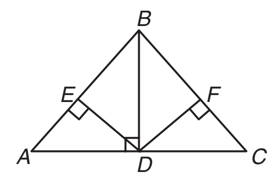
$x = 37°$

6

Complete each two-column proof.

13. Given: Isosceles $\triangle ABC$ with $\overline{AB} \cong \overline{CB}$, $\overline{BD} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$, and $\overline{DF} \perp \overline{CB}$

Prove: $\overline{ED} \cong \overline{FD}$



Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	1. Given
2. $\overline{BD} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{CB}$	2. Given
3. $\angle AED$ and $\angle CFD$ are right angles.	3. Definition of perpendicular lines
4. $\triangle AED$ and $\triangle CFD$ are right triangles.	4. Definition of right triangle
5. $\angle A \cong \angle C$	5. Base Angle Theorem
6. $\overline{AD} \cong \overline{CD}$	6. Isosceles Triangle Base Theorem
7. $\triangle AED \cong \triangle CFD$	7. HA Congruence Theorem
8. $\overline{ED} \cong \overline{FD}$	8. CPCTC

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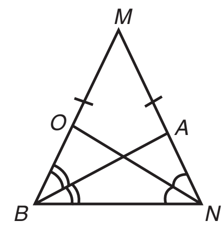
14. Given: Isosceles $\triangle MNB$ with $\overline{MN} \cong \overline{MB}$, $\overline{NO}$ bisects $\angle ANB$, $\overline{BA}$ bisects $\angle OBN$

Prove: $\triangle BAN \cong \triangle NOB$

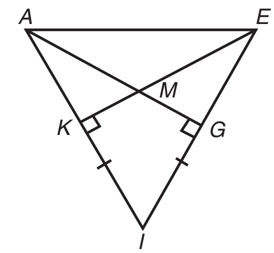


Statements	Reasons
1. $\overline{MN} \cong \overline{MB}$	1. Given
2. $\angle OBN \cong \angle ANB$	2. Base Angle Theorem
3. $\overline{NO}$ bisects $\angle ANB$, $\overline{BA}$ bisects $\angle OBN$	3. Given
4. $\angle OBA \cong \angle ABN$, $\angle ANO \cong \angle ONB$	4. Definition of angle bisector
5. $m\angle OBN = m\angle ANB$	5. Definition of congruent angles
6. $m\angle OBA = m\angle ABN$, $m\angle ANO = m\angle ONB$	6. Definition of congruent angles
7. $m\angle OBN = m\angle OBA + m\angle ABN$	7. Angle Addition Postulate
8. $m\angle ANB = m\angle ANO + m\angle ONB$	8. Angle Addition Postulate
9. $m\angle OBA + m\angle ABN = m\angle ANO + m\angle ONB$	9. Substitution Property
10. $m\angle ABN + m\angle ABN = m\angle ONB + m\angle ONB$	10. Substitution Property
11. $2(m\angle ABN) = 2(m\angle ONB)$	11. Factoring
12. $m\angle ABN = m\angle ONB$	12. Division Property of Equality
13. $\angle ABN \cong \angle ONB$	13. Definition of congruent angles
14. $\overline{BN} \cong \overline{BN}$	14. Reflexive Property of $\cong$
15. $\overline{BA} \cong \overline{NO}$	15. Isos. Triangle Angle Bisector to Congruent Sides Theorem
16. $\triangle BAN \cong \triangle NOB$	16. SAS Congruence Theorem

6

15. Given: Isosceles $\triangle IAE$ with $\overline{IA} \cong \overline{IE}$, $\overline{AG} \perp \overline{IE}$, $\overline{EK} \perp \overline{IA}$

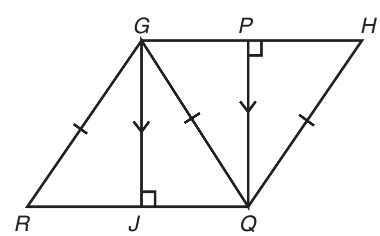
Prove: $\triangle IGA \cong \triangle IKE$



Statements	Reasons
1. $\overline{IA} \cong \overline{IE}$	1. Given
2. $\overline{AG} \perp \overline{IE}$, $\overline{EK} \perp \overline{IA}$	2. Given
3. $\angle IGA$ and $\angle IKE$ are right angles.	3. Definition of perpendicular lines
4. $\triangle IGA$ and $\triangle IKE$ are right triangles.	4. Definition of right triangle
5. $\overline{AG} \cong \overline{EK}$	5. Isos. Triangle Altitude to Congruent Sides Theorem
6. $\triangle IGA \cong \triangle IKE$	6. HL Congruence Theorem

