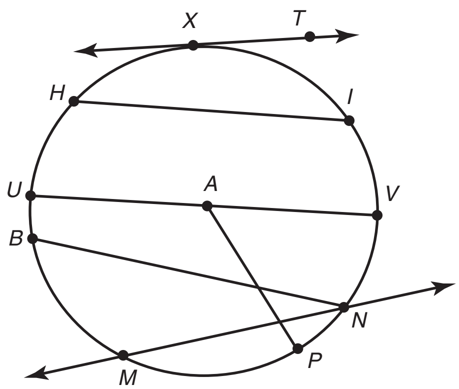
Name ______________________________ Date ____________

Riding a Ferris Wheel
Introduction to Circles

Vocabulary

Identify an instance of each term in the diagram.



1. center of the circle
   point A

2. chord
   $\overline{HI}$, $\overline{UV}$, $\overline{BN}$, or $\overline{MN}$

3. secant of the circle
   $\overleftrightarrow{MN}$

4. tangent of the circle
   $\overleftrightarrow{XT}$

5. point of tangency
   point X

6. central angle
   $\angle VAP$

7. inscribed angle
   $\angle MNB$

8. arc
   Sample answer: $\overset{\frown}{HI}$

9. major arc
   Sample answer: $\overset{\frown}{HBV}$

10. minor arc
    Sample answer: $\overset{\frown}{MP}$

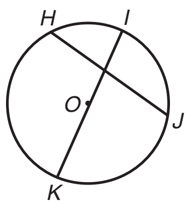
11. diameter
    $\overline{UV}$

12. semicircle
    $\overline{UV}$

Problem Set

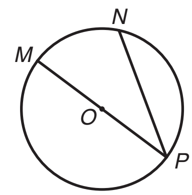
Identify the indicated part of each circle. Explain your answer.

1. O



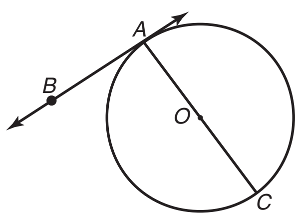
Point O is the center of the circle. It is a point that is the same distance from each point on the circle.

2. $\overline{NP}$



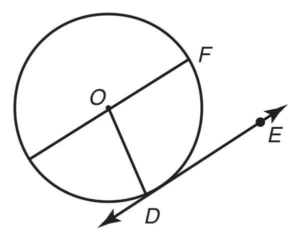
Segment NP is a chord. It is a line segment that connects two points on the circle.

3. $\overleftrightarrow{AB}$



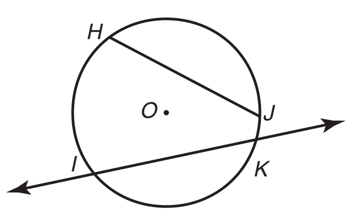
Line AB is a tangent. It is a line that intersects the circle at exactly one point, A.

4. D



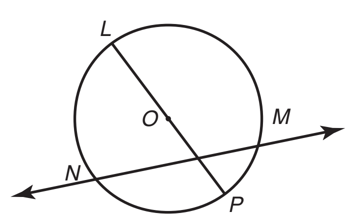
Point D is a point of tangency. It is the point at which the tangent $\overleftrightarrow{DE}$ intersects the circle.

5. $\overline{JH}$



Segment JH is a chord. It is a line segment that connects two points on the circle.

6. $\overleftrightarrow{MN}$



Line MN is a secant. It is a line that intersects the circle at two points.

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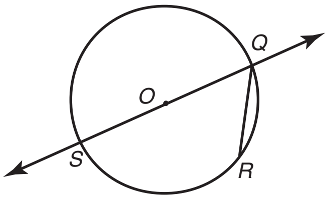
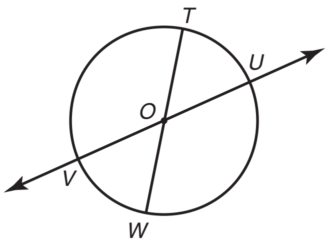
7. ∠*SQR*



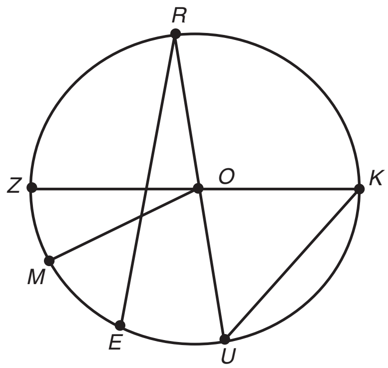
Angle *SQR* is an inscribed angle. It is an angle whose vertex lies on the circle.

8. ∠*TOU*



Angle *TOU* is a central angle. It is an angle whose vertex is the center of the circle.

Identify each angle as an inscribed angle or a central angle.



9. ∠*URE*

Angle *URE* is an inscribed angle.

10. ∠*ZOM*

Angle *ZOM* is a central angle.

11. ∠*KOM*

Angle *KOM* is a central angle.

12. ∠*ZKU*

Angle *ZKU* is an inscribed angle.

13. ∠*MOU*

Angle *MOU* is a central angle.

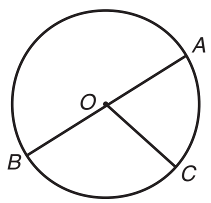
14. ∠*ROK*

Angle *ROK* is a central angle.

9

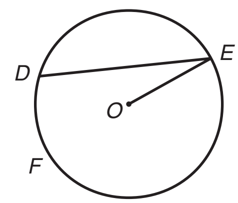
Classify each arc as a major arc, a minor arc, or a semicircle.

15. $\overset{\frown}{AC}$



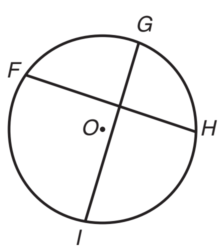
Arc *AC* is a minor arc.

16. $\overset{\frown}{DE}$



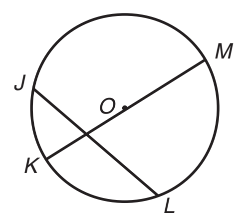
Arc *DE* is a minor arc.

17. $\overset{\frown}{FHI}$



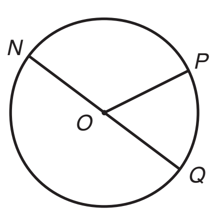
Arc *FHI* is a major arc.

18. $\overset{\frown}{JML}$



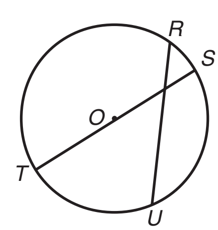
Arc *JML* is a major arc.

19. $\overset{\frown}{NPQ}$



Arc *NPQ* is a semicircle.

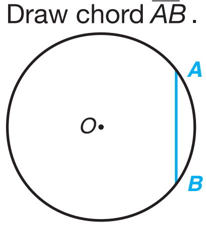
20. $\overset{\frown}{TRS}$



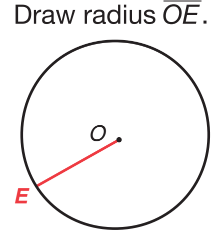
Arc *TRS* is a semicircle.

Draw the part of a circle that is described.

21. Draw chord $\overline{AB}$.



22. Draw radius $\overline{OE}$.



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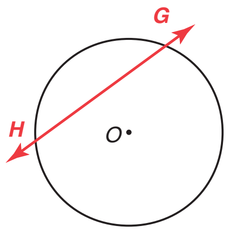
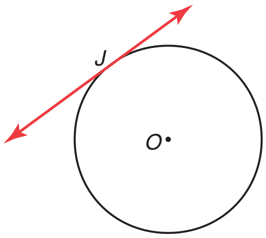
23. Draw secant $\overleftrightarrow{GH}$.

Answers will vary.



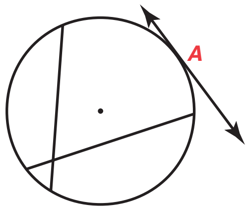
24. Draw a tangent at point *J*.

Answers will vary.



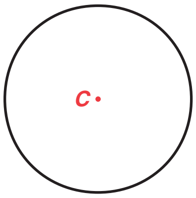
25. Label the point of tangency *A*.

Answers will vary.



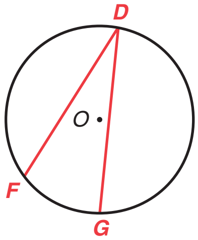
26. Label center *C*.

Answers will vary.



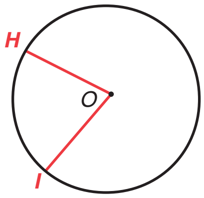
27. Draw inscribed angle $\angle FDG$.

Answers will vary.



28. Draw central angle $\angle HOI$.

Answers will vary.



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Name ________________________________ Date ____________

Take the Wheel
Central Angles, Inscribed Angles, and Intercepted Arcs

Vocabulary

Define each term in your own words.

1. degree measure of a minor arc

   The degree measure of a minor arc is the same as the measure of its central angle.

2. adjacent arcs

   Adjacent arcs are two arcs of the same circle sharing a common endpoint.

3. Arc Addition Postulate

   The Arc Addition Postulate states: "The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs."

4. intercepted arc

   An intercepted arc is an arc associated with and determined by angles of a circle. An intercepted arc is a portion of the circumference of the circle located on the interior of the angle whose endpoints lie on the sides of the angle.

5. Inscribed Angle Theorem

   The Inscribed Angle Theorem states: "The measure of an inscribed angle is equal to one-half the measure of its intercepted arc."

6. Parallel Lines–Congruent Arcs Theorem

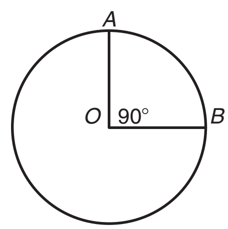
   The Parallel Lines-Congruent Arcs Theorem states: "Parallel lines intercept congruent arcs on a circle."

9

Problem Set

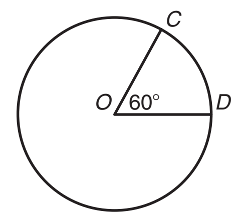
Determine the measure of each minor arc.

1. $\overset{\frown}{AB}$



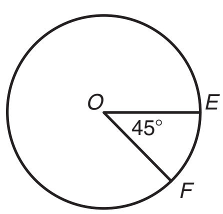
The measure of $\overset{\frown}{AB}$ is 90°.

