Name _____ Date _____

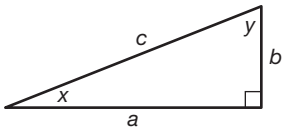
Three Angle Measure

Introduction to Trigonometry

Vocabulary

Use the diagram to complete each sentence.

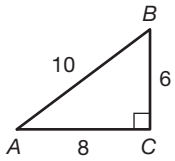
1. If b is the opposite side, then x is the reference angle.
2. If y is the reference angle, then b is the adjacent side.
3. If x is the reference angle, then b is the opposite side.



Problem Set

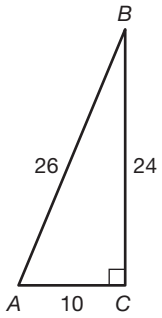
Determine the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.

1.



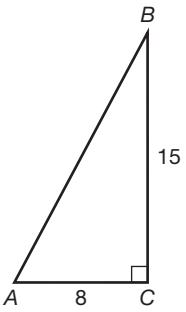
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

2.



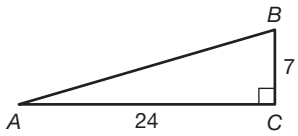
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13}$

3.



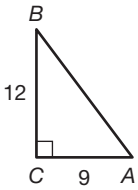
$c^2 = a^2 + b^2$
$c^2 = 15^2 + 8^2$
$c^2 = 225 + 64 = 289$
$c = \sqrt{289} = 17$
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{17}$

4.



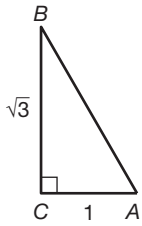
$c^2 = a^2 + b^2$
$c^2 = 7^2 + 24^2$
$c^2 = 49 + 576 = 625$
$c = \sqrt{625} = 25$
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{25}$

5.



$c^2 = a^2 + b^2$
$c^2 = 12^2 + 9^2$
$c^2 = 144 + 81 = 225$
$c = \sqrt{225} = 15$
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

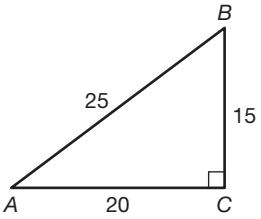
6.



$c^2 = a^2 + b^2$
$c^2 = 1^2 + (\sqrt{3})^2$
$c^2 = 1 + 3 = 4$
$c = \sqrt{4} = 2$
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$

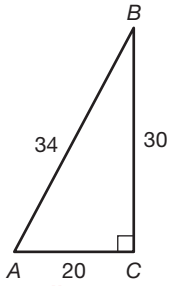
Determine the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.

7.



$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{25} = \frac{4}{5}$

8.



$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{16}{34} = \frac{8}{17}$

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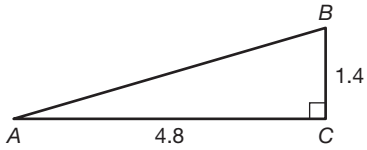
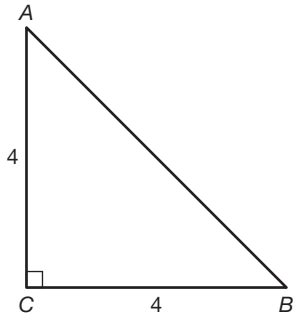
9.



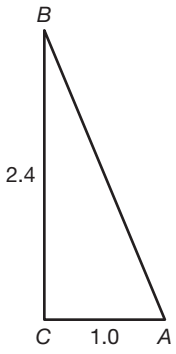
$c^2 = a^2 + b^2$
$c^2 = 1.4^2 + 4.8^2$
$c^2 = 1.96 + 23.04 = 25.0$
$c = \sqrt{25.0} = 5.0$
$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4.8}{5.0} = \frac{24}{25}$

10.



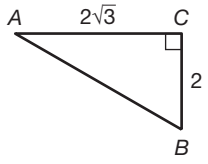
$c^2 = a^2 + b^2$
$c^2 = 4^2 + 4^2$
$c^2 = 16 + 16 = 32$
$c = \sqrt{32} = 4\sqrt{2}$
$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

11.



$c^2 = a^2 + b^2$
$c^2 = 2.4^2 + 1.0^2$
$c^2 = 1.00 + 5.76 = 6.76$
$c = \sqrt{6.76} = 2.6$
$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1.0}{2.6} = \frac{5}{13}$

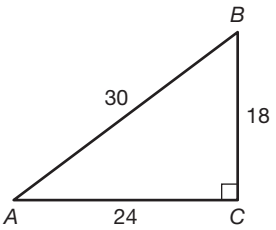
12.



$c^2 = a^2 + b^2$
$c^2 = 2^2 + (2\sqrt{3})^2$
$c^2 = 4 + 12 = 16$
$c = \sqrt{16} = 4$
$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Determine the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.

13.

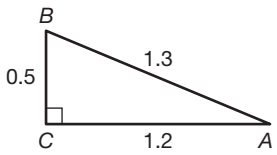


$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{18}{30} = \frac{3}{5}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{30} = \frac{4}{5}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{18}{24} = \frac{3}{4}$

14.

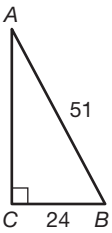


$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{0.5}{1.3} = \frac{5}{13}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1.2}{1.3} = \frac{12}{13}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{0.5}{1.2} = \frac{5}{12}$

15.



$b^2 = c^2 - a^2$

$b^2 = 51^2 - 24^2$

$b^2 = 2601 - 576 = 2025$

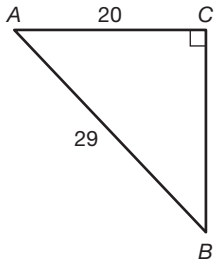
$b = \sqrt{2025} = 45$

$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{51} = \frac{8}{17}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{45}{51} = \frac{15}{17}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{24}{45} = \frac{8}{15}$

16.



$a^2 = c^2 - b^2$

$a^2 = 29^2 - 20^2$

$a^2 = 841 - 400 = 441$

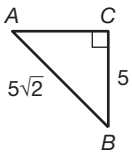
$a = \sqrt{441} = 21$

$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{21}{29}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{29}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{21}{20}$

Name _____ Date _____

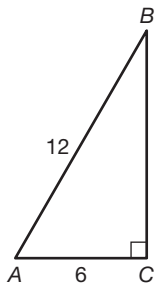
17. 

$a^2 = c^2 - b^2$
$b^2 = (5\sqrt{2})^2 - 5^2$
$b^2 = 50 - 25 = 25$
$b = \sqrt{25} = 5$

$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{\sqrt{2}}$ or $\frac{5\sqrt{2}}{2}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{\sqrt{2}}$ or $\frac{5\sqrt{2}}{2}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{5}{5} = 1$

18. 

$a^2 = c^2 - b^2$
$a^2 = 12^2 - 6^2$
$a^2 = 144 - 36 = 108$
$a = \sqrt{108} = \sqrt{36} \cdot \sqrt{3} = 6\sqrt{3}$

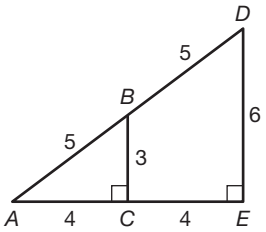
$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{12} = \frac{1}{2}$

$\frac{\text{opposite}}{\text{adjacent}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$

In each figure, triangles *ABC* and *DEF* are similar by the *AA* Similarity Theorem. Calculate the indicated ratio twice, first using $\triangle ABC$ and then using $\triangle ADE$.

19. $\frac{\text{opposite}}{\text{hypotenuse}}$ for reference angle *A*

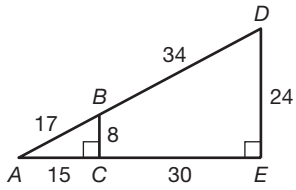


$AE = 4 + 4 = 8$
$AD = 5 + 5 = 10$

In $\triangle ABC$, $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$.

In $\triangle ADE$, $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$.

20. $\frac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle *A*

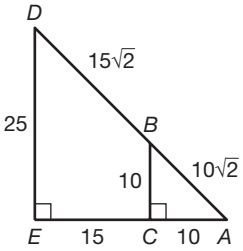


$AE = 15 + 30 = 45$
$AD = 17 + 34 = 51$

In $\triangle ABC$, $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$.

In $\triangle ADE$, $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{45}{51} = \frac{15}{17}$.

21. $\frac{\text{opposite}}{\text{hypotenuse}}$ for reference angle A



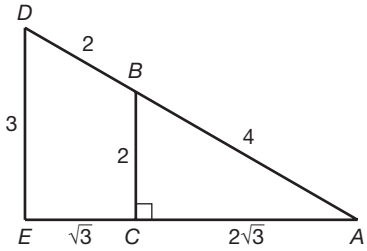
$AE = 10 + 15 = 25$

$AD = 10\sqrt{2} + 15\sqrt{2} = 25\sqrt{2}$

In $\triangle ABC$, $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$.

In $\triangle ADE$, $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$.