16. Given: Isosceles $\triangle GQR$ with $\overline{GR} \cong \overline{GQ}$,
Isosceles $\triangle QGH$ with $\overline{GQ} \cong \overline{QH}$,
$\overline{GJ} \perp \overline{QR}$, $\overline{QP} \perp \overline{GH}$, and $\overline{GJ} \parallel \overline{QP}$

Prove: $\overline{RJ} \cong \overline{HP}$



Statements	Reasons
1. $\overline{GR} \cong \overline{GQ}$, $\overline{GQ} \cong \overline{QH}$	1. Given
2. $\overline{GJ} \perp \overline{QR}$, $\overline{QP} \perp \overline{GH}$	2. Given
3. $\overline{GJ} \parallel \overline{QP}$	3. Given
4. $\overline{GR} \cong \overline{QH}$	4. Transitive Property of $\cong$
5. $\angle RJG$ and $\angle HPQ$ are right angles.	5. Definition of perpendicular lines
6. $\triangle RJG$ and $\triangle HPQ$ are right triangles.	6. Definition of right triangle
7. $\angle RGJ \cong \angle QGJ$, $\angle HQP \cong \angle GQP$	7. Isos. Triangle Vertex Angle Theorem
8. $\angle QGJ \cong \angle GQP$	8. Alternate Interior Angle Theorem
9. $\angle RGJ \cong \angle HQP$	9. Substitution Property of $\cong$
10. $\triangle RJG \cong \triangle HPQ$	10. HA Congruence Theorem
11. $\overline{RJ} \cong \overline{HP}$	11. CPCTC

6

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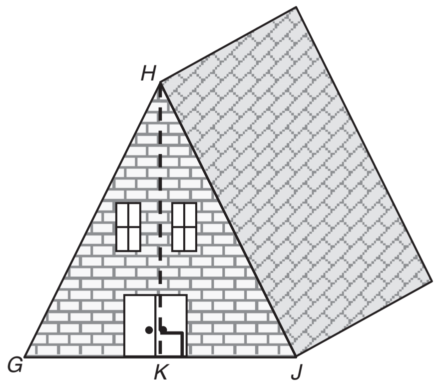
Use the given information to answer each question.

17. The front of an A-frame house is in the shape of an isosceles triangle, as shown in the diagram. In the diagram, $\overline{HK} \perp \overline{GJ}$, $\overline{GH} \cong \overline{JH}$, and $m\angle HGJ = 68.5°$. Use this information to determine the measure of $\angle GHJ$. Explain.

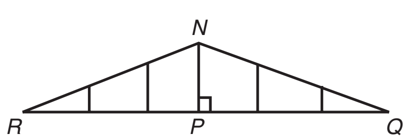


The measure of $\angle GHJ$ is 43°.

By the Triangle Sum Theorem, $m\angle GHK = 180° - (90° + 68.5°) = 21.5°$.

By the Isosceles Triangle Vertex Angle Theorem, $m\angle GHK = m\angle JHK$.
Therefore, $m\angle GHJ = 21.5° + 21.5° = 43°$.

18. When building a house, rafters are used to support the roof. The rafter shown in the diagram has the shape of an isosceles triangle. In the diagram, $\overline{NP} \perp \overline{RQ}$, $\overline{NR} \cong \overline{NQ}$, $NP = 12$ feet, and $RP = 16$ feet. Use this information to determine the length of $\overline{NQ}$. Explain.

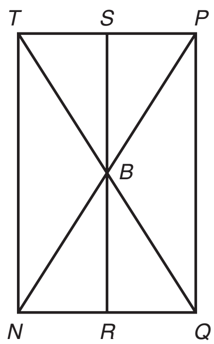


The length of $\overline{NQ}$ is 20 feet.