2. $\overset{\frown}{CD}$



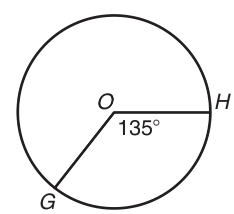
The measure of $\overset{\frown}{CD}$ is 60°.

3. $\overset{\frown}{EF}$



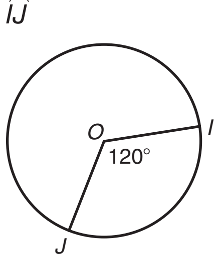
The measure of $\overset{\frown}{EF}$ is 45°.

4. $\overset{\frown}{GH}$



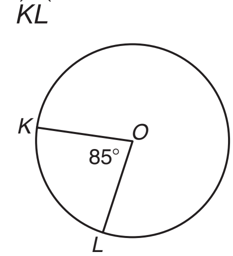
The measure of $\overset{\frown}{GH}$ is 135°.

5. $\overset{\frown}{IJ}$



The measure of $\overset{\frown}{IJ}$ is 120°.

6. $\overset{\frown}{KL}$



The measure of $\overset{\frown}{KL}$ is 85°.

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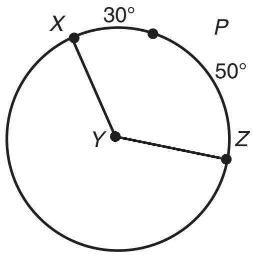
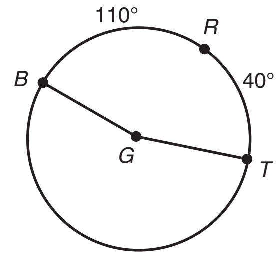
Determine the measure of each central angle.

7. $m\angle XYZ$



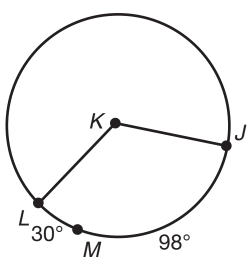
$m\angle XYZ = 80°$

8. $m\angle BGT$



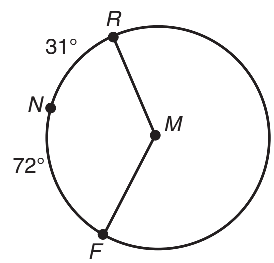
$m\angle BGT = 150°$

9. $m\angle LKJ$



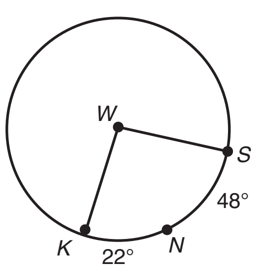
$m\angle LKJ = 128°$

10. $m\angle FMR$



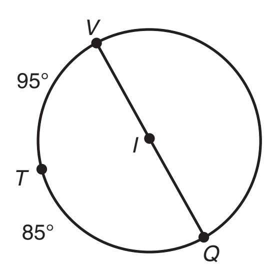
$m\angle FMR = 103°$

11. $m\angle KWS$



$m\angle KWS = 70°$

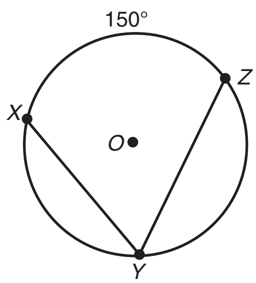
12. $m\angle VIQ$



$m\angle VIQ = 180°$

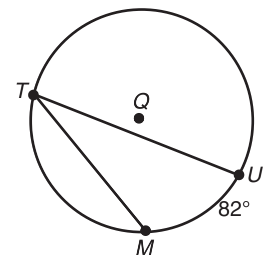
Determine the measure of each inscribed angle.

13. $m\angle XYZ$



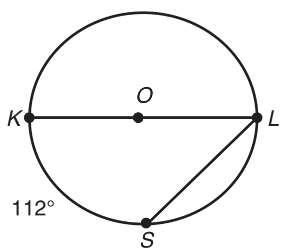
$m\angle XYZ = 75°$

14. $m\angle MTU$



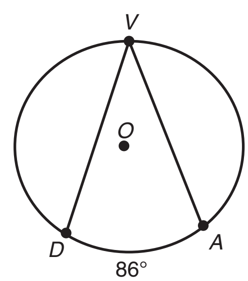
$m\angle MTU = 41°$

15. $m\angle KLS$



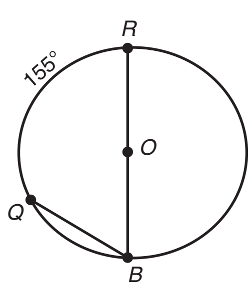
$m\angle KLS = 56°$

16. $m\angle DVA$



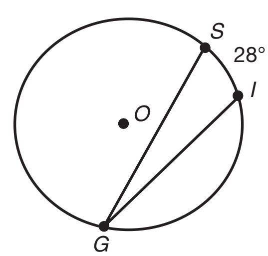
$m\angle DVA = 43°$

17. $m\angle QBR$



$m\angle QBR = 77.5°$

18. $m\angle SGI$



$m\angle SGI = 14°$

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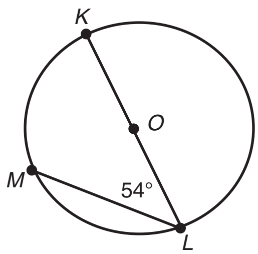
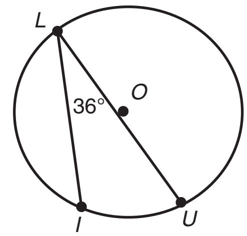
Determine the measure of each intercepted arc.

19. $m\overset{\frown}{KM}$



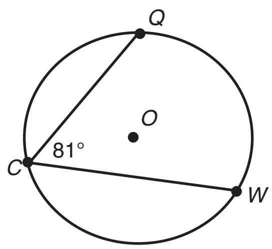
$m\overset{\frown}{KM} = 108°$

20. $m\overset{\frown}{IU}$



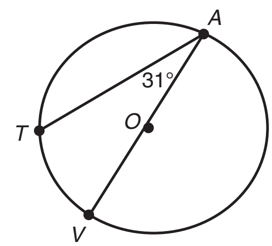
$m\overset{\frown}{IU} = 72°$

21. $m\overset{\frown}{QW}$



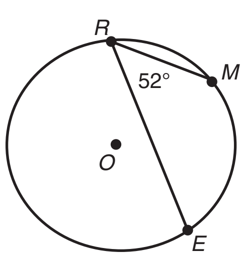
$m\overset{\frown}{QW} = 162°$

22. $m\overset{\frown}{TV}$



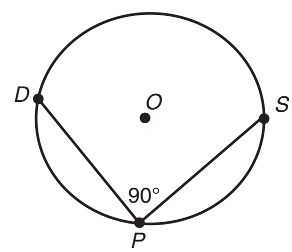
$m\overset{\frown}{TV} = 62°$

23. $m\overset{\frown}{ME}$



$m\overset{\frown}{ME} = 104°$

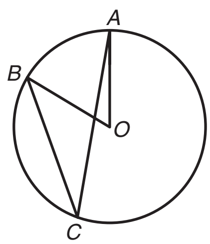
24. $m\overset{\frown}{DS}$



$m\overset{\frown}{DS} = 180°$

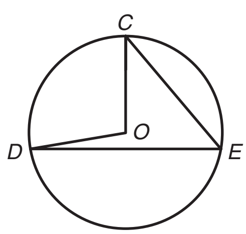
Calculate the measure of each angle.

25. The measure of $\angle AOB$ is 62°. What is the measure of $\angle ACB$?



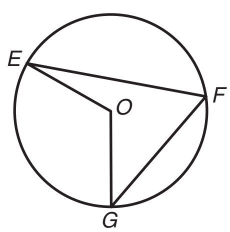
$m\angle ACB = \frac{1}{2}(m\angle AOB) = \frac{62°}{2} = 31°$

26. The measure of $\angle COD$ is 98°. What is the measure of $\angle CED$?



$m\angle CED = \frac{1}{2}(m\angle COD) = \frac{98}{2°} = 49°$

27. The measure of $\angle EOG$ is 128°. What is the measure of $\angle EFG$?



$m\angle EFG = \frac{1}{2}(m\angle EOG) = \frac{128°}{2} = 64°$

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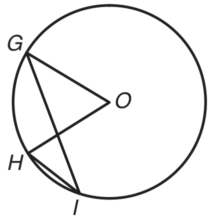
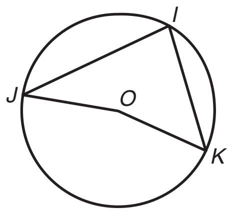
28. The measure of $\angle GOH$ is 74°. What is the measure of $\angle GIH$?



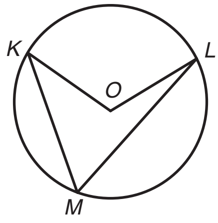
$m\angle GIH = \frac{1}{2}(m\angle GOH) = \frac{74°}{2} = 37°$

29. The measure of $\angle JOK$ is 168°. What is the measure of $\angle JIK$?



$m\angle JIK = \frac{1}{2}(m\angle JOK) = \frac{168°}{2} = 84°$

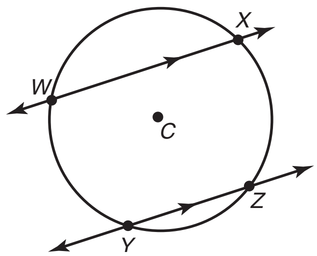
30. The measure of $\angle KOL$ is 148°. What is the measure of $\angle KML$?



$m\angle KML = \frac{1}{2}(m\angle KOL) = \frac{148°}{2} = 74°$

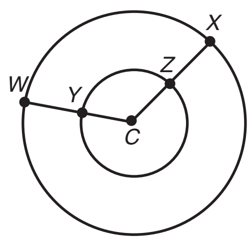
Use the given information to answer each question.