22. $\frac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle A



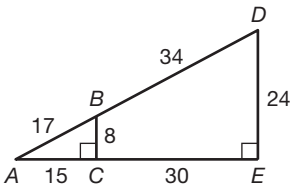
$AE = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$AD = 4 + 2 = 6$

In $\triangle ADE$, $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$.

In $\triangle ADE$, $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$.

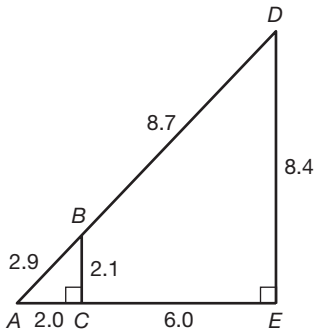
23. $\frac{\text{opposite}}{\text{adjacent}}$ for reference angle A



In $\triangle ABC$, $\frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$.

In $\triangle ADE$, $\frac{\text{opposite}}{\text{adjacent}} = \frac{24}{45} = \frac{8}{15}$.

24. $\frac{\text{opposite}}{\text{adjacent}}$ for reference angle A



In $\triangle ABC$, $\frac{\text{opposite}}{\text{adjacent}} = \frac{2.1}{2.0} = \frac{21}{20}$.

In $\triangle ADE$, $\frac{\text{opposite}}{\text{adjacent}} = \frac{8.4}{8.0} = \frac{21}{20}$.

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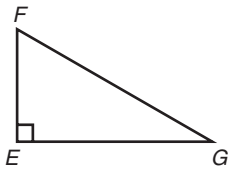
The Tangent Ratio

Tangent Ratio, Cotangent Ratio, and Inverse Tangent

Vocabulary

Match each description to its corresponding term for triangle *EFG*.



1. $\frac{EG}{EF}$ in relation to $\angle G$

b. cotangent

2. $\frac{EF}{EG}$ in relation to $\angle G$

a. tangent

3. $\tan^{-1}\left(\frac{EF}{EG}\right)$ in relation to $\angle G$

c. inverse tangent

a. tangent

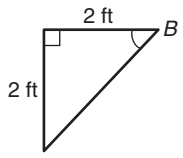
b. cotangent

c. inverse tangent

Problem Set

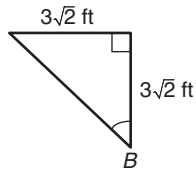
Calculate the tangent of the indicated angle in each triangle. Write your answers in simplest form.

1.



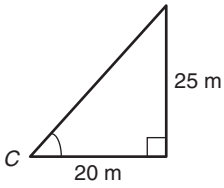
$\tan B = \frac{2}{2} = 1$

2.



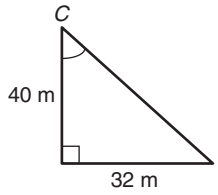
$\tan B = \frac{3\sqrt{2}}{3\sqrt{2}} = 1$

3.



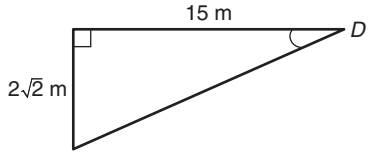
$\tan C = \frac{25}{20} = \frac{5}{4}$

4.



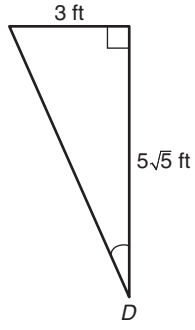
$\tan C = \frac{32}{40} = \frac{4}{5}$

5.



$\tan D = \frac{2\sqrt{2}}{15}$

6.



$\tan D = \frac{3}{5\sqrt{5}} = \frac{3\sqrt{5}}{25}$

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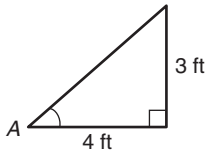
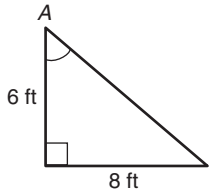
Calculate the cotangent of the indicated angle in each triangle. Write your answers in simplest form.

7.



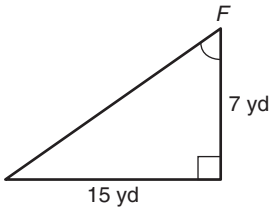
$\cot A = \frac{4}{3}$

8.



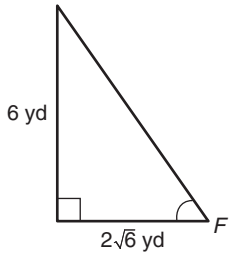
$\cot A = \frac{6}{8} = \frac{3}{4}$

9.



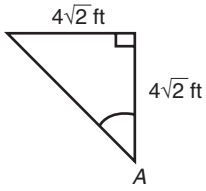
$\cot F = \frac{7}{15}$

10.



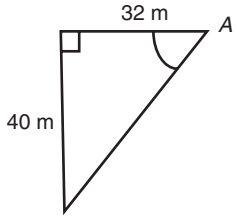
$\cot F = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

11.



$\cot A = \frac{4\sqrt{2}}{4\sqrt{2}} = 1$

12.



$\cot A = \frac{32}{40} = \frac{4}{5}$

Use a calculator to approximate each tangent ratio. Round your answers to the nearest hundredth.

13. tan 30°
0.58

14. tan 45°
1

15. tan 60°
1.73

16. tan 15°
0.27

17. tan 75°
3.73

18. tan 89°
57.29

Use a calculator to approximate each cotangent ratio. Round your answers to the nearest hundredth.

19. cot 60°

0.58

20. cot 15°

3.73

21. cot 45°

1

22. cot 75°

0.27

23. cot 10°

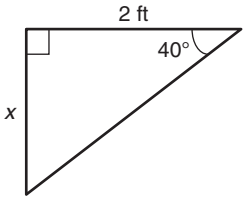
5.67

24. cot 30°

1.73

Use a tangent ratio or a cotangent ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

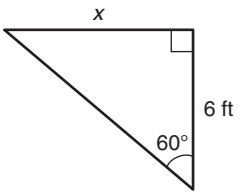


$\tan 40° = \frac{x}{2}$

$2 \tan 40° = x$

$x \approx 1.68$ ft

26.

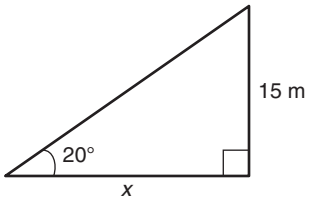


$\tan 60° = \frac{x}{6}$

$6 \tan 60° = x$

$x \approx 10.39$ ft

27.

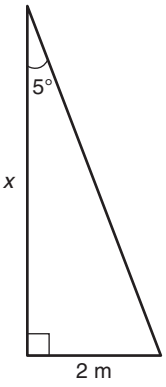


$\tan 20° = \frac{15}{x}$

$x = \frac{15}{\tan 20°}$

$x \approx 41.21$ m

28.



$\tan 5° = \frac{2}{x}$

$x = \frac{2}{\tan 5°}$

$x \approx 22.86$ m

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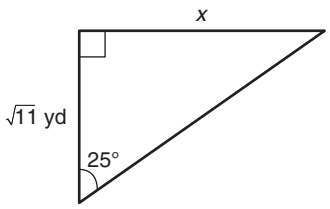
29.

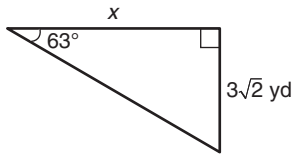


$\tan 25° = \frac{x}{\sqrt{11}}$

$\sqrt{11} \tan 25° = x$

$x \approx 1.55$ yd

30.



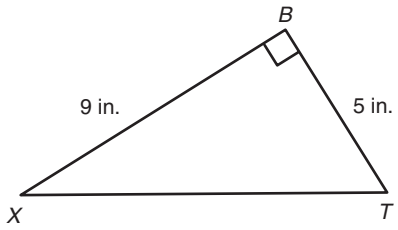
$\tan 63° = \frac{3\sqrt{2}}{x}$

$x = \frac{3\sqrt{2}}{\tan 63°}$

$x \approx 2.16$ yd

Calculate the measure of angle X for each triangle. Round your answers to the nearest hundredth.

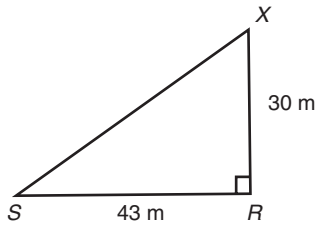
31.



$\tan X = \frac{5}{9}$

$m\angle X = \tan^{-1}\left(\frac{5}{9}\right) \approx 29.05°$

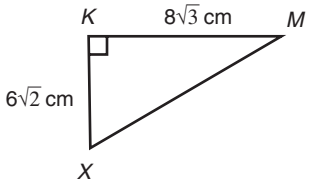
32.



$\tan X = \frac{43}{30}$

$m\angle X = \tan^{-1}\left(\frac{43}{30}\right) \approx 55.10°$

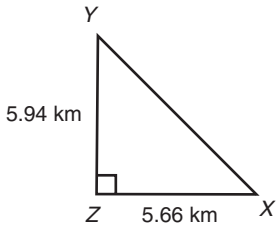
33.