By the Isosceles Triangle Perpendicular Bisector Theorem, $\overline{RP}$ and $\overline{QP}$ have the same length. Using the Pythagorean Theorem with $NP = 12$ feet and $QP = 16$ feet:

$$(NP)^2 + (QP)^2 = (NQ)^2$$
$$12^2 + 16^2 = (NQ)^2$$
$$400 = (NQ)^2$$
$$20 = NQ$$

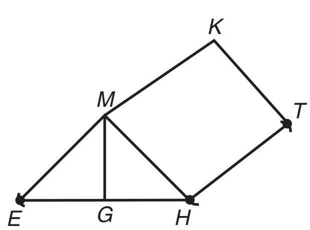
19. Stained glass windows are constructed using different pieces of colored glass held together by lead. The stained glass window in the diagram is rectangular with six different colored glass pieces represented by $\triangle TBS$, $\triangle PBS$, $\triangle PBQ$, $\triangle QBR$, $\triangle NBR$, and $\triangle NBT$. Triangle TBP with altitude $\overline{SB}$ and $\triangle QBN$ with altitude $\overline{RB}$, are congruent isosceles triangles. If the measure of $\angle NBR$ is 20°, what is the measure of $\angle STB$? Explain.



The measure of $\angle STB$ is 70°.

Since $\triangle TBP$ and $\triangle QBN$ are congruent isosceles triangles, $\angle TBP \cong \angle QBN$. Altitudes $\overline{SB}$ and $\overline{RB}$ each bisect the vertex angle of the triangle, creating four congruent angles. In $\triangle STB$, the measure of $\angle TSB$ is 90°. By CPCTC, the measure of $\angle SBT$ is 20°. By the Triangle Sum Theorem, $m\angle STB = 180° - (90° + 20°) = 70°$.

20. While growing up, Nikki often camped out in her back yard in a pup tent. A pup tent has two rectangular sides made of canvas, and a front and back in the shape of two isosceles triangles also made of canvas. The zipper in front, represented by $\overline{MG}$ in the diagram, is the height of the pup tent and the altitude of isosceles $\triangle EMH$. If the length of $\overline{EG}$ is 2.5 feet, what is the length of $\overline{HG}$? Explain.



The length of $\overline{HG}$ is 2.5 feet.

Since $\overline{MG}$ is the altitude of isosceles $\triangle EMH$, by the Isosceles Triangle Perpendicular Bisector Theorem, $\overline{EG} \cong \overline{HG}$. Therefore $HG = 2.5$ feet.

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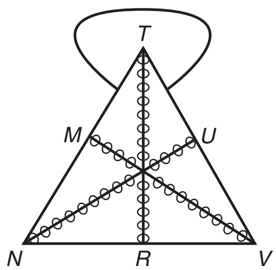
21. A beaded purse is in the shape of an isosceles triangle. In the diagram, $\overline{TN} \cong \overline{TV}$, $\overline{VM} \perp \overline{TN}$, and $\overline{NU} \perp \overline{TV}$. How long is the line of beads represented by $\overline{NU}$, if TV is 13 inches and TM is 5 inches? Explain.



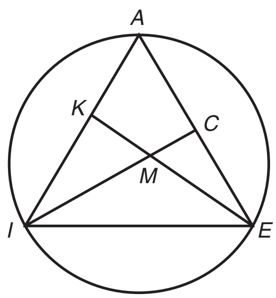
By the Isosceles Triangle Altitude to Congruent Sides Theorem, $\overline{NU}$ and $\overline{VM}$ have the same length. The line of beads represented by $\overline{NU}$ is 12 inches long.

Using the Pythagorean Theorem with $TV = 13$ inches and $TM = 5$ inches:

$$(VM)^2 + (TM)^2 = (TV)^2$$
$$(VM)^2 + 5^2 = 13^2$$
$$(VM)^2 + 25 = 169$$
$$(VM)^2 = 144$$
$$VM = 12$$

6

22. A kaleidoscope is a cylinder with mirrors inside and an assortment of loose colored beads. When a person looks through the kaleidoscope, different colored shapes and patterns are created as the kaleidoscope is rotated. Suppose that the diagram represents the shapes that a person sees when they look into the kaleidoscope. Triangle *AEI* is an isosceles triangle with $\overline{AE} \cong \overline{AI}$. $\overline{EK}$ bisects $\angle AEI$ and $\overline{IC}$ bisects $\angle AIE$. What is the length of $\overline{IC}$, if one half the length of $\overline{EK}$ is 14 centimeters? Explain.



The length of $\overline{IC}$ is 28 centimeters.

By the Isosceles Triangle Angle Bisector to Congruent Sides Theorem, $\overline{IC}$ and $\overline{EK}$ are congruent. Since half the length of $\overline{EK}$ is 14 centimeters, its full length is 28 centimeters. Therefore, the length of $\overline{EK}$ is 28 centimeters. So, the length of $\overline{IC}$ is 28 centimeters.