31. In circle C, $m\widehat{XZ} = 86°$. What is $m\widehat{WY}$?



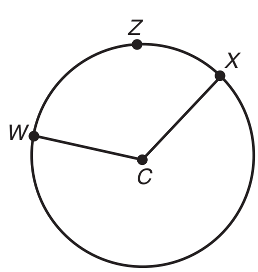
$m\widehat{WY} = 86°$

32. In circle C, $m\angle WCX = 102°$. What is $m\widehat{YZ}$?



$m\widehat{YZ} = 102°$

33. In circle C, $m\widehat{WZ} = 65°$ and $m\widehat{XZ} = 38°$. What is $m\angle WCX$?



$m\angle WCX = 65° + 38° = 103°$

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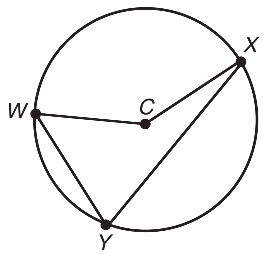
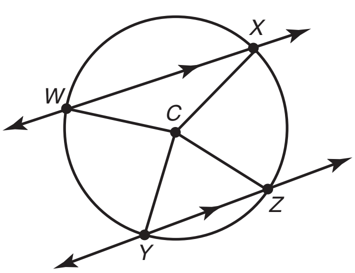
34. In circle C, $m\angle WCX = 105°$. What is $m\angle WYX$?



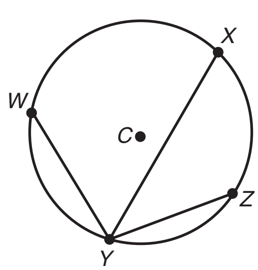
$m\angle WYX = \frac{1}{2}(105°) = 52.5°$

35. In circle C, $m\angle WCY = 83°$. What is $m\angle XCZ$?



$m\angle XCZ = 83°$

36. In circle C, $m\angle WYX = 50°$ and $m\angle XYZ = 30°$. What is $m\overset{\frown}{WXZ}$?



$m\overset{\frown}{WXZ} = 2(50° + 30°) = 2(80°) = 160°$

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Manhole Covers
Measuring Angles Inside and Outside of Circles

Vocabulary

Define each theorem in your own words.

1. Interior Angles of a Circle Theorem
   The Interior Angles of a Circle Theorem states that an interior angle measure is one half of the sum of the intercepted arcs.

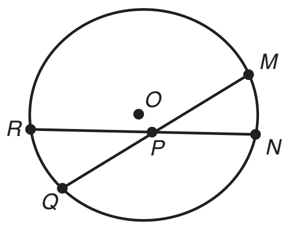
2. Exterior Angles of a Circle Theorem
   The Exterior Angles of a Circle Theorem states that an exterior angle measure is one half of the difference of the intercepted arcs.

3. Tangent to a Circle Theorem
   The Tangent to a Circle Theorem states that a line drawn tangent to a circle is perpendicular to a radius of the circle drawn to the point of tangency.

Problem Set

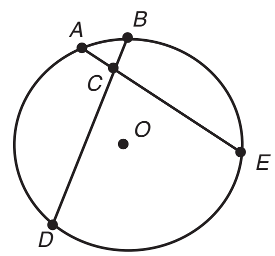
Write an expression for the measure of the given angle.

1. $m\angle RPM$



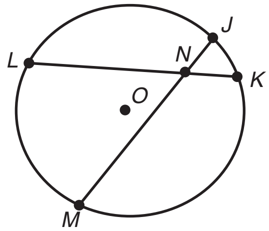
$m\angle RPM = \frac{1}{2}(m\widehat{RM} + m\widehat{QN})$

2. $m\angle ACD$



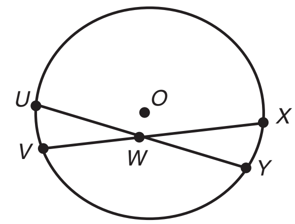
$m\angle ACD = \frac{1}{2}(m\widehat{AD} + m\widehat{BE})$

3. $m\angle JNK$



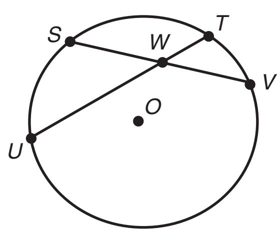
$m\angle JNK = \frac{1}{2}(m\overset{\frown}{LM} + m\overset{\frown}{JK})$

4. $m\angle UWV$



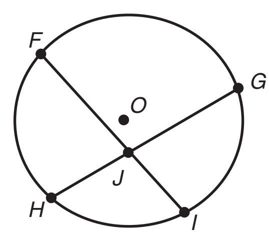
$m\angle UWV = \frac{1}{2}(m\overset{\frown}{UV} + m\overset{\frown}{XY})$

5. $m\angle SWT$



$m\angle SWT = \frac{1}{2}(m\overset{\frown}{ST} + m\overset{\frown}{VU})$

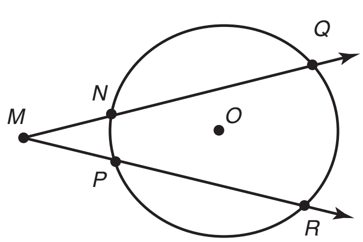
6. $m\angle HJI$



$m\angle HJI = \frac{1}{2}(m\overset{\frown}{FG} + m\overset{\frown}{HI})$

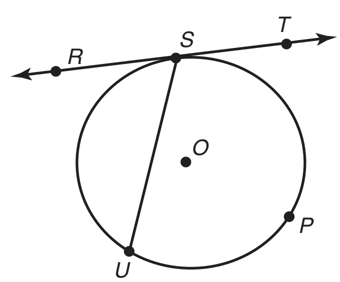
List the intercepted arc(s) for the given angle.

7. $\angle QMR$



$\overset{\frown}{NP}$, $\overset{\frown}{QR}$

8. $\angle RSU$



$\overset{\frown}{SU}$

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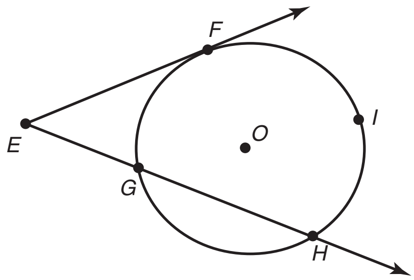
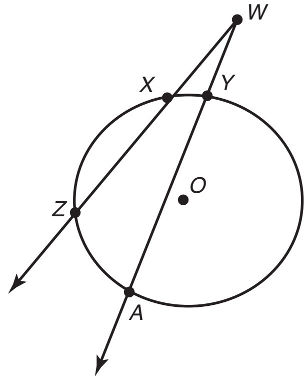
9. $\angle FEH$



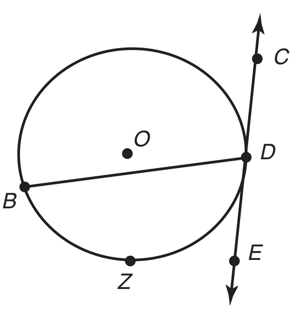
$\overset{\frown}{FIH}$, $\overset{\frown}{GF}$

10. $\angle ZWA$



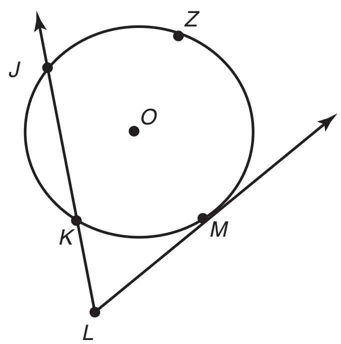
$\overset{\frown}{ZA}$, $\overset{\frown}{XY}$

11. $\angle BDE$



$\overset{\frown}{BZD}$

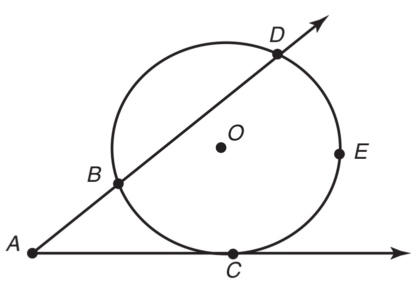
12. $\angle JLM$



$\overset{\frown}{JZM}$, $\overset{\frown}{KM}$

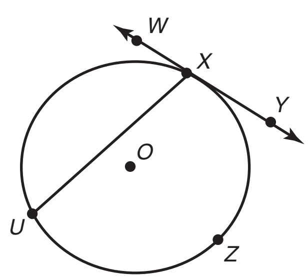
Write an expression for the measure of the given angle.

13. $m\angle DAC$



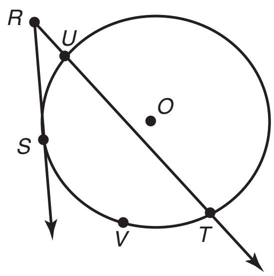
$m\angle DAC = \frac{1}{2}(m\overset{\frown}{DEC} - m\overset{\frown}{BC})$

14. $m\angle UXY$



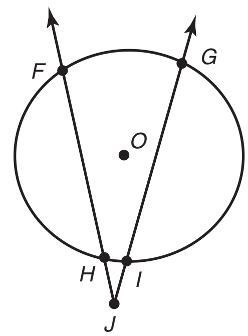
$m\angle UXY = \frac{1}{2}(m\overset{\frown}{XZU})$

15. $m\angle SRT$



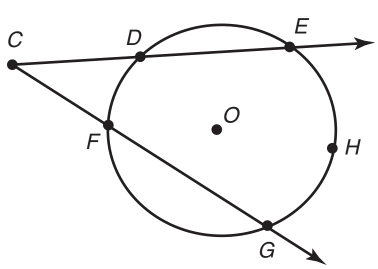
$m\angle SRT = \frac{1}{2}(m\widehat{SVT} - m\widehat{SU})$

16. $m\angle FJG$



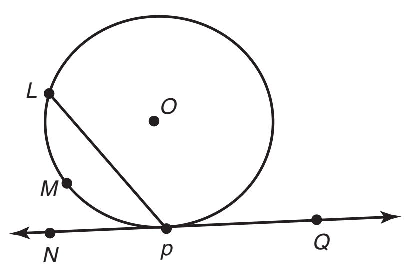
$m\angle FJG = \frac{1}{2}(m\widehat{FG} - m\widehat{HI})$

17. $m\angle ECG$



$m\angle ECG = \frac{1}{2}(m\widehat{EHG} - m\widehat{DF})$

18. $m\angle LPN$



$m\angle LPN = \frac{1}{2}(m\widehat{LMP})$

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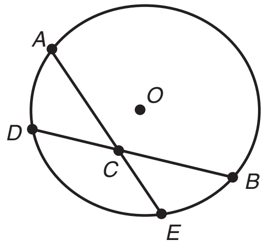
Create a proof to prove each statement.