$\tan X = \frac{8\sqrt{3}}{6\sqrt{2}}$

$m\angle X = \tan^{-1}\left(\frac{8\sqrt{3}}{6\sqrt{2}}\right) \approx 58.52°$

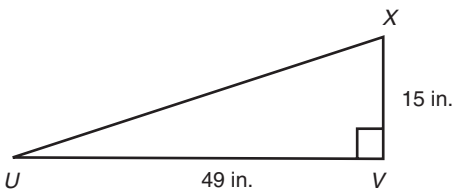
34.



$\tan X = \frac{5.94}{5.66}$

$m\angle X = \tan^{-1}\left(\frac{5.94}{5.66}\right) \approx 46.38°$

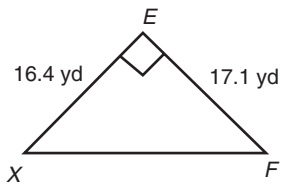
35.



$\tan X = \frac{49}{15}$

$m\angle X = \tan^{-1}\left(\frac{49}{15}\right) \approx 72.98°$

36.

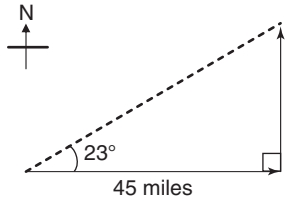


$\tan X = \frac{17.1}{16.4}$

$m\angle X = \tan^{-1}\left(\frac{17.1}{16.4}\right) \approx 46.20°$

Solve each problem. Round your answers to the nearest hundredth.

37. A boat travels in the following path. How far north did it travel?

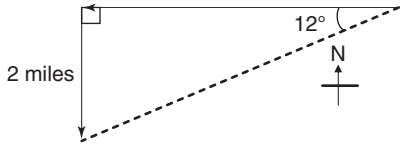


$\tan 23° = \frac{N}{45}$

$45 \tan 23° = N$

$N \approx 19.10$ mi

38. During a group hike, a park ranger makes the following path. How far west did they travel?

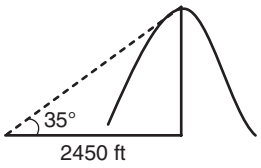


$\tan 12° = \frac{2}{W}$

$W = \frac{2}{\tan 12°}$

$W \approx 9.41$ mi

39. A surveyor makes the following diagram of a hill. What is the height of the hill?



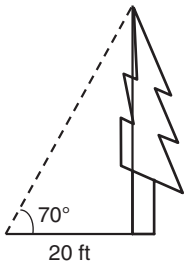
$\tan 35° = \frac{h}{2450}$

$2450 \tan 35° = h$

$h \approx 1715.51$ ft

Name _____ Date _____

40. To determine the height of a tree, a botanist makes the following diagram. What is the height of the tree?

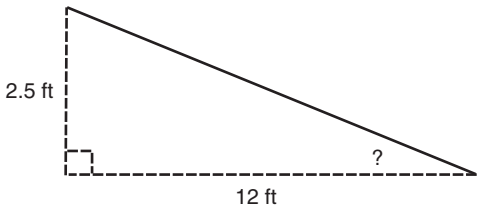


$\tan 70° = \frac{h}{20}$

$20 \tan 70° = h$

$h \approx 54.95$ ft

41. A moving truck is equipped with a ramp that extends from the back of the truck to the ground. When the ramp is fully extended, it touches the ground 12 feet from the back of the truck. The height of the ramp is 2.5 feet. Calculate the measure of the angle formed by the ramp and the ground.

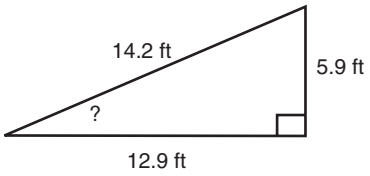


$\tan x = \frac{2.5}{12}$

$x = \tan^{-1}\left(\frac{2.5}{12}\right) \approx 11.77°$

The angle formed by the ramp and the ground is approximately 11.77°.

42. A park has a skateboard ramp with a length of 14.2 feet and a length along the ground of 12.9 feet. The height is 5.9 feet. Calculate the measure of the angle formed by the ramp and the ground.

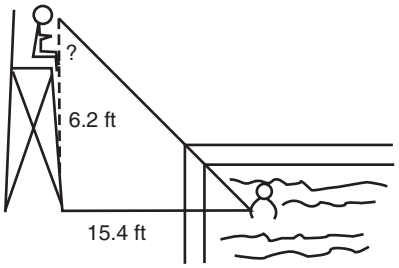


$\tan x = \frac{5.9}{12.9}$

$x = \tan^{-1}\left(\frac{5.9}{12.9}\right) \approx 24.58°$

The angle formed by the ramp and the ground is approximately 24.58°.

43. A lifeguard is sitting on an observation chair at a pool. The lifeguard's eye level is 6.2 feet from the ground. The chair is 15.4 feet from a swimmer. Calculate the measure of the angle formed when the lifeguard looks down at the swimmer.

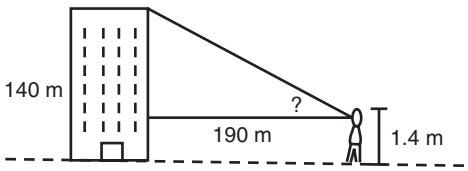


$\tan x = \frac{15.4}{6.2}$

$x = \tan^{-1}\left(\frac{15.4}{6.2}\right) \approx 68.07°$

The lifeguard is looking down at an angle of approximately 68.07°.

44. A surveyor is looking up at the top of a building that is 140 meters tall. His eye level is 1.4 meters above the ground, and he is standing 190 meters from the building. Calculate the measure of the angle from his eyes to the top of the building.



$\tan x = \frac{140 - 1.4}{190}$

$x = \tan^{-1}\left(\frac{138.6}{190}\right) \approx 36.11°$

The surveyor is looking up at an angle of approximately 36.11°.

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The Sine Ratio

Sine Ratio, Cosecant Ratio, and Inverse Sine

Vocabulary

Write the term from the box that best completes each statement.

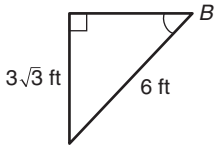
sine	cosecant	inverse sine

1. The cosecant of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of a side that is opposite the angle.
2. The inverse sine of x is the measure of an acute angle whose sine is x.
3. The sine of an acute angle in a right triangle is the ratio of the length of the side that is opposite the angle to the length of the hypotenuse.

Problem Set

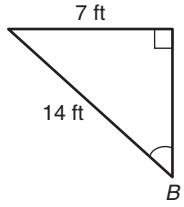
Calculate the sine of the indicated angle in each triangle. Write your answers in simplest form.

1.



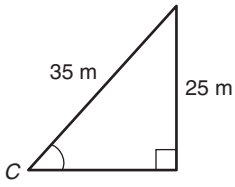
$\sin B = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

2.



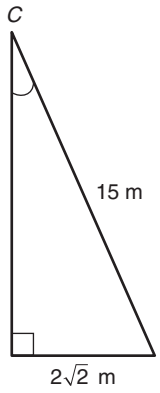
$\sin B = \frac{7}{14} = \frac{1}{2}$

3.



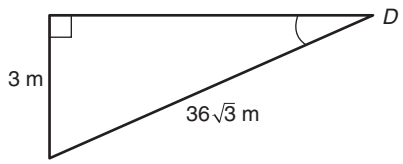
$\sin C = \frac{25}{35} = \frac{5}{7}$

4.



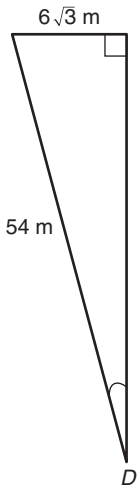
$\sin C = \frac{2\sqrt{2}}{15}$

5.



$\sin D = \frac{3}{36\sqrt{3}} = \frac{\sqrt{3}}{36}$

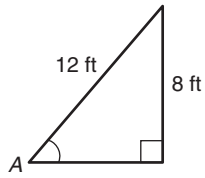
6.



$\sin D = \frac{6\sqrt{3}}{54} = \frac{\sqrt{3}}{9}$

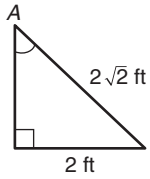
Calculate the cosecant of the indicated angle in each triangle. Write your answers in simplest form.

7.



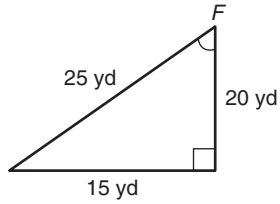
$\csc A = \frac{12}{8} = \frac{3}{2}$

8.



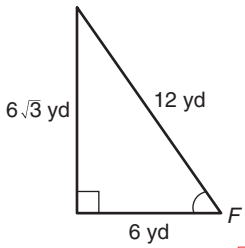
$\csc A = \frac{2\sqrt{2}}{2} = \sqrt{2}$

9.



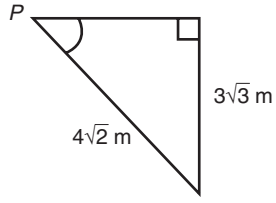
$\csc F = \frac{25}{15} = \frac{5}{3}$

10.