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Making Some Assumptions

Inverse, Contrapositive, Direct Proof, and Indirect Proof

Vocabulary

Define each term in your own words.

1. inverse

   The inverse of the conditional statement "If p, then q," is the statement "If not p, then not q."

2. contrapositive

   The contrapositive of the conditional statement "If p, then q," is the statement "If not q, then not p."

3. direct proof

   A direct proof is a proof that begins with the given information and works to the desired conclusion directly through the use of givens, definitions, properties, postulates, and theorems.

4. indirect proof (or proof by contradiction)

   An indirect proof, or proof by contradiction, is a proof that uses the contrapositive. If you prove the contrapositive true, then the statement is true.

5. Hinge Theorem

   If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first pair is larger than the included angle of the second pair, then the third side of the first triangle is longer than the third side of the second triangle.

6. Hinge Converse Theorem

   If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first pair of sides is larger than the included angle of the second pair of sides.

Problem Set

Write the converse of each conditional statement. Then, determine whether the converse is true.

1. If two lines do not intersect and are not parallel, then they are skew lines.

   The converse of the conditional would be:

   If two lines are skew lines, then they do not intersect and are not parallel.

   The converse is true.

2. If two lines are coplanar and do not intersect, then they are parallel lines.

   The converse of the conditional would be:

   If two lines are parallel lines, then they are coplanar and do not intersect.

   The converse is true.

3. If a triangle has one angle whose measure is greater than 90°, then the triangle is obtuse.

   The converse of the conditional would be:

   If a triangle is obtuse, then the measure of one of its angles is greater than 90°.

   The converse is true.

4. If a triangle has two sides with equal lengths, then it is an isosceles triangle.

   The converse of the conditional would be:

   If a triangle is an isosceles triangle, then it has two sides with equal lengths.

   The converse is true.

5. If the lengths of the sides of a triangle measure 5 mm, 12 mm, and 13 mm, then it is a right triangle.

   The converse of the conditional would be:

   If a triangle is a right triangle, then the lengths of its sides are 5 mm, 12 mm, and 13 mm.

   The converse is not true.

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6. If the lengths of the sides of a triangle are 3 cm, 4 cm, and 5 cm, then the triangle is a right triangle.

   The converse of the conditional would be:

   If a triangle is a right triangle, then the lengths of its sides are 3 cm, 4 cm, and 5 cm.

   The converse is not true.

7. If the corresponding sides of two triangles are congruent, then the triangles are congruent.

   The converse of the conditional would be:

   If two triangles are congruent, then the corresponding sides of the two triangles are congruent.

   The converse is true.

8. If the corresponding angles of two triangles are congruent, then the triangles are similar.

   The converse of the conditional would be:

   If two triangles are similar, then the corresponding angles of the two triangles are congruent.

   The converse is true.

Write the inverse of each conditional statement. Then, determine whether the inverse is true.

9. If a triangle is an equilateral triangle, then it is an isosceles triangle.

   The inverse of the conditional would be:

   If a triangle is not an equilateral triangle, then it is not an isosceles triangle.

   The inverse is not true.

10. If a triangle is a right triangle, then the sum of the measures of its acute angles is 90°.

    The inverse of the conditional would be:

    If a triangle is not a right triangle, then the sum of the measures of its acute angles is not 90°.

    The inverse is true.

11. If the sum of the internal angles of a polygon is 180°, then the polygon is a triangle.

The inverse of the conditional would be:

If the sum of the internal angles of a polygon is not 180°, then the polygon is not a triangle.

The inverse is true.

12. If a polygon is a triangle, then the sum of its exterior angles is 360°.

The inverse of the conditional would be:

If a polygon is not a triangle, then the sum of its exterior angles is not 360°.

The inverse is not true.

13. If two angles are the acute angles of a right triangle, then they are complementary.

The inverse of the conditional would be:

If two angles are not the acute angles of a right triangle, then they are not complementary.

The inverse is not true.

14. If two angles are complementary, then the sum of their measures is 90°.

The inverse of the conditional would be:

If two angles are not complementary, then the sum of their measures is not 90°.

The inverse is true.

15. If a polygon is a square, then it is a rhombus.

The inverse of the conditional would be:

If a polygon is not a square, then it is not a rhombus.