19. Given: Chords $\overline{AE}$ and $\overline{BD}$ intersect at point C.

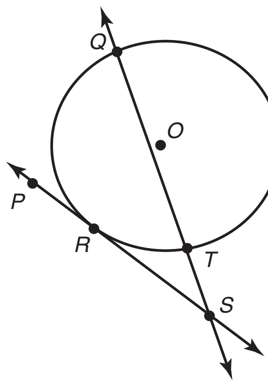
Prove: $m\angle ACB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DE})$



Statements	Reasons
1. Chords $\overline{AE}$ and $\overline{BD}$ intersect at point C.	1. Given
2. Draw chord $\overline{AD}$	2. Construction
3. $m\angle ACB = m\angle D + m\angle A$	3. Exterior Angle Theorem
4. $m\angle A = \frac{1}{2}m\overset{\frown}{DE}$	4. Inscribed Angle Theorem
5. $m\angle D = \frac{1}{2}m\overset{\frown}{AB}$	5. Inscribed Angle Theorem
6. $m\angle ACB = \frac{1}{2}m\overset{\frown}{DE} + \frac{1}{2}m\overset{\frown}{AB}$	6. Substitution
7. $m\angle ACB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DE})$	7. Distributive Property

20. Given: Secant $\overleftrightarrow{QT}$ and tangent $\overleftrightarrow{SR}$ intersect at point S.

Prove: $m\angle QSR = \frac{1}{2}(m\overset{\frown}{QR} - m\overset{\frown}{RT})$



Statements	Reasons
1. Secant $\overleftrightarrow{QT}$ and tangent $\overleftrightarrow{SR}$ intersect at point S.	1. Given
2. Draw chord $\overline{QR}$	2. Construction
3. $m\angle QRP = m\angle RQS + m\angle QSR$	3. Exterior Angle Theorem
4. $m\angle QSR = m\angle QRP - m\angle RQS$	4. Subtraction Property of Equality
5. $m\angle QRP = \frac{1}{2}m\overset{\frown}{RQ}$	5. Exterior Angle of a Circle Theorem
6. $m\angle RQS = \frac{1}{2}m\overset{\frown}{RT}$	6. Inscribed Angle Theorem
7. $m\angle QSR = \frac{1}{2}m\overset{\frown}{RQ} - \frac{1}{2}m\overset{\frown}{RT}$	7. Substitution
8. $m\angle QSR = \frac{1}{2}(m\overset{\frown}{RQ} - m\overset{\frown}{RT})$	8. Distributive Property

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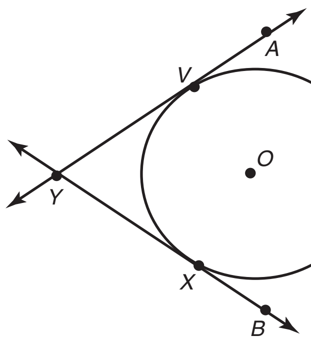
21. Given: Tangents $\overleftrightarrow{VY}$ and $\overleftrightarrow{XY}$ intersect at point Y.

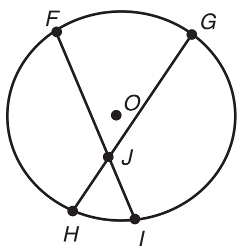
Prove: $m\angle Y = \frac{1}{2}(m\widehat{VWX} - m\widehat{VX})$



Statements	Reasons
1. Tangents $\overleftrightarrow{VY}$ and tangent $\overleftrightarrow{XY}$ intersect at point Y.	1. Given
2. Draw chord $\overline{VX}$	2. Construction
3. $m\angle VXB = m\angle Y + m\angle YVX$	3. Exterior Angle Theorem
4. $m\angle Y = m\angle VXB - m\angle YVX$	4. Subtraction Property of Equality
5. $m\angle VXB = \frac{1}{2} m\widehat{VWX}$	5. Exterior Angle of a Circle Theorem
6. $m\angle YVX = \frac{1}{2} m\widehat{VX}$	6. Exterior Angle of a Circle Theorem
7. $m\angle Y = \frac{1}{2} m\widehat{VWX} - \frac{1}{2} m\widehat{VX}$	7. Substitution
8. $m\angle Y = \frac{1}{2}(m\widehat{VWX} - m\widehat{VX})$	8. Distributive Property

22. Given: Chords $\overline{FI}$ and $\overline{GH}$ intersect at point J.

Prove: $m\angle FJH = \frac{1}{2}(m\overset{\frown}{FH} + m\overset{\frown}{GI})$



Statements	Reasons
1. Chords $\overline{FI}$ and $\overline{GH}$ intersect at point J.	1. Given
2. Draw chord $\overline{FG}$	2. Construction
3. $m\angle FJH = m\angle G + m\angle F$	3. Exterior Angle Theorem
4. $m\angle F = \frac{1}{2} m\overset{\frown}{GI}$	4. Inscribed Angle Theorem
5. $m\angle G = \frac{1}{2} m\overset{\frown}{FH}$	5. Inscribed Angle Theorem
6. $m\angle FJH = \frac{1}{2} m\overset{\frown}{FH} + \frac{1}{2} m\overset{\frown}{GI}$	6. Substitution
7. $m\angle FJH = \frac{1}{2}(m\overset{\frown}{FH} + m\overset{\frown}{GI})$	7. Distributive Property

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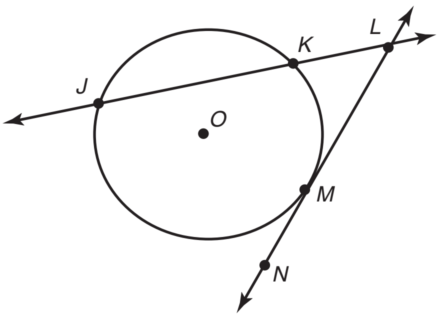
23. Given: Secant $\overleftrightarrow{JL}$ and tangent $\overleftarrow{NL}$ intersect at point L.

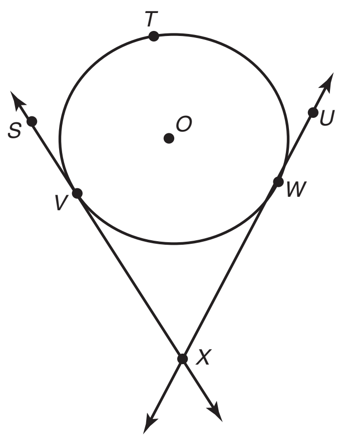
Prove: $m\angle L = \frac{1}{2}(m\overset{\frown}{JM} - m\overset{\frown}{KM})$



Statements	Reasons
1. Secant $\overleftrightarrow{JL}$ and tangent $\overleftarrow{NL}$ intersect at point L.	1. Given
2. Draw chord $\overline{JM}$	2. Construction
3. $m\angle JMN = m\angle MJL + m\angle L$	3. Exterior Angle Theorem
4. $m\angle L = m\angle JMN - m\angle MJL$	4. Subtraction Property of Equality
5. $m\angle JMN = \frac{1}{2}m\overset{\frown}{JM}$	5. Exterior Angle of a Circle Theorem
6. $m\angle MJL = \frac{1}{2}m\overset{\frown}{KM}$	6. Inscribed Angle Theorem
7. $m\angle L = \frac{1}{2}m\overset{\frown}{JM} - \frac{1}{2}m\overset{\frown}{KM}$	7. Substitution
8. $m\angle L = \frac{1}{2}(m\overset{\frown}{JM} - m\overset{\frown}{KM})$	8. Distributive Property

24. Given: Tangents $\overleftrightarrow{SX}$ and $\overleftrightarrow{UX}$ intersect at point X.

Prove: $m\angle X = \frac{1}{2}(m\overset{\frown}{VTW} - m\overset{\frown}{VW})$



Statements	Reasons
1. Tangents $\overleftrightarrow{SX}$ and $\overleftrightarrow{UX}$ intersect at point X.	1. Given
2. Draw chord $\overline{VW}$	2. Construction
3. $m\angle SVW = m\angle X + m\angle VWX$	3. Exterior Angle Theorem
4. $m\angle X = m\angle SVW - m\angle VWX$	4. Subtraction Property of Equality
5. $m\angle SVW = \frac{1}{2}m\overset{\frown}{VTW}$	5. Exterior Angle of a Circle Theorem
6. $m\angle VWX = \frac{1}{2}m\overset{\frown}{VW}$	6. Exterior Angle of a Circle Theorem
7. $m\angle X = \frac{1}{2}m\overset{\frown}{VTW} - \frac{1}{2}m\overset{\frown}{VW}$	7. Substitution
8. $m\angle X = \frac{1}{2}(m\overset{\frown}{VTW} - m\overset{\frown}{VW})$	8. Distributive Property

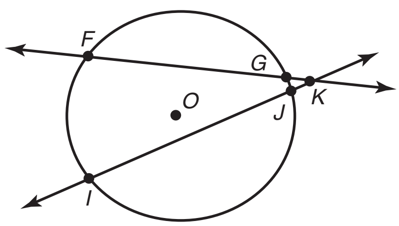
Name ____ Date ______________

Use the diagram shown to determine the measure of each angle or arc.

25. Determine $m\overset{\frown}{FI}$.

$m\angle K = 20°$

$m\overset{\frown}{GJ} = 80°$



The measure of arc *FI* is 120 degrees.

$m\angle K = \frac{1}{2}(m\overset{\frown}{FI} = m\overset{\frown}{GJ})$

$20 = \frac{1}{2}(m\overset{\frown}{FI} - 80)$

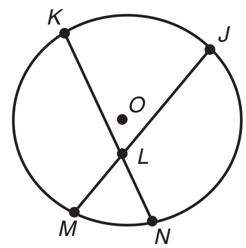
$40 = m\overset{\frown}{FI} - 80$

$m\overset{\frown}{FI} = 120$

26. Determine $m\angle KLJ$.

$m\overset{\frown}{KM} = 120°$

$m\overset{\frown}{JN} = 100°$



The measure of angle *KLJ* is 70 degrees.