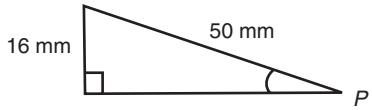
$\csc F = \frac{12}{6\sqrt{3}} = \frac{12\sqrt{3}}{18} = \frac{2\sqrt{3}}{3}$

11.



$\csc P = \frac{4\sqrt{2}}{3\sqrt{3}} = \frac{4\sqrt{6}}{9}$

12.



$\csc P = \frac{50}{16} = \frac{25}{8}$

Name _____ Date _____

Use a calculator to approximate each sine ratio. Round your answers to the nearest hundredth.

13. sin 30°

0.5

14. sin 45°

0.71

15. sin 60°

0.87

16. sin 15°

0.26

17. sin 75°

0.97

18. sin 5°

0.09

Use a calculator to approximate each cosecant ratio. Round your answers to the nearest hundredth.

19. csc 45°

1.41

20. csc 90°

1

21. csc 120°

1.15

22. csc 30°

2

23. csc 15°

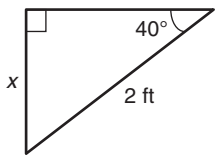
3.86

24. csc 60°

1.15

Use a sine ratio or a cosecant ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

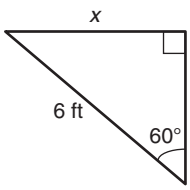


$\sin 40° = \frac{x}{2}$

$2 \sin 40° = x$

$x \approx 1.29$ ft

26.

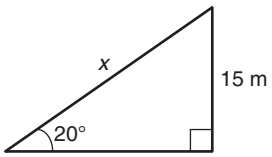


$\sin 60° = \frac{x}{6}$

$6 \sin 60° = x$

$x \approx 5.20$ ft

27.

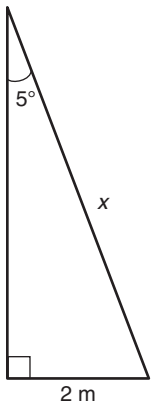


$\sin 20° = \frac{15}{x}$

$x = \frac{15}{\sin 20°}$

$x \approx 43.86 \text{ m}$

28.

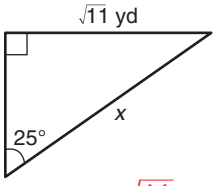


$\sin 5° = \frac{2}{x}$

$x = \frac{2}{\sin 5°}$

$x \approx 22.95 \text{ m}$

29.

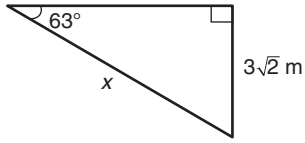


$\sin 25° = \frac{\sqrt{11}}{x}$

$x = \frac{\sqrt{11}}{\sin 25°}$

$x \approx 7.85 \text{ yd}$

30.



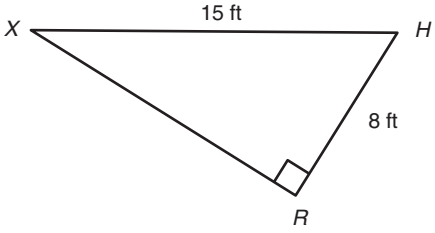
$\sin 63° = \frac{3\sqrt{2}}{x}$

$x = \frac{3\sqrt{2}}{\sin 63°}$

$x \approx 4.76 \text{ m}$

Calculate the measure of angle X for each triangle. Round your answers to the nearest hundredth.

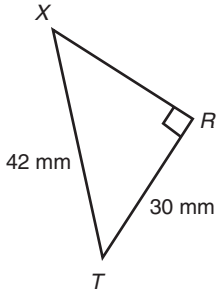
31.



$\sin X = \frac{8}{15}$

$m\angle X = \sin^{-1}\left(\frac{8}{15}\right) \approx 32.23°$

32.

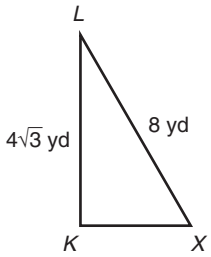


$\sin X = \frac{30}{42}$

$m\angle X = \sin^{-1}\left(\frac{30}{42}\right) \approx 45.58°$

Name _____ Date _____

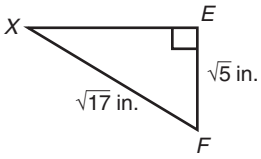
33.



$\sin X = \frac{4\sqrt{3}}{8}$

$m\angle X = \sin^{-1}\left(\frac{4\sqrt{3}}{8}\right) \approx 60°$

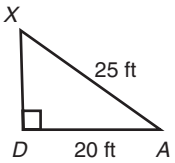
34.



$\sin X = \frac{\sqrt{5}}{\sqrt{17}}$

$m\angle X = \sin^{-1}\left(\frac{\sqrt{5}}{\sqrt{17}}\right) \approx 32.84°$

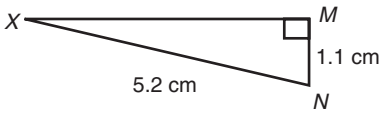
35.



$\sin X = \frac{20}{25}$

$m\angle X = \sin^{-1}\left(\frac{20}{25}\right) \approx 53.13°$

36.

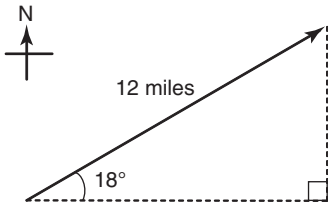


$\sin X = \frac{1.1}{5.2}$

$m\angle X = \sin^{-1}\left(\frac{1.1}{5.2}\right) \approx 12.21°$

Solve each problem. Round your answers to the nearest hundredth.

37. A scout troop traveled 12 miles from camp, as shown on the map below. How far north did they travel?

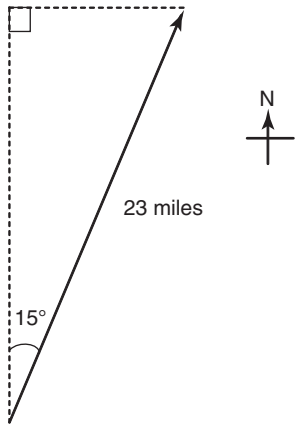


$\sin 18° = \frac{N}{12}$

$12 \sin 18° = N$

$N \approx 3.71$ mi

38. An ornithologist tracked a Cooper's hawk that traveled 23 miles. How far east did the bird travel?

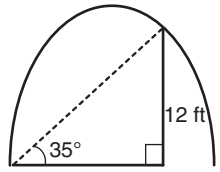


$\sin 15° = \frac{E}{23}$

$23 \sin 15° = E$

$E \approx 5.95$ mi

39. An architect needs to use a diagonal support in an arch. Her company drew the following diagram. How long does the diagonal support have to be?

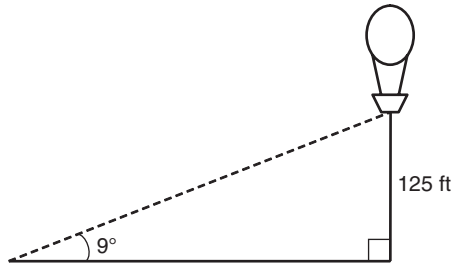


$\sin 35° = \frac{12}{l}$

$l = \frac{12}{\sin 35°}$

$l \approx 20.92$ ft

40. A hot air balloon lifts 125 feet into the air. The diagram below shows that the hot air balloon was blown to the side. How long is the piece of rope that connects the balloon to the ground?



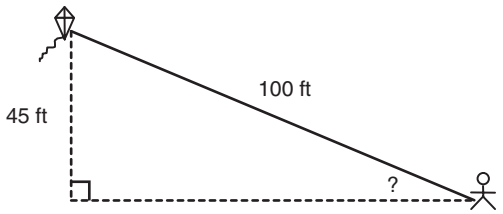
$\sin 9° = \frac{125}{l}$

$l = \frac{125}{\sin 9°}$

$l \approx 799.06$ ft

Name _____ Date _____

41. Jerome is flying a kite on the beach. The kite is attached to a 100-foot string and is flying 45 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the string and the ground.

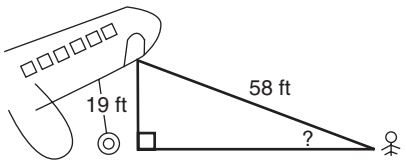


$\sin x = \frac{45}{100}$

$x = \sin^{-1}\left(\frac{45}{100}\right) \approx 26.74°$

The angle formed by the string and the ground is approximately 26.74°.

42. An airplane ramp is 58 feet long and reaches the cockpit entrance 19 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the ramp and the ground.

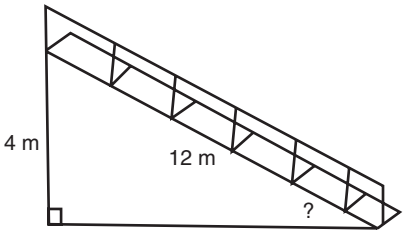


$\sin x = \frac{19}{58}$

$x = \sin^{-1}\left(\frac{19}{58}\right) \approx 19.12°$

The angle formed by the ramp and the ground is approximately 19.12°.