The inverse is not true.

16. If a polygon is a trapezoid, then it is a quadrilateral.

The inverse of the conditional would be:

If a polygon is not a trapezoid, then it is not a quadrilateral.

The inverse is not true.

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Write the contrapositive of each conditional statement. Then, determine whether the contrapositive is true.

17. If one of the acute angles of a right triangle measures 45°, then it is an isosceles right triangle.

The contrapositive of the conditional would be:

If a triangle is not an isosceles right triangle, then it is not a right triangle with an acute angle that measures 45°.

The contrapositive is true.

18. If one of the acute angles of a right triangle measures 30°, then it is a 30°–60°–90° triangle.

The contrapositive of the conditional would be:

If a triangle is not a 30°–60°–90° triangle, then it is not a right triangle with an acute angle that measures 30°.

The contrapositive is true.

19. If a quadrilateral is a rectangle, then it is a parallelogram.

The contrapositive of the conditional would be:

If a quadrilateral is not a parallelogram, then it is not a rectangle.

The contrapositive is true.

20. If a quadrilateral is an isosceles trapezoid, then it has two pairs of congruent base angles.

The contrapositive of the conditional would be:

If a quadrilateral does not have two pairs of congruent base angles, then it is not an isosceles trapezoid.

The contrapositive is true.

21. If the sum of the measures of two angles is 180°, then the angles are supplementary.

The contrapositive of the conditional would be:

If two angles are not supplementary, then the sum of their measures is not 180°.

The contrapositive is true.

22. If two angles are supplementary, then the sum of their measures is 180°.

The contrapositive of the conditional would be:

If the sum of the measures of two angles is not 180°, then the angles are not supplementary.

The contrapositive is true.

23. If the radius of a circle is 8 meters, then the diameter of the circle is 16 meters.

The contrapositive of the conditional would be:

If the diameter of a circle is not 16 meters, then the radius of the circle is not 8 meters.

The contrapositive is true.

24. If the diameter of a circle is 12 inches, then the radius of the circle is 6 inches.

The contrapositive of the conditional would be:

If the radius of a circle is not 6 inches, then the diameter of the circle is not 12 inches.

The contrapositive is true.

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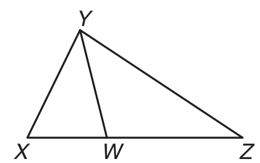
Create an indirect proof to prove each statement.

25. Given: $\overline{WY}$ bisects $\angle XYZ$ and $\overline{XW} \ncong \overline{ZW}$

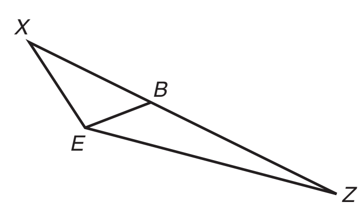
Prove: $\overline{XY} \ncong \overline{ZY}$



Statements	Reasons
1. $\overline{XY} \cong \overline{ZY}$	1. Assumption
2. $\overline{WY}$ bisects $\angle XYZ$	2. Given
3. $\angle XYW \cong \angle ZYW$	3. Definition of angle bisector
4. $\overline{YW} \cong \overline{YW}$	4. Reflexive Property of $\cong$
5. $\triangle XYW \cong \triangle ZYW$	5. SAS Congruence Theorem
6. $\overline{XW} \cong \overline{ZW}$	6. CPCTC
7. $\overline{XW} \ncong \overline{ZW}$	7. Given
8. $\overline{XY} \cong \overline{ZY}$ is false.	8. Step 7 contradicts Step 6. The assumption is false.
9. $\overline{XY} \ncong \overline{ZY}$ is true.	9. Proof by contradiction

26. Given: $m\angle EBX \neq m\angle EBZ$

Prove: $\overline{EB}$ is not an altitude of $\triangle EZX$.



Statements	Reasons
1. $\overline{EB}$ is an altitude of $\triangle EZX$.	1. Assumption
2. $\angle EBX$ and $\angle EBZ$ are right angles.	2. Definition of altitude
3. $\angle EBX \cong \angle EBZ$	3. Right Angles Congruence Theorem
4. $m\angle EBX = m\angle EBZ$	4. Definition of congruent angles
5. $m\angle EBX \neq m\angle EBZ$	5. Given
6. $\overline{EB}$ is an altitude of $\triangle EZX$ is false.	6. Step 5 contradicts Step 4. The assumption is false.
7. $\overline{EB}$ is not an altitude of $\triangle EZX$ is true.	7. Proof by contradiction

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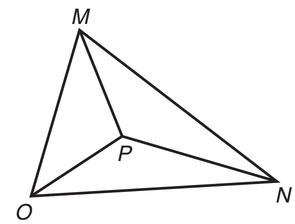
27. Given: $\angle OMP \cong \angle MOP$ and $\overline{NP}$ does not bisect $\angle ONM$.