First, I determined that the sum of the measures of arcs *KJ* and *MN* is 140 degrees

$m\overset{\frown}{KM} + m\overset{\frown}{JN} + m\overset{\frown}{KJ} + m\overset{\frown}{MN} = 360$

$120 + 100 + m\overset{\frown}{KJ} + m\overset{\frown}{MN} = 360$

$220 + m\overset{\frown}{KJ} + m\overset{\frown}{MN} = 360$

$m\overset{\frown}{KJ} + m\overset{\frown}{MN} = 140$

Then, I calculated the measure of angle *KLJ*.

$m\angle KLJ = \frac{1}{2}(m\overset{\frown}{KJ} + m\overset{\frown}{MN})$

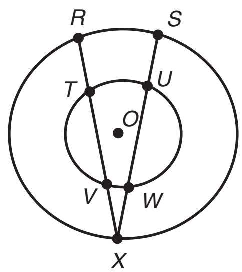
$= \frac{1}{2}(140)$

$= 70$

27. Determine $m\angle X$.

$m\overset{\frown}{VW} = 50°$

$m\overset{\frown}{TU} = 85°$



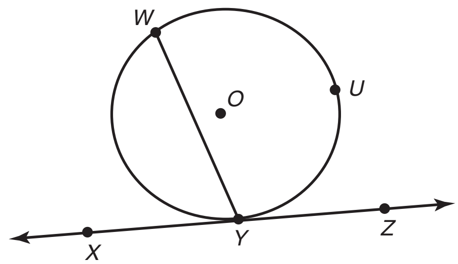
The measure of angle *X* is 17.5 degrees.

To solve the problem, I established that angle *X* is an exterior angle of the smaller circle and calculated its measure.

$$\begin{aligned} m\angle X &= \frac{1}{2}(m\overset{\frown}{TU} - m\overset{\frown}{VW}) \\ &= \frac{1}{2}(85 - 50) \\ &= \frac{1}{2}(35) \\ &= 17.5 \end{aligned}$$

28. Determine $m\angle WYX$.

$m\overset{\frown}{WUY} = 300°$



The measure of angle *WYX* is 30 degrees.

To solve the problem, I calculated the measure of angle *WYZ* first. Then, I used the fact that angle *WYZ* and angle *WYX* are supplementary to calculate the measure of angle *WYX*.

$$\begin{aligned} m\angle WYZ &= \frac{1}{2}(m\overset{\frown}{WUY}) \\ &= \frac{1}{2}(300) \\ &= 150 \end{aligned}$$

$$\begin{aligned} m\angle WYX + m\angle WYZ &= 180 \\ m\angle WYX + 150 &= 180 \\ m\angle WYX &= 30 \end{aligned}$$

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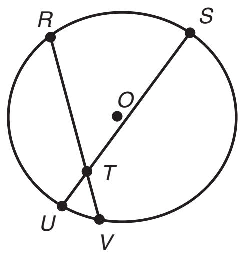
29. Determine $m\widehat{RS}$.

$m\widehat{UV} = 30°$

$m\angle RTS = 80°$



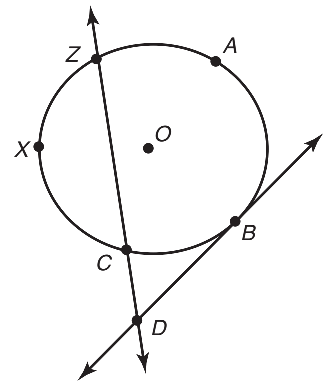
The measure of arc *RS* is 130 degrees.

$$m\angle RTS = \frac{1}{2}(m\widehat{RS} - m\widehat{UV})$$
$$80 = \frac{1}{2}(m\widehat{RS} + 30)$$
$$160 = m\widehat{RS} + 30$$
$$m\widehat{RS} = 130$$

30. Determine $m\angle D$.

$m\widehat{ZXC} = 150°$

$m\widehat{CB} = 30°$



The measure of angle *D* is 75 degrees.

To solve the problem, I calculated the measure of arc *ZAB* first. Then, I calculated the measure of angle *D*.

$$m\widehat{ZXC} + m\widehat{CB} + m\widehat{ZAB} = 360$$
$$150 + 30 + m\widehat{ZAB} = 360$$
$$180 + m\widehat{ZAB} = 360$$
$$m\widehat{ZAB} = 180$$

$$m\angle D = \frac{1}{2}(m\widehat{ZAB} - m\widehat{CB})$$
$$= \frac{1}{2}(180 - 30)$$
$$= \frac{1}{2}(150)$$
$$= 75$$

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Color Theory
Chords

Vocabulary

Match each definition with its corresponding term.

1. Diameter–Chord Theorem g
2. Equidistant Chord Theorem c
3. Equidistant Chord Converse Theorem e
4. Congruent Chord–Congruent Arc Theorem a
5. Congruent Chord–Congruent Arc Converse Theorem d
6. segments of a chord b
7. Segment–Chord Theorem f

a. If two chords of the same circle or congruent circles are congruent, then their corresponding arcs are congruent.

b. The segments formed on a chord when two chords of a circle intersect

c. If two chords of the same circle or congruent circles are congruent, then they are equidistant from the center of the circle.

d. If two arcs of the same circle or congruent circles are congruent, then their corresponding chords are congruent.

e. If two chords of the same circle or congruent circles are equidistant from the center of the circle, then the chords are congruent.

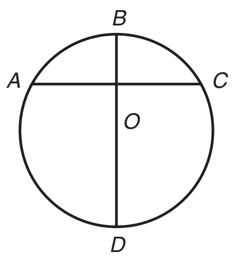
f. If two chords of a circle intersect, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments in the second chord.

g. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and bisects the arc determined by the chord.

Problem Set

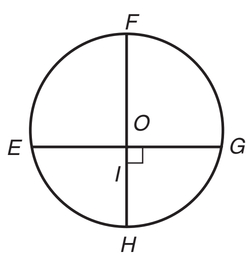
Use the given information to answer each question. Explain your answer.

1. If diameter $\overline{BD}$ bisects $\overline{AC}$, what is the angle of intersection?



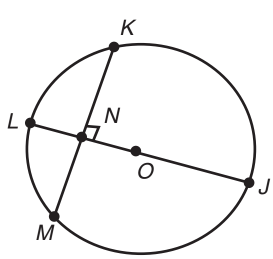
The angle of intersection is 90° because diameters that bisect chords are perpendicular bisectors.

2. If diameter $\overline{FH}$ intersects $\overline{EG}$ at a right angle, how does the length of $\overline{EI}$ compare to the length of $\overline{IG}$?



The length of $\overline{EI}$ is equal to the length of $\overline{IG}$ because a diameter that intersects a chord at a right angle is a perpendicular bisector.

3. How does the measure of $\overset{\frown}{KL}$ and $\overset{\frown}{LM}$ compare?



The measure of $\overset{\frown}{KL}$ is equal to the measure of $\overset{\frown}{LM}$ because a diameter that intersects a chord at a right angle bisects the arc formed by the chord.

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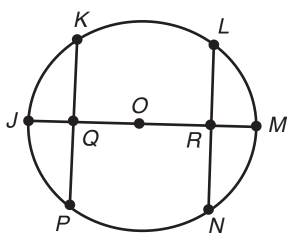
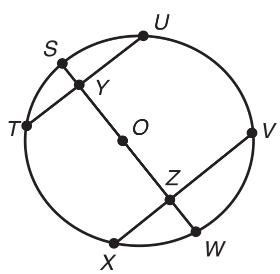
4. If $\overline{KP} \cong \overline{LN}$, how does the length of $\overline{QO}$ compare to the length of $\overline{RO}$?



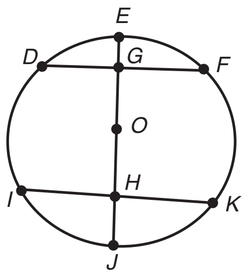
The length of $\overline{QO}$ is equal to the length of $\overline{RO}$ because congruent chords are the same distance from the center of the circle.

5. If $\overline{YO} \cong \overline{ZO}$, what is the relationship between $\overline{TU}$ and $\overline{XV}$?



Chords $\overline{TU}$ and $\overline{XV}$ are congruent because chords that are the same distance from the center of the circle are congruent.

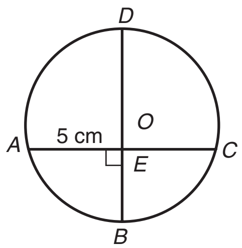
6. If $\overline{GO} \cong \overline{HO}$ and diameter $\overline{EJ}$ is perpendicular to both, what is the relationship between $\overline{GF}$ and $\overline{HK}$?



The length of $\overline{GF}$ is equal to the length of $\overline{HK}$ because chords the same distance from the center are congruent and chords that intersect a diameter at a right angle are bisected.

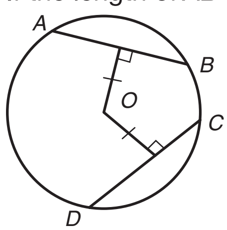
Determine each measurement.

7. If $\overline{BD}$ is a diameter, what is the length of $\overline{EC}$?



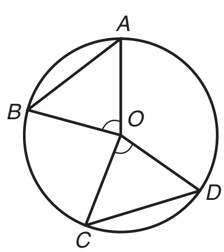
$EC = EA = 5$ cm

8. If the length of $\overline{AB}$ is 13 millimeters, what is the length of $\overline{CD}$?



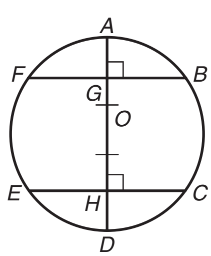
$CD = AB = 13$ mm

9. If the length of $\overline{AB}$ is 24 centimeters, what is the length of $\overline{CD}$?



$CD = AB = 24$ cm

10. If the length of $\overline{BF}$ is 32 inches, what is the length of $\overline{CH}$?



$CH = \frac{1}{2}BF = \frac{32}{2} = 16$ inches

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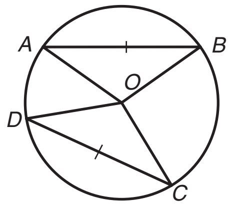
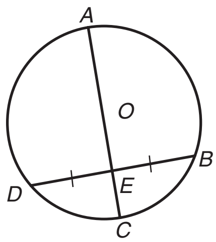
11. If the measure of $\angle AOB = 155°$, what is the measure of $\angle DOC$?