43. Bleachers in a stadium are 4 meters tall and have a length of 12 meters, as shown in the diagram. Calculate the measure of the angle formed by the bleachers and the ground.

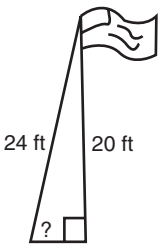


$\sin x = \frac{4}{12}$

$x = \sin^{-1}\left(\frac{4}{12}\right) \approx 19.47°$

The angle formed by the bleachers and the ground is approximately 19.47°.

44. A 20-foot flagpole is raised by a 24-foot rope, as shown in the diagram. Calculate the measure of the angle formed by the rope and the ground.



$\sin x = \frac{20}{24}$

$x = \sin^{-1}\left(\frac{20}{24}\right) \approx 56.44°$

The angle formed by the rope and the ground is approximately 56.44°.

Name _____ Date _____

The Cosine Ratio

Cosine Ratio, Secant Ratio, and Inverse Cosine

Vocabulary

Describe the similarities and differences between the pair of terms.

1. cosine ratio and secant ratio

Both the cosine ratio and the secant ratio are ratios of sides of a right triangle. The cosine of an angle in a right triangle is the ratio of the side adjacent to the angle to the hypotenuse, while the secant of an angle in a right triangle is the ratio of the hypotenuse to the side adjacent to the angle. So, they are reciprocals of each other.

Define the term in your own words.

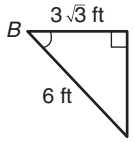
2. inverse cosine

The inverse cosine of x is defined as the measure of an acute angle whose cosine is x.

Problem Set

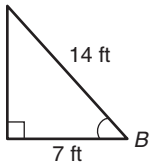
Calculate the cosine of the indicated angle in each triangle. Write your answers in simplest form.

1.



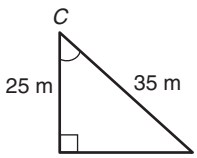
$\cos B = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

2.



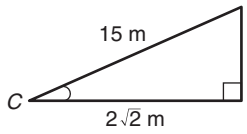
$\cos B = \frac{7}{14} = \frac{1}{2}$

3.



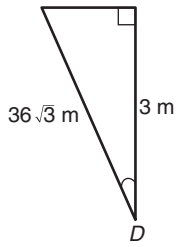
$\cos C = \frac{25}{35} = \frac{5}{7}$

4.



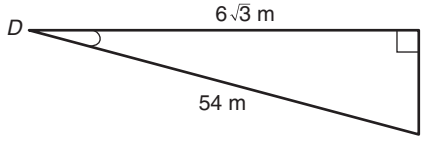
$\cos C = \frac{2\sqrt{2}}{15}$

5.



$\cos D = \frac{3}{36\sqrt{3}} = \frac{\sqrt{3}}{36}$

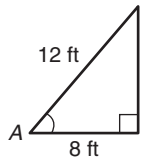
6.



$\cos D = \frac{6\sqrt{3}}{54} = \frac{\sqrt{3}}{9}$

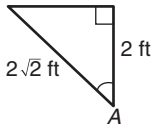
Calculate the secant of the indicated angle in each triangle. Write your answers in simplest form.

7.



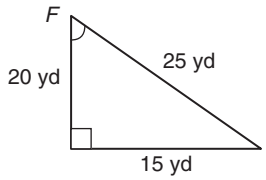
$\sec A = \frac{12}{8} = \frac{3}{2}$

8.



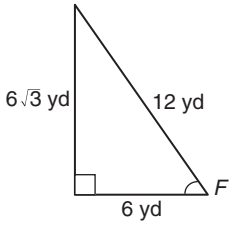
$\sec A = \frac{2\sqrt{2}}{2} = \sqrt{2}$

9.



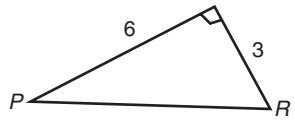
$\sec F = \frac{25}{20} = \frac{5}{4}$

10.



$\sec F = \frac{12}{6} = 2$

11.



$\sec P = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$ because

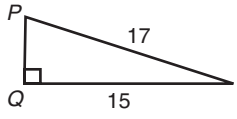
$PR^2 = 6^2 + 3^2$

$PR^2 = 36 + 9$

$PR^2 = 45$

$PR = \sqrt{45} = 3\sqrt{5}$

12.



$\sec P = \frac{17}{8}$ because

$15^2 + PQ^2 = 17^2$

$225 + PQ^2 = 289$

$PQ^2 = 64$

$PQ = 8$

Name _____ Date _____

Use a calculator to approximate each cosine ratio. Round your answers to the nearest hundredth.

13. cos 30°

0.87

14. cos 45°

0.71

15. cos 60°

0.5

16. cos 15°

0.97

17. cos 75°

0.26

18. cos 89°

0.02

Use a calculator to approximate each secant ratio. Round your answers to the nearest hundredth.

19. sec 45°

$\frac{1}{\cos(45°)} = 1.41$

20. sec 25°

$\frac{1}{\cos(25°)} = 1$

21. sec 75°

$\frac{1}{\cos(75°)} = 3.86$

22. sec 30°

$\frac{1}{\cos(30°)} = 1.15$

23. sec 15°

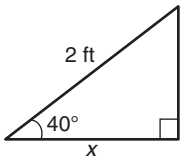
$\frac{1}{\cos(15°)} = 1.04$

24. sec 60°

$\frac{1}{\cos(60°)} = 2$

Use a cosine ratio or a secant ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

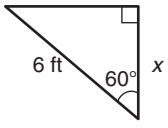


$\cos 40° = \frac{x}{2}$

$2 \cos 40° = x$

$x \approx 1.53$ ft

26.

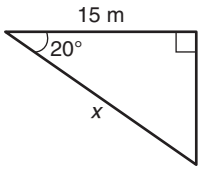


$\cos 60° = \frac{x}{6}$

$6 \cos 60° = x$

$x = 3$ ft

27.

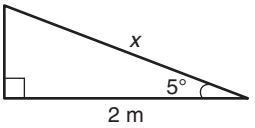


$\cos 20° = \frac{15}{x}$

$x = \frac{15}{\cos 20°}$

$x \approx 15.96$ m

28.

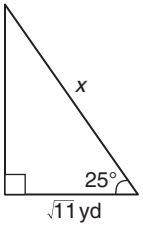


$\cos 5° = \frac{2}{x}$

$x = \frac{2}{\cos 5°}$

$x \approx 2.01$ m

29.

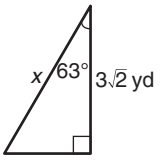


$\cos 25° = \frac{\sqrt{11}}{x}$

$x = \frac{\sqrt{11}}{\cos 25°}$

$x \approx 3.66$ yd

30.



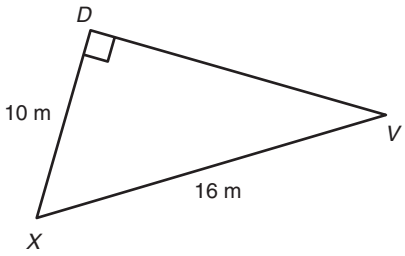
$\cos 63° = \frac{3\sqrt{2}}{x}$

$x = \frac{3\sqrt{2}}{\cos 63°}$

$x \approx 9.35$ yd

Calculate the measure of angle X for each triangle. Round your answers to the nearest hundredth.

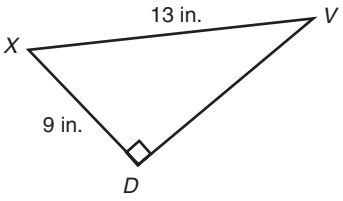
31.



$\cos X = \frac{10}{16}$

$m\angle X = \cos^{-1}\left(\frac{10}{16}\right) \approx 51.32°$

32.

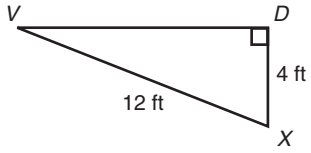


$\cos X = \frac{9}{13}$

$m\angle X = \cos^{-1}\left(\frac{9}{13}\right) \approx 46.19°$

Name _____ Date _____

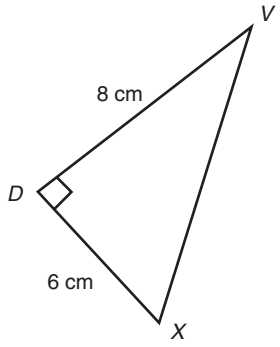
33.



$\cos X = \frac{4}{12}$

$m\angle X = \cos^{-1}\left(\frac{4}{12}\right) \approx 70.53°$

34.



$XV^2 = 6^2 + 8^2$

$XV^2 = 36 + 64$

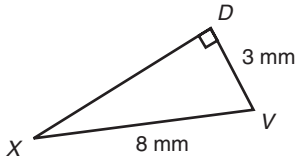
$XV^2 = 100$

$XV = 10$

$\cos X = \frac{6}{10}$

$m\angle X = \cos^{-1}\left(\frac{6}{10}\right) \approx 53.13°$

35.