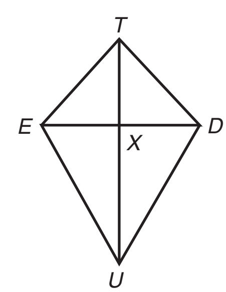
Prove: $\overline{NM} \ncong \overline{NO}$



Statements	Reasons
1. $\overline{NM} \cong \overline{NO}$	1. Assumption
2. $\angle OMP \cong \angle MOP$	2. Given
3. $\overline{NP}$ does not bisect $\angle ONM$.	3. Given
4. $\overline{MP} \cong \overline{OP}$	4. Isosceles Triangle Base Angle Converse Theorem
5. $\overline{PN} \cong \overline{PN}$	5. Reflexive Property of $\cong$
6. $\triangle ONP \cong \triangle MNP$	6. SSS Congruence Theorem
7. $\angle ONP \cong \angle MNP$	7. CPCTC
8. $\overline{NP}$ bisects $\angle ONM$.	8. Definition of angle bisector
9. $\overline{NM} \cong \overline{NO}$ is false.	9. Step 8 contradicts Step 3. The assumption is false.
10. $\overline{NM} \ncong \overline{NO}$ is true.	10. Proof by contradiction

28. Given: $\overline{ET} \cong \overline{DT}$ and $\overline{EU} \ncong \overline{DU}$

Prove: $\overline{EX} \ncong \overline{DX}$



Paragraph Proof:

You are given that $\overline{ET} \cong \overline{DT}$ and $\overline{EU} \ncong \overline{DU}$. Begin by assuming that $\overline{EX} \cong \overline{DX}$. By the Reflexive Property of Congruence, $\overline{TX} \cong \overline{TX}$ and $\overline{UX} \cong \overline{UX}$. By the SSS Congruence Theorem, $\triangle ETX \cong \triangle DTX$. By CPCTC, $\angle EXT \cong \angle DXT$. By the Vertical Angles Theorem, $\angle EXT \cong \angle DXU$ and $\angle TXD \cong \angle EXU$. Using the Transitivity Property of Congruence, $\triangle EXU \cong \triangle DXU$. By the SAS Congruence Theorem, $\triangle EXU \cong \triangle DXU$. As a result, CPCTC justifies the conclusion that $\overline{EU} \cong \overline{DU}$, which contradicts the given information ($\overline{EU} \ncong \overline{DU}$), so the assumption is false. Therefore, by contradiction, $\overline{EX} \ncong \overline{DX}$.

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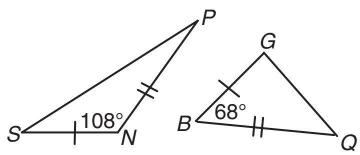
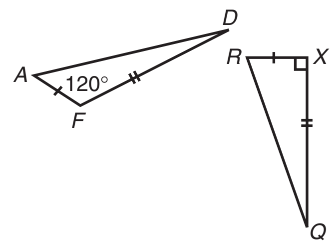
For each pair of triangles, use the Hinge Theorem or its converse to write a conclusion using an inequality,

29.



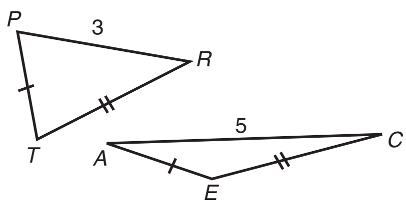
$SP > GQ$

30.



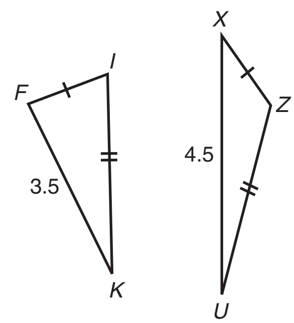
$AD > RQ$

31.



$m\angle E > m\angle T$

32.



$m\angle Z > m\angle I$

6

6