$m\angle DOC = m\angle AOB = 155°$

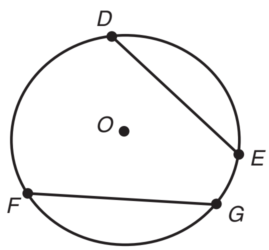
12. If segment $\overline{AC}$ is a diameter, what is the measure of $\angle AED$?



$m\angle AED = 90°$

Compare each measurement. Explain your answer.

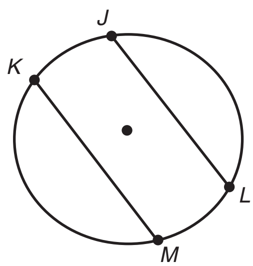
13. If $\overline{DE} \cong \overline{FG}$, how does the measure of $\overset{\frown}{DE}$ and $\overset{\frown}{FG}$ compare?



The measure of $\overset{\frown}{DE}$ is equal to the measure of $\overset{\frown}{FG}$ because the corresponding arcs of congruent chords are congruent.

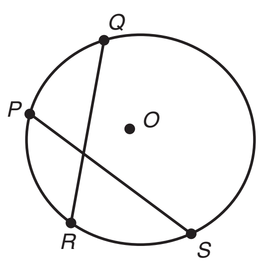
9

14. If $\overset{\frown}{KM} \cong \overset{\frown}{JL}$, how does the measure of $\overline{JL}$ and $\overline{KM}$ compare?



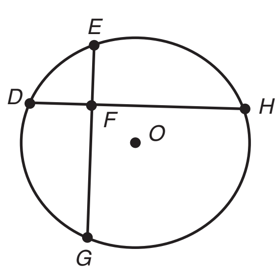
The measure of $\overline{KM}$ is equal to the measure of $\overline{JL}$ because the corresponding chords of congruent arcs are congruent.

15. If $\overline{QR} \cong \overline{PS}$, how does the measure of $\overset{\frown}{QPR}$ and $\overset{\frown}{PRS}$ compare?



The measure of $\overset{\frown}{QPR}$ is equal to the measure of $\overset{\frown}{PRS}$ because the corresponding arcs of congruent chords are congruent.

16. If $\overset{\frown}{EDG} \cong \overset{\frown}{DEH}$, how does the measure of $\overline{EG}$ and $\overline{DH}$ compare?



The measure of $\overline{EG}$ is equal to the measure of $\overline{DH}$ because the corresponding chords of congruent arcs are congruent.

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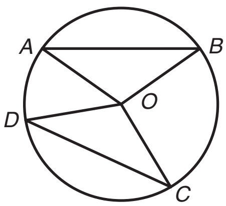
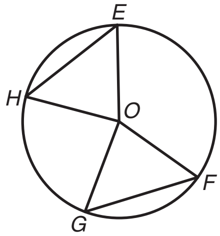
17. If $\angle AOB \cong \angle DOC$, what is the relationship between $\overline{AB}$ and $\overline{DC}$?



Segment *AB* is congruent to segment *DC* because the corresponding chords of congruent arcs are congruent.

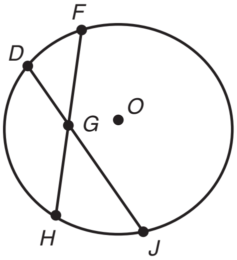
18. If $\angle EOH \cong \angle GOF$, what is the relationship between $\overset{\frown}{EH}$ and $\overset{\frown}{FG}$?



Arc *EH* is congruent to arc *FG* because the corresponding intercepted arcs of congruent central angles are congruent.

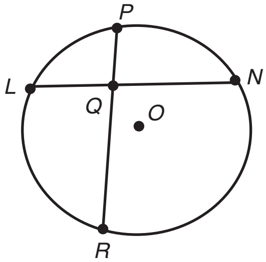
Use each diagram and the Segment Chord Theorem to write an equation involving the segments of the chords.

19.



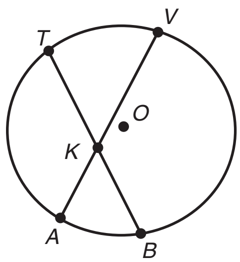
$DG \cdot GJ = FG \cdot GH$

20.



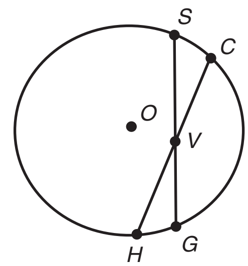
$LQ \cdot QN = PQ \cdot QR$

21.



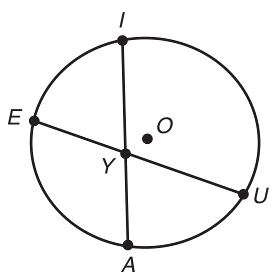
TK • KB = AK • KV

22.



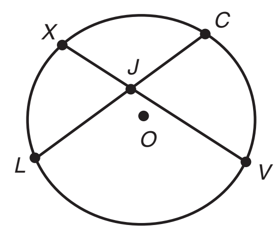
SV • VG = HV • VC

23.



EY • YU = IY • YA

24.



XJ • JV = LJ • JC

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Solar Eclipses
Tangents and Secants

Vocabulary

Write the term from the box that best completes each statement.

external secant segment	Secant Segment Theorem	tangent segment
secant segment	Secant Tangent Theorem	Tangent Segment Theorem

1. A(n) tangent segment is the segment that is formed from an exterior point of a circle to the point of tangency.

2. The Tangent Segment Theorem states that if two tangent segments are drawn from the same point on the exterior of the circle, then the tangent segments are congruent.

3. When two secants intersect in the exterior of a circle, the segment that begins at the point of intersection, continues through the circle, and ends on the other side of the circle is called a(n) secant segment.

4. When two secants intersect in the exterior of a circle, the segment that begins at the point of intersection and ends where the secant enters the circle is called a(n) external secant segment.

5. The Secant Segment Theorem states that if two secants intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the product of the lengths of the second secant segment and its external secant segment.

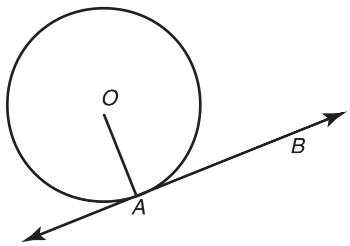
6. The Secant Tangent Theorem states that if a tangent and a secant intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the square of the length of the tangent segment.

9

Problem Set

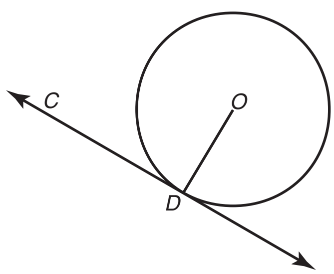
Calculate the measure of each angle. Explain your reasoning.

1. If $\overline{OA}$ is a radius, what is the measure of $\angle OAB$?



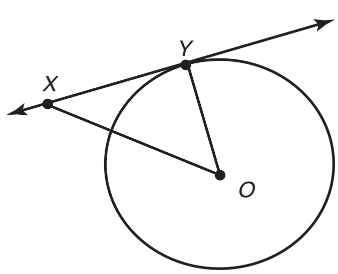
The measure of angle *OAB* is 90 degrees because a tangent line and the radius that ends at the point of tangency are perpendicular.

2. If $\overline{OD}$ is a radius, what is the measure of $\angle ODC$?



The measure of angle *ODC* is 90 degrees because a tangent line and the radius that ends at the point of tangency are perpendicular.

3. If $\overline{YO}$ is a radius, what is the measure of $\angle XYO$?



The measure of angle *XYO* is 90 degrees because a tangent line and the radius that ends at the point of tangency are perpendicular.

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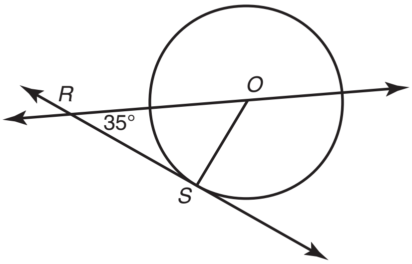
4. If $\overline{RS}$ is a tangent segment and $\overline{OS}$ is a radius, what is the measure of $\angle ROS$?

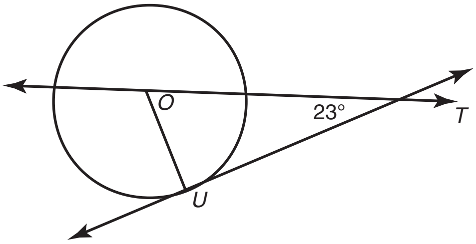


The measure of angle *ROS* is 55 degrees.

To determine the measure of angle *ROS*, I used the fact that the interior angles of a triangle sum to 180 degrees and that the measure of angle *OSR* is 90 degrees.

$$m\angle SRO + m\angle OSR + m\angle ROS = 180$$
$$35 + 90 + m\angle ROS = 180$$
$$125 + m\angle ROS = 180$$
$$m\angle ROS = 55$$

5. If $\overline{UT}$ is a tangent segment and $\overline{OU}$ is a radius, what is the measure of $\angle TOU$?

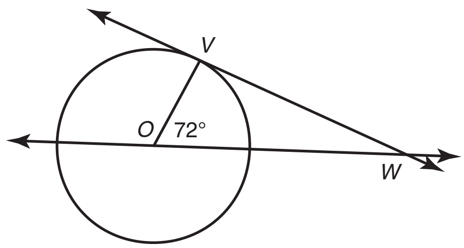


The measure of angle *TOU* is 67 degrees.

To determine the measure of angle *TOU*, I used the fact that the interior angles of a triangle sum to 180 degrees and that the measure of angle *OUT* is 90 degrees.

$$m\angle UTO + m\angle OUT + m\angle TOU = 180$$
$$23 + 90 + m\angle TOU = 180$$
$$113 + m\angle TOU = 180$$
$$m\angle TOU = 67$$

6. If $\overline{VW}$ is a tangent segment and $\overline{OV}$ is a radius, what is the measure of $\angle VWO$?



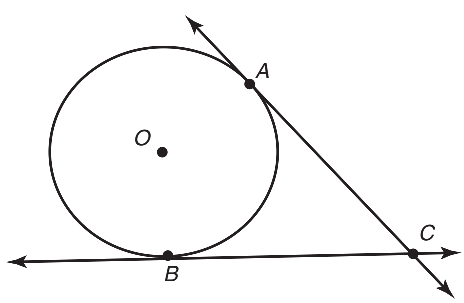
The measure of angle *VWO* is 18 degrees.