$XD^2 + 3^2 = 8^2$

$XD^2 + 9 = 64$

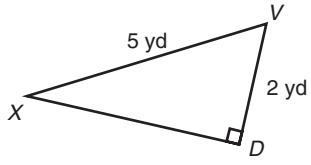
$XD^2 = 55$

$XD = \sqrt{55}$

$\cos X = \frac{\sqrt{55}}{8}$

$m\angle X = \cos^{-1}\left(\frac{\sqrt{55}}{8}\right) \approx 22.02°$

36.



$XD^2 + 2^2 = 5^2$

$XD^2 + 4 = 25$

$XD^2 = 21$

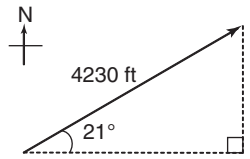
$XD = \sqrt{21}$

$\cos X = \frac{\sqrt{21}}{5}$

$m\angle X = \cos^{-1}\left(\frac{\sqrt{21}}{5}\right) \approx 23.58°$

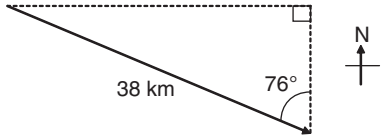
Solve each problem. Round your answers to the nearest hundredth.

37. The path of a model rocket is shown below. How far east did the rocket travel?



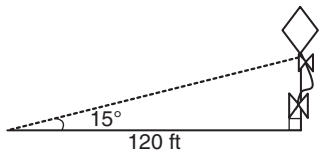
$$\cos 21° = \frac{E}{4230}$$
$$4230 \cos 21° = E$$
$$E \approx 3949.05 \text{ ft}$$

38. An ichthyologist tags a shark and charts its path. Examine his chart below. How far south did the shark travel?



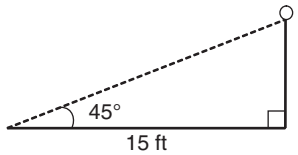
$$\cos 76° = \frac{S}{38}$$
$$38 \cos 76° = S$$
$$S \approx 9.19 \text{ km}$$

39. A kite is flying 120 feet away from the base of its string, as shown below. How much string is let out?



$$\cos 15° = \frac{120}{s}$$
$$s = \frac{120}{\cos 15°}$$
$$s \approx 124.23 \text{ ft}$$

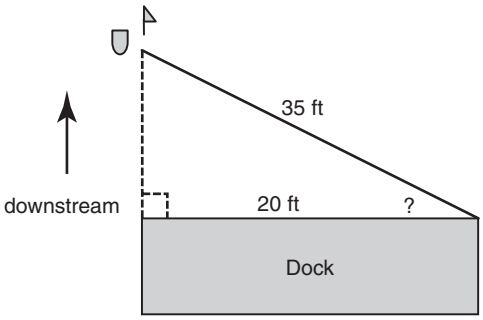
40. A pole has a rope tied to its top and to a stake 15 feet from the base. What is the length of the rope?



$$\cos 45° = \frac{15}{l}$$
$$l = \frac{15}{\cos 45°}$$
$$x \approx 21.21 \text{ ft}$$

Name _____ Date _____

41. You park your boat at the end of a 20-foot dock. You tie the boat to the opposite end of the dock with a 35-foot rope. The boat drifts downstream until the rope is extended as far as it will go, as shown in the diagram. What is the angle formed by the rope and the dock?

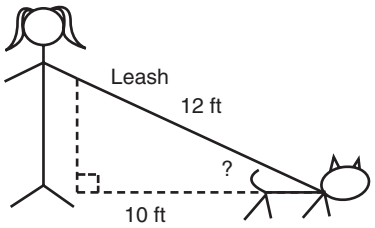


$\cos x = \frac{20}{35}$

$x = \cos^{-1}\left(\frac{20}{35}\right) \approx 55.15°$

The angle formed by the rope and the dock is approximately 55.15°.

42. Rennie is walking her dog. The dog's leash is 12 feet long and is attached to the dog 10 feet horizontally from Rennie's hand, as shown in the diagram. What is the angle formed by the leash and the horizontal at the dog's collar?

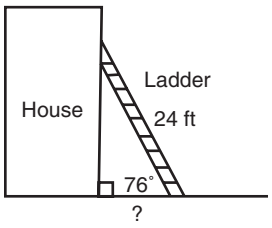


$\cos x = \frac{10}{12}$

$x = \cos^{-1}\left(\frac{10}{12}\right) \approx 35.56°$

The angle formed by the leash and the horizontal at the dog's collar is approximately 33.56°.

43. A ladder is leaning against the side of a house, as shown in the diagram. The ladder is 24 feet long and makes a 76° angle with the ground. How far from the edge of the house is the base of the ladder?



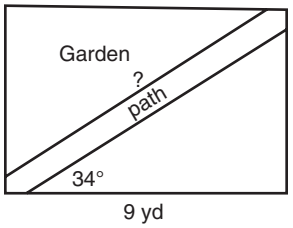
$\cos 76° = \frac{x}{24}$

$x = 24 \cos 76°$

$x \approx 5.81$ ft

The base of the ladder is approximately 5.81 feet from the edge of the house.

44. A rectangular garden 9 yards long has a diagonal path going through it, as shown in the diagram. The path makes a 34° angle with the longer side of the garden. Determine the length of the path.



$\cos 34° = \frac{9}{x}$

$x \cos 34° = 9$

$x = \frac{9}{\cos 34°} \approx 10.86$ yd

The length of the path is approximately 10.86 yd.

Name _____ Date _____

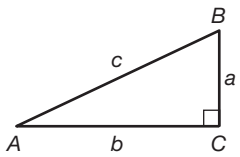
We Complement Each Other!

Complement Angle Relationships

Problem Set

For each right triangle, name the given ratio in two different ways.

1.

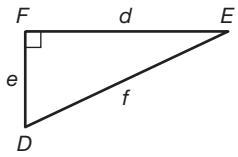


$\frac{a}{c}$

$\sin \angle A = \frac{a}{c}$

$\cos \angle B = \frac{a}{c}$

2.

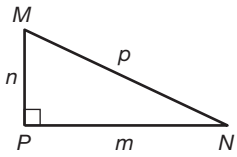


$\frac{d}{e}$

$\tan \angle D = \frac{d}{e}$

$\cot \angle E = \frac{d}{e}$

3.

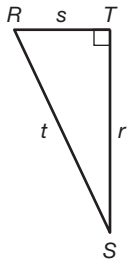


$\frac{p}{m}$

$\sec \angle N = \frac{p}{m}$

$\csc \angle M = \frac{p}{m}$

4.

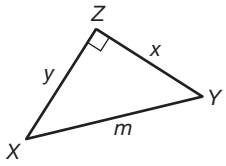


$\frac{s}{r}$

$\tan \angle S = \frac{s}{r}$

$\cot \angle R = \frac{s}{r}$

5.

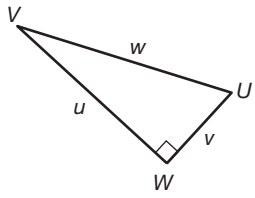


$\frac{y}{z}$

$\sin \angle Y = \frac{y}{z}$

$\cos \angle X = \frac{y}{z}$

6.



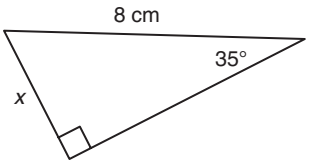
$\frac{w}{v}$

$\sec \angle U = \frac{w}{v}$

$\csc \angle V = \frac{w}{v}$

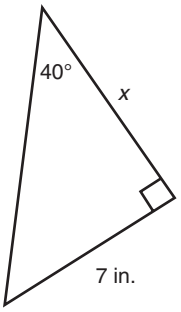
Determine the trigonometric ratio that you would use to solve for x in each triangle. Explain your reasoning. You do not need to solve for x.

7.



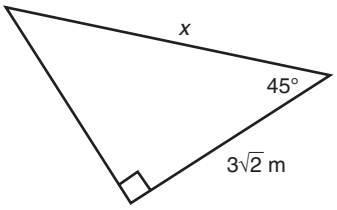
I would use the sine ratio because the hypotenuse is given and the length of the side opposite the given angle needs to be determined.

8.



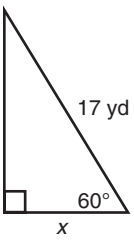
I would use the cotangent ratio because the side opposite the given angle is given and the length of the side adjacent to the given angle needs to be determined.

9.



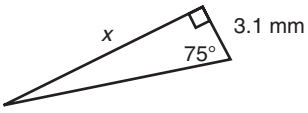
I would use the secant ratio because the side adjacent to the given angle is given and the length of the hypotenuse needs to be determined.

10.



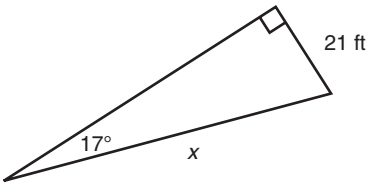
I would use the cosine ratio because the hypotenuse is given and the length of the side adjacent to the given angle needs to be determined.