To determine the measure of angle *VWO*, I used the fact that the interior angles of a triangle sum to 180 degrees and that the measure of angle *OVW* is 90 degrees.

$$m\angle WOV + m\angle OVW + m\angle VWO = 180$$
$$72 + 90 + m\angle VWO = 180$$
$$162 + m\angle VWO = 180$$
$$m\angle VWO = 18$$

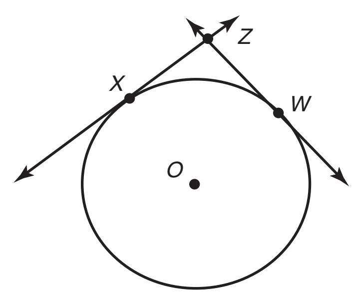
Write a statement to show the congruent segments.

7. $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ are tangent to circle *O*.



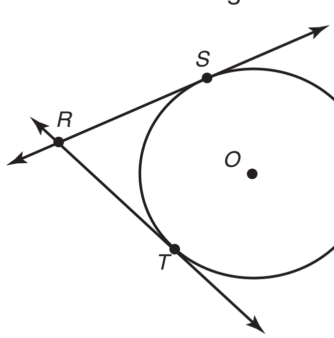
$\overline{AC} \cong \overline{CB}$

8. $\overleftrightarrow{XZ}$ and $\overleftrightarrow{ZW}$ are tangent to circle *O*.



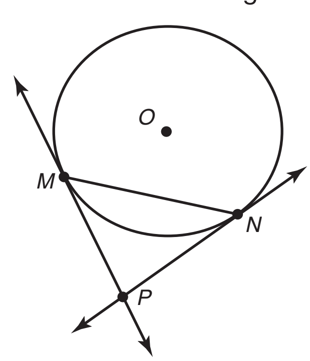
$\overline{XZ} \cong \overline{ZW}$

9. $\overleftrightarrow{RS}$ and $\overleftrightarrow{RT}$ are tangent to circle *O*.



$\overline{RS} \cong \overline{RT}$

10. $\overleftrightarrow{MP}$ and $\overleftrightarrow{NP}$ are tangent to circle *O*.



$\overline{MP} \cong \overline{PN}$

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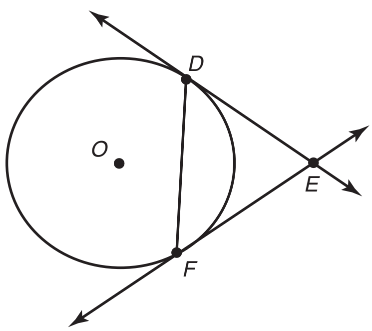
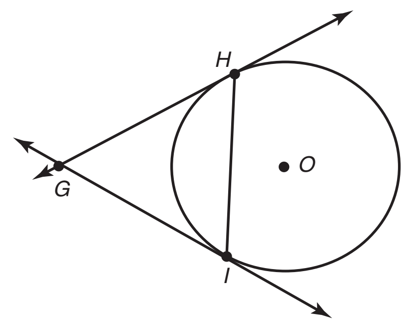
11. $\overleftrightarrow{DE}$ and $\overleftrightarrow{FE}$ are tangent to circle O.



$\overline{DE} \cong \overline{EF}$

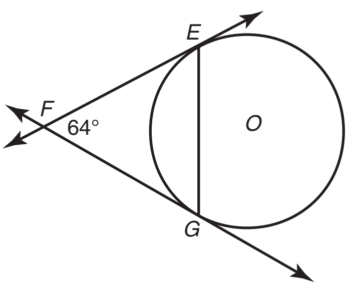
12. $\overleftrightarrow{GH}$ and $\overleftrightarrow{GI}$ are tangent to circle O.



$\overline{GH} \cong \overline{GI}$

Calculate the measure of each angle. Explain your reasoning.

13. If $\overline{EF}$ and $\overline{GF}$ are tangent segments, what is the measure of $\angle EGF$?



The measure of angle *EGF* is 58 degrees.

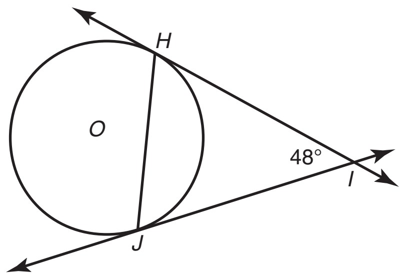
I know triangle *EFG* is isosceles and its base angles are congruent.

Let x represent the measure of angle *FEG* and the measure of angle *EGF*.

$$m\angle F + x + x = 180$$
$$64 + 2x = 180$$
$$2x = 116$$
$$x = 58$$

9

14. If $\overline{HI}$ and $\overline{JI}$ are tangent segments, what is the measure of $\angle HJI$?



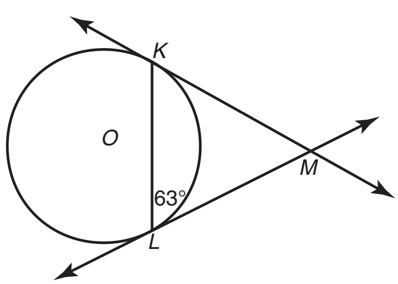
The measure of angle *HJI* is 66 degrees.

I know triangle *HJI* is isosceles and its base angles are congruent.

Let *x* represent the measure of angle *HJI* and the measure of angle *IHJ*.

$$m\angle I + x + x = 180$$
$$48 + 2x = 180$$
$$2x = 132$$
$$x = 66$$

15. If $\overline{KM}$ and $\overline{LM}$ are tangent segments, what is the measure of $\angle KML$?



The measure of angle *KML* is 54 degrees.

I know triangle *KML* is isosceles and its base angles are congruent.

$$m\angle M + m\angle MLK + m\angle LKM = 180$$
$$m\angle M + 63 + 63 = 180$$
$$m\angle M + 126 = 180$$
$$m\angle M = 54$$

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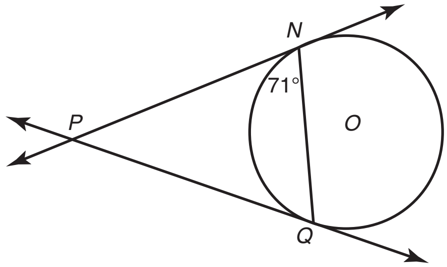
16. If $\overline{NP}$ and $\overline{QP}$ are tangent segments, what is the measure of $\angle NPQ$?

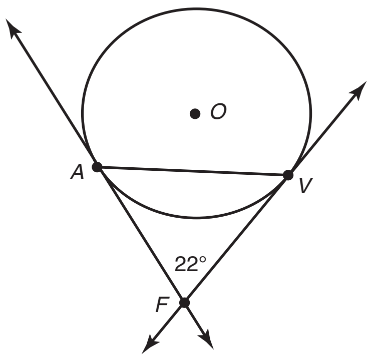


The measure of angle *NPQ* is 38 degrees.

I know triangle *NPQ* is isosceles and its base angles are congruent.

$$m\angle NPQ + m\angle PQN + m\angle QNP = 180$$
$$m\angle NPQ + 71 + 71 = 180$$
$$m\angle NPQ + 142 = 180$$
$$m\angle NPQ = 38$$

17. If $\overline{AF}$ and $\overline{VF}$ are tangent segments, what is the measure of $\angle AVF$?



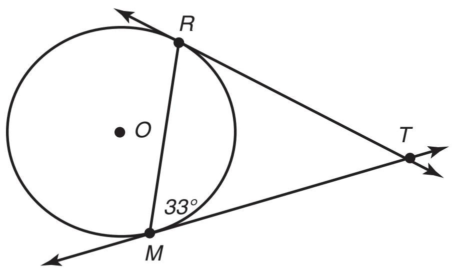
The measure of angle *AVF* is 79 degrees.

I know triangle *AFV* is isosceles and its base angles are congruent.

Let x represent the measure of angle *FVA* and the measure of angle *VAF*.

$$m\angle P + x + x = 180$$
$$22 + 2x = 180$$
$$2x = 158$$
$$x = 79$$

18. If $\overline{RT}$ and $\overline{MT}$ are tangent segments, what is the measure of $\angle RTM$?



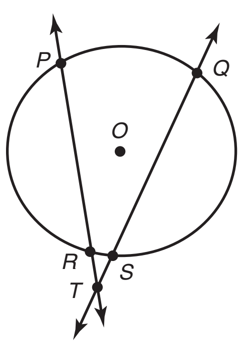
The measure of angle *RTM* is 114 degrees.

I know triangle *RTM* is isosceles and its base angles are congruent.

$$m\angle RTM + m\angle TMR + m\angle MRT = 180$$
$$m\angle RTM + 33 + 33 = 180$$
$$m\angle RTM + 66 = 180$$
$$m\angle RTM = 114$$

Name two secant segments and two external secant segments for circle *O*.

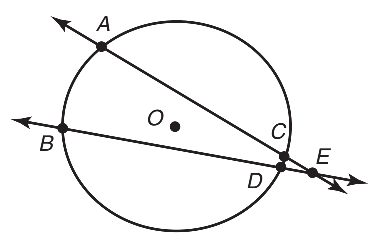
19.



Secant segments: $\overline{PT}$ and $\overline{QT}$

External secant segments: $\overline{RT}$ and $\overline{ST}$

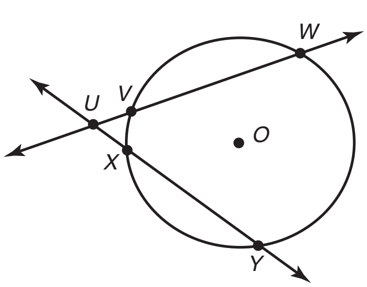
20.



Secant segments: $\overline{AE}$ and $\overline{BE}$

External secant segments: $\overline{CE}$ and $\overline{DE}$

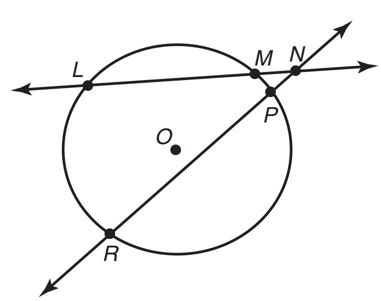
21.



Secant segments: $\overline{WU}$ and $\overline{UY}$

External secant segments: $\overline{UV}$ and $\overline{UX}$

22.



Secant segments: $\overline{LN}$ and $\overline{RN}$

External secant segments: $\overline{MN}$ and $\overline{NP}$

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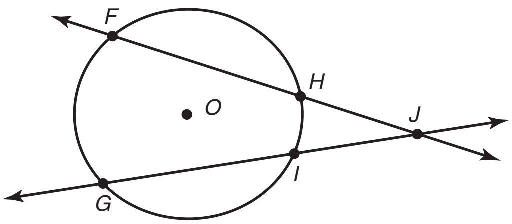
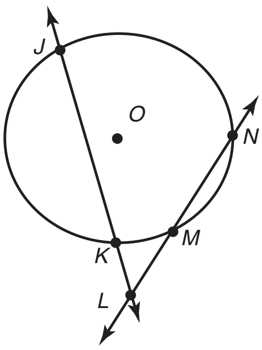
23.



Secant segments: $\overline{FJ}$ and $\overline{GJ}$

External secant segments: $\overline{HJ}$ and $\overline{IJ}$

24.

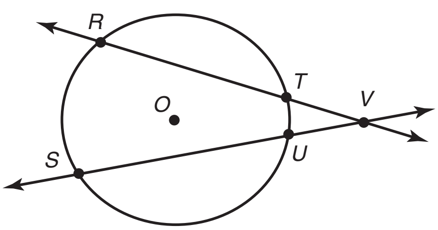


Secant segments: $\overline{JL}$ and $\overline{NL}$

External secant segments: $\overline{KL}$ and $\overline{ML}$

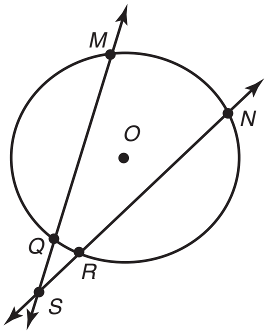
Use each diagram and the Secant Segment Theorem to write an equation involving the secant segments.

25.



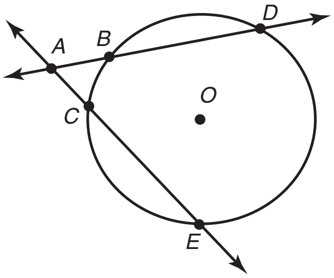
$RV \cdot TV = SV \cdot UV$

26.



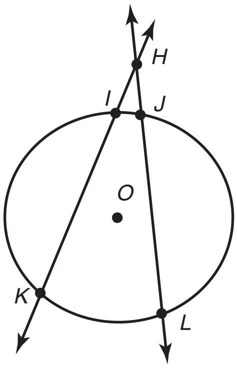
$MS \cdot QS = NS \cdot RS$

27.



$AD \cdot AB = AE \cdot AC$

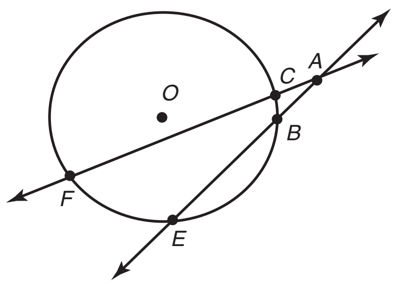
28.



$HK \cdot HI = HL \cdot HJ$

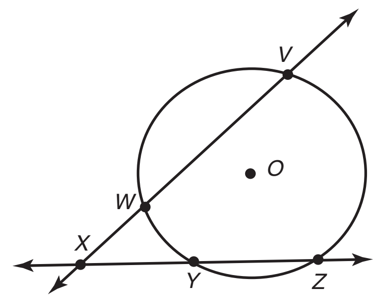
9

29.



$FA \cdot CA = EA \cdot BA$

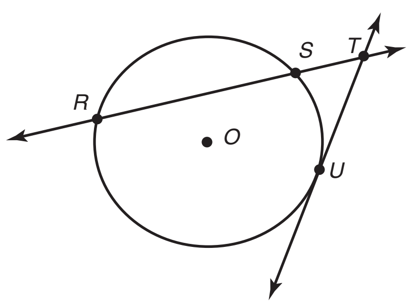
30.



$XV \cdot WV = XZ \cdot XY$

Name a tangent segment, a secant segment, and an external secant segment for circle *O*.

31.

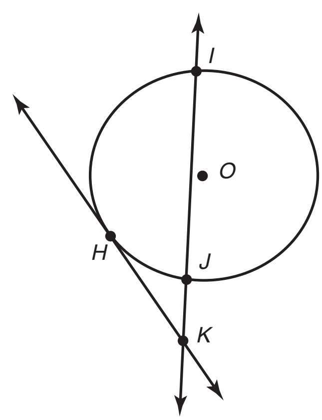


Tangent segment: $\overline{TU}$

Secant segment: $\overline{RT}$

External secant segment: $\overline{ST}$

32.

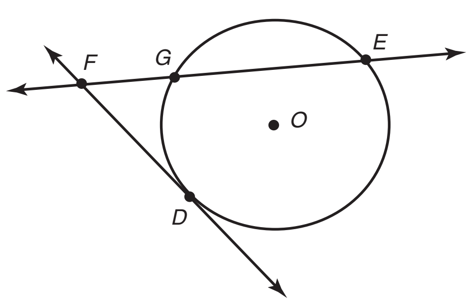


Tangent segment: $\overline{HK}$

Secant segment: $\overline{IK}$

External secant segment: $\overline{JK}$

33.

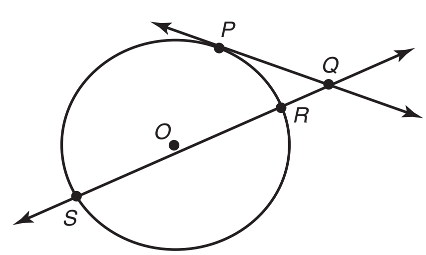


Tangent segment: $\overline{FD}$

Secant segment: $\overline{FE}$

External secant segment: $\overline{FG}$

34.



Tangent segment: $\overline{PQ}$

Secant segment: $\overline{SQ}$

External secant segment: $\overline{QR}$

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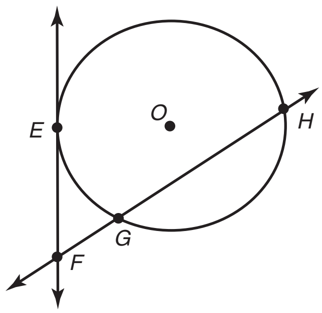
35.

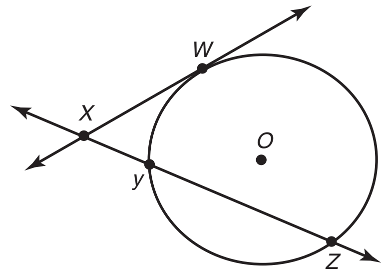


Tangent segment: $\overline{EF}$

Secant segment: $\overline{FH}$

External secant segment: $\overline{FG}$

36.



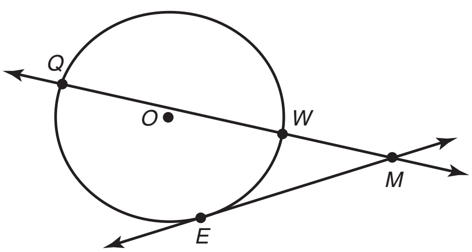
Tangent segment: $\overline{XW}$

Secant segment: $\overline{XZ}$

External secant segment: $\overline{XY}$

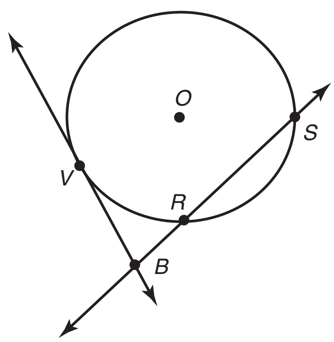
Use each diagram and the Secant Tangent Theorem to write an equation involving the secant and tangent segments.

37.



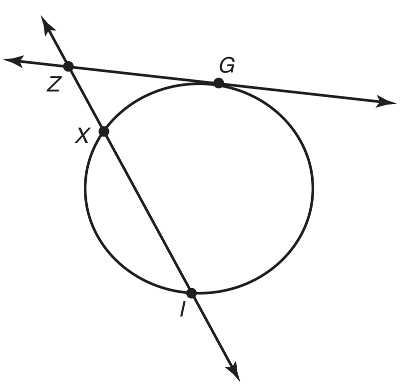
$(EM)^2 = QM \cdot WM$

38.



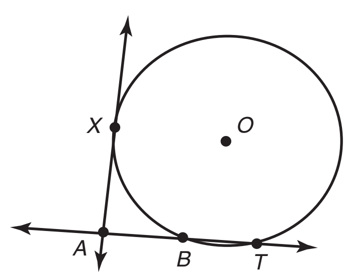
$(VB)^2 = BR \cdot BS$

39.



$(ZG)^2 = ZI \cdot ZX$

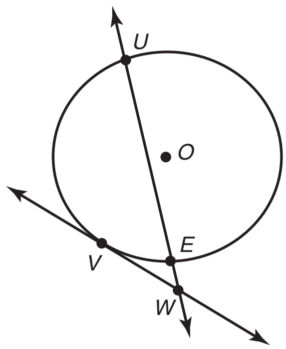
40.



$(XA)^2 = AT \cdot AB$

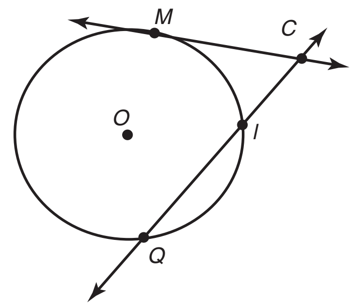
9

41.



$(VW)^2 = WU \cdot WE$

42.



$(MC)^2 = CQ \cdot CI$