11.



I would use the tangent ratio because the side adjacent to the given angle is given and the length of the side opposite the given angle needs to be determined.

12.



I would use the cosecant ratio because the side opposite the given angle is given and the length of the hypotenuse needs to be determined.

Name _____ Date _____

Solve each problem. Round your answers to the nearest hundredth.

13. You are standing 40 feet away from a building. The angle of elevation from the ground to the top of the building is 57°. What is the height of the building?

$\tan 57° = \frac{h}{40}$

$40 \tan 57° = h$

$h \approx 61.59$ ft

14. A surveyor is 3 miles from a mountain. The angle of elevation from the ground to the top of the mountain is 15°. What is the height of the mountain?

$\tan 15° = \frac{h}{3}$

$3 \tan 15° = h$

$h \approx 0.80$ mi

15. The angle of elevation from a ship to a 135-foot-tall lighthouse is 2°. How far is the ship from the lighthouse?

$\tan 2° = \frac{135}{d}$

$d = \frac{135}{\tan 2°}$

$d \approx 3865.89$ ft

16. The Statue of Liberty is about 151 feet tall. If the angle of elevation from a tree in Liberty State Park to the statue's top is 1.5°, how far is the tree from the statue?

$\tan 1.5° = \frac{151}{d}$

$d = \frac{151}{\tan 1.5°}$

$d \approx 5766.46$ ft

17. The angle of elevation from the top of a person's shadow on the ground to the top of the person is 45°. The top of the shadow is 50 inches away from the person. How tall is the person?

$$\tan 45° = \frac{h}{50}$$
$$50 \tan 45° = h$$
$$h = 50 \text{ in.}$$

18. A plane is spotted above a hill that is 12,000 feet away. The angle of elevation to the plane is 28°. How high is the plane?

$$\tan 28° = \frac{h}{12{,}000}$$
$$12{,}000 \tan 28° = h$$
$$h \approx 6380.51 \text{ ft}$$

19. During the construction of a house, a 6-foot-long board is used to support a wall. The board has an angle of elevation from the ground to the wall of 67°. How far is the base of the wall from the board?

$$\cos 67° = \frac{d}{6}$$
$$6 \cos 67° = d$$
$$d \approx 2.34 \text{ ft}$$

20. Museums use metal rods to position the bones of dinosaurs. If an angled rod needs to be placed 1.3 meters away from a bone, with an angle of elevation from the ground of 51°, what must the length of the rod be?

$$\cos 51° = \frac{1.3}{r}$$
$$r = \frac{1.3}{\cos 51°}$$
$$r \approx 2.07 \text{ m}$$

Name _____ Date _____

Solve each problem. Round your answers to the nearest hundredth.

21. The angle of depression from the top of a building to a telephone line is 34°. If the building is 25 feet tall, how far from the building does the telephone line reach the ground?

$$\tan 34° = \frac{25}{d}$$

$$d = \frac{25}{\tan 34°}$$

$$d \approx 37.06 \text{ ft}$$

22. An airplane flying 3500 feet from the ground sees an airport at an angle of depression of 77°. How far is the airplane from the airport?

$$\tan 77° = \frac{3500}{d}$$

$$d = \frac{3500}{\tan 77°}$$

$$d \approx 808.04 \text{ ft}$$

23. To determine the depth of a well's water, a hydrologist measures the diameter of the well to be 3 feet. She then uses a flashlight to point down to the water on the other side of the well. The flashlight makes an angle of depression of 79°. What is the depth of the well water?

$$\tan 79° = \frac{d}{3}$$

$$3 \tan 79° = d$$

$$d \approx 15.43 \text{ ft}$$

24. A zip wire from a tree to the ground has an angle of depression of 18°. If the zip wire ends 250 feet from the base of the tree, how far up the tree does the zip wire start?

$$\tan 18° = \frac{h}{250}$$

$$250 \tan 18° = h$$

$$h \approx 81.23 \text{ ft}$$

25. From a 50-foot-tall lookout tower, a park ranger sees a fire at an angle of depression of 1.6°. How far is the fire from the tower?

$\tan 1.6° = \frac{50}{d}$

$d = \frac{50}{\tan 1.6°}$

$d \approx 1790.03$ ft

26. The Empire State Building is 448 meters tall. The angle of depression from the top of the Empire State Building to the base of the UN building is 74°. How far is the UN building from the Empire State Building?

$\tan 74° = \frac{448}{d}$

$d = \frac{448}{\tan 74°}$

$d \approx 128.46$ m

27. A factory conveyor has an angle of depression of 18° and drops 10 feet. How long is the conveyor?

$\sin 18° = \frac{10}{l}$

$l = \frac{10}{\sin 18°}$

$l \approx 32.36$ ft

28. A bicycle race organizer needs to put up barriers along a hill. The hill is 300 feet tall and from the top makes an angle of depression of 26°. How long does the barrier need to be?

$\sin 26° = \frac{300}{l}$

$l = \frac{300}{\sin 26°}$

$l \approx 684.35$ ft

Name _____ Date _____

Time to Derive!

Deriving the Triangle Area Formula, the Law of Sines, and the Law of Cosines

Vocabulary

Define each term in your own words.

1. Law of Sines

   The Law of Sines states that the ratios of the sines of each angle measure to the opposite sides are equal: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

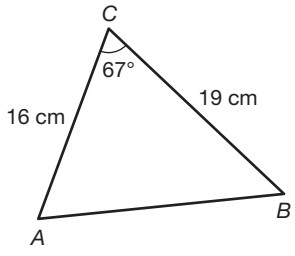
2. Law of Cosines

   The Law of Cosines describes the lengths of the squares of the sides of any triangle in terms of the cosines of their corresponding angles and the lengths of the other two sides:

   $$a^2 = c^2 + b^2 - 2bc \cdot \cos A$$
   $$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
   $$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Problem Set

Determine the area of each triangle. Round your answers to the nearest tenth.

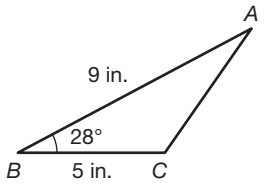
1. 

   $A = \frac{1}{2}ab \sin C$

   $A = \frac{1}{2}(19)(16)(\sin 67°)$

   $A \approx 139.9$

   The area of the triangle is approximately 139.9 square centimeters.

2. 

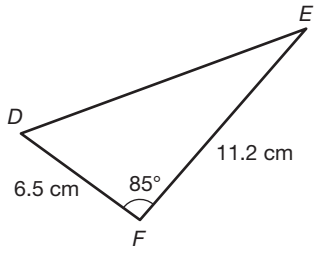
   $A = \frac{1}{2}ac \sin B$

   $A = \frac{1}{2}(5)(9)(\sin 28°)$

   $A \approx 10.6$

   The area of the triangle is approximately 10.6 square inches.

3.



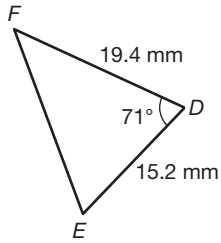
$A = \frac{1}{2} df \sin E$

$A = \frac{1}{2}(11.2)(6.5)(\sin 85°)$

$A \approx 36.3$

The area of the triangle is approximately 36.3 square centimeters.

4.



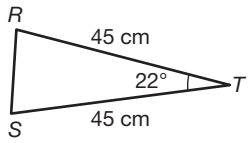
$A = \frac{1}{2} ef \sin D$

$A = \frac{1}{2}(19.4)(15.2)(\sin 71°)$

$A \approx 139.4$

The area of the triangle is approximately 139.4 square millimeters.

5.



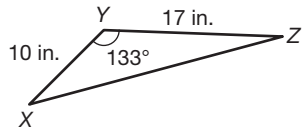
$A = \frac{1}{2} rs \sin T$

$A = \frac{1}{2}(45)(45)(\sin 22°)$

$A \approx 379.3$

The area of the triangle is approximately 379.3 square centimeters.

6.



$A = \frac{1}{2} xz \sin Y$

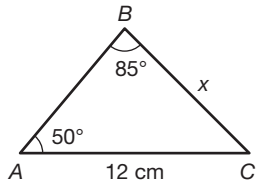
$A = \frac{1}{2}(17)(10)(\sin 133°)$

$A \approx 62.2$

The area of the triangle is approximately 62.2 square inches.

Determine the unknown side length x by using the Law of Sines. Round your answers to the nearest tenth.

7.



$\frac{\sin A}{a} = \frac{\sin B}{b}$

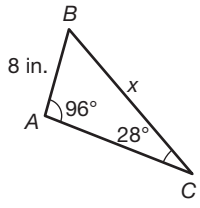
$\frac{\sin 50°}{x} = \frac{\sin 85°}{12}$

$12 \sin 50° = x \sin 85°$

$x = \frac{12 \sin 50°}{\sin 85°}$

$x \approx 9.2$ cm

8.



$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 96°}{x} = \frac{\sin 28°}{8}$

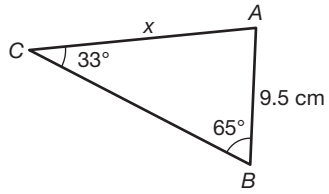
$8 \sin 96° = x \sin 28°$

$x = \frac{8 \sin 96°}{\sin 28°}$

$x \approx 16.9$ in.

Name _____ Date _____

9.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

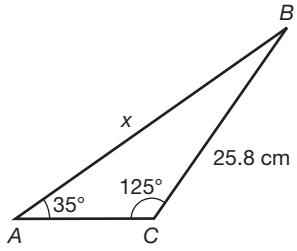
$\frac{\sin 65°}{x} = \frac{\sin 33°}{9.5}$

$9.5 \sin 65° = x \sin 33°$

$x = \frac{9.5 \sin 65°}{\sin 33°}$

$x \approx 15.8 \text{ cm}$

10.



$\frac{\sin A}{a} = \frac{\sin C}{c}$

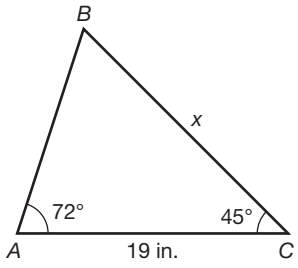
$\frac{\sin 35°}{25.8} = \frac{\sin 125°}{x}$

$x \sin 35° = 25.8 \sin 125°$

$x = \frac{25.8 \sin 125°}{\sin 35°}$

$x \approx 36.8 \text{ cm}$

11.



$m\angle B = 180° - 72° - 45° = 63°$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

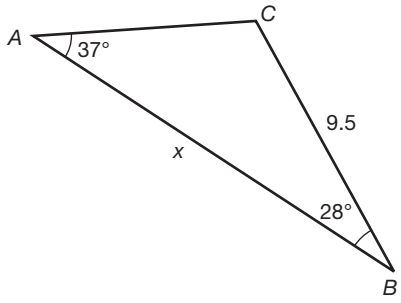
$\frac{\sin 72°}{x} = \frac{\sin 63°}{19}$

$19 \sin 72° = x \sin 63°$

$x = \frac{19 \sin 72°}{\sin 63°}$

$x \approx 20.3 \text{ in.}$

12.



$m\angle C = 180° - 37° - 28° = 115°$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 37°}{9.5} = \frac{\sin 115°}{x}$

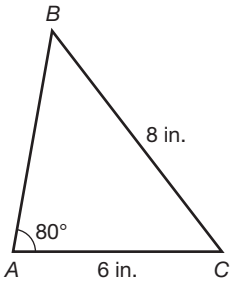
$x \sin 37° = 9.5 \sin 115°$

$x = \frac{9.5 \sin 115°}{\sin 37°}$

$x \approx 14.3 \text{ in.}$

Determine $m\angle B$ by using the Law of Sines. Round your answers to the nearest tenth.

13.



$\frac{\sin B}{b} = \frac{\sin A}{a}$

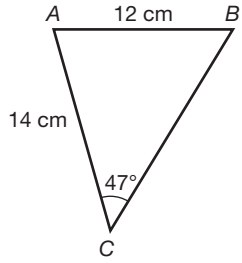
$\frac{\sin B}{6} = \frac{\sin 80°}{8°}$

$8 \sin B = 6 \sin 80°$

$\sin B = \frac{6 \sin 80°}{8} \approx 0.739$

$m\angle B = \sin^{-1}(0.739) \approx 47.6°$

14.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

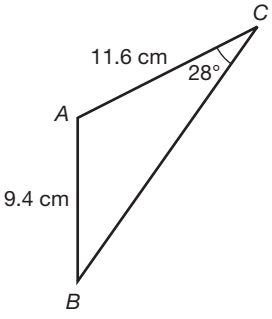
$\frac{\sin B}{14} = \frac{\sin 47°}{12°}$

$12 \sin B = 14 \sin 47°$

$\sin B = \frac{14 \sin 80°}{12} \approx 0.853$

$m\angle B = \sin^{-1}(0.853) \approx 58.5°$

15.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

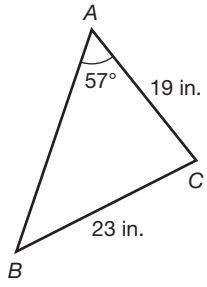
$\frac{\sin B}{11.6} = \frac{\sin 28°}{9.4°}$

$9.4 \sin B = 11.6 \sin 28°$

$\sin B = \frac{11.6 \sin 28°}{9.4} \approx 0.579$

$m\angle B = \sin^{-1}(0.579) \approx 35.4°$

16.



$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{19} = \frac{\sin 57°}{23°}$

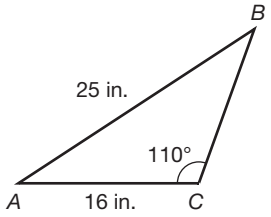
$23 \sin B = 19 \sin 57°$

$\sin B = \frac{19 \sin 57°}{23} \approx 0.693$

$m\angle B = \sin^{-1}(0.693) \approx 43.9°$

Name _____ Date _____

17.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

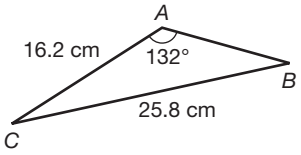
$\frac{\sin B}{16} = \frac{\sin 110°}{25°}$

$25 \sin B = 16 \sin 110°$

$\sin B = \frac{16 \sin 110°}{25} \approx 0.601$

$m\angle B = \sin^{-1}(0.601) \approx 36.9°$

18.



$\frac{\sin B}{B} = \frac{\sin A}{a}$

$\frac{\sin B}{16.2} = \frac{\sin 132°}{25.8°}$

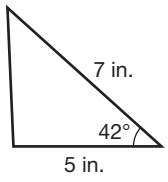
$25.8 \sin B = 16.2 \sin 132°$

$\sin B = \frac{16.2 \sin 132°}{25.8} \approx 0.467$

$m\angle B = \sin^{-1}(0.467) \approx 27.8°$

Determine the unknown side length by using the Law of Cosines. Round your answers to the nearest tenth.

19.



$b^2 = a^2 + c^2 - 2ac \cos B$

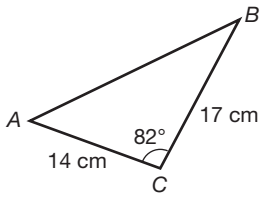
$b^2 = 5^2 + 7^2 - 2(5)(7)\cos 42°$

$b^2 = 25 + 49 - 70 \cos 42° \approx 21.98$

$b = \sqrt{21.98}$

$b \approx 4.7$ in.

20.



$c^2 = a^2 + b^2 - 2ab \cos C$

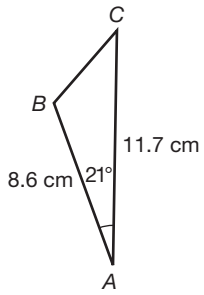
$c^2 = 14^2 + 17^2 - 2(14)(17)\cos 82°$

$c^2 = 196 + 289 - 476 \cos 82° \approx 418.75$

$c = \sqrt{418.75}$

$c \approx 20.5$ cm

21.



$a^2 = b^2 + c^2 - 2bc \cos A$

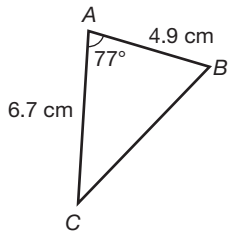
$a^2 = 11.7^2 + 8.6^2 - 2(11.7)(8.67)\cos 21°$

$a^2 = 136.89 + 73.96 - 201.24 \cos 21° \approx 22.98$

$a = \sqrt{22.98}$

$a \approx 4.8$ cm

22.



$a^2 = b^2 + c^2 - 2bc \cos A$

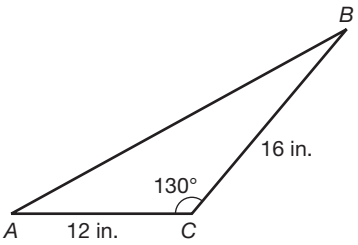
$a^2 = 6.7^2 + 4.9^2 - 2(6.7)(4.9)\cos 77°$

$a^2 = 44.89 + 24.01 - 65.66 \cos 77° \approx 54.13$

$a = \sqrt{54.13}$

$a \approx 7.4$ cm

23.



$c^2 = a^2 + b^2 - 2ab \cos C$

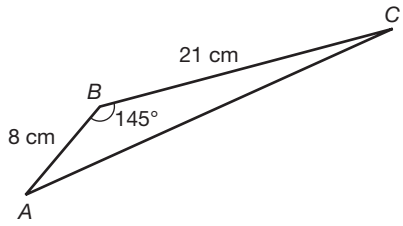
$a^2 = 16^2 + 12^2 - 2(16)(12)\cos 130°$

$a^2 = 256 + 144 - 384 \cos 130° \approx 646.83$

$a = \sqrt{646.83}$

$a \approx 25.4$ in.

24.



$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 21^2 + 8^2 - 2(21)(8)\cos 145°$

$b^2 = 441 + 64 - 336 \cos 145° \approx 780.24$

$b = \sqrt{780.24}$

$b \approx 27.9$ cm