Name ____________________ Date ________

Replacement for a Carpenter's Square

Inscribed and Circumscribed Triangles and Quadrilaterals

Vocabulary

Answer each question.

1. How are inscribed polygons and circumscribed polygons different?

   Inscribed polygons are drawn inside of a circle with all vertices touching the circle. Circumscribed polygons are drawn outside of a circle with all sides tangent to the circle.

2. Describe how you can use the Inscribed Right Triangle–Diameter Theorem to show an inscribed triangle is a right triangle.

   If an inscribed triangle has one side that is a diameter of the circle, then the triangle must be a right triangle.

3. What does the Converse of the Inscribed Right Triangle–Diameter Theorem help to show in a circle?

   If a right triangle is inscribed in a circle then one of the sides of the triangle must be the diameter.

4. What information about a quadrilateral inscribed in a circle does the Inscribed Quadrilateral–Opposite Angles Theorem give?

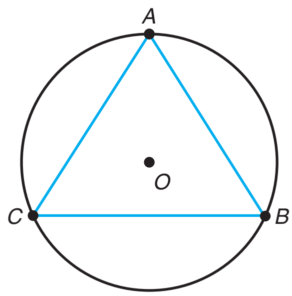
   If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

© Carnegie Learning

Problem Set

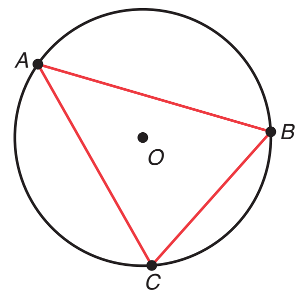
Draw a triangle inscribed in the circle through the three points. Then determine if the triangle is a right triangle.

1.



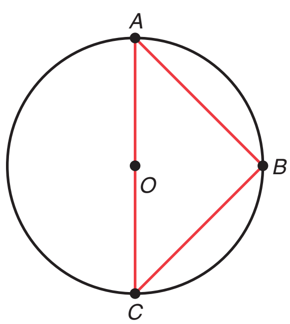
No. The triangle is not a right triangle. None of the sides of the triangle is a diameter of the circle.

2.



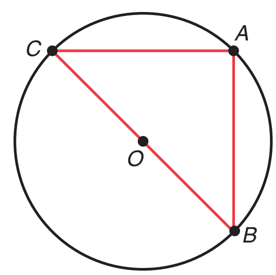
No. The triangle is not a right triangle. None of the sides of the triangle is a diameter of the circle.

3.



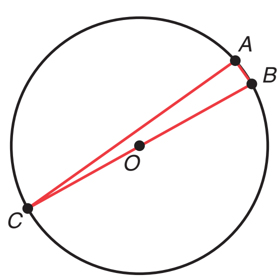
Yes. The triangle is a right triangle. Line segment *AC* is a diameter of the circle.

4.



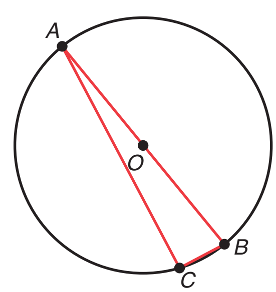
Yes. The triangle is a right triangle. Line segment *BC* is a diameter of the circle.

5.



Yes. The triangle is a right triangle. Line segment *BC* is a diameter of the circle.

6.

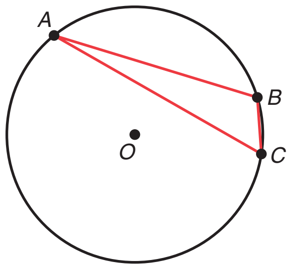


Yes. The triangle is a right triangle. Line segment *AB* is a diameter of the circle.

© Carnegie Learning

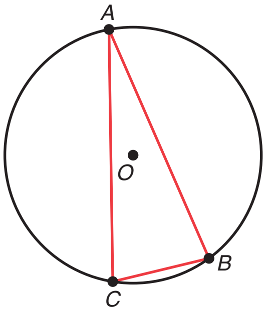
Name ____________________ Date ____________

7.



No. The triangle is not a right triangle. None of the sides of the triangle is a diameter of the circle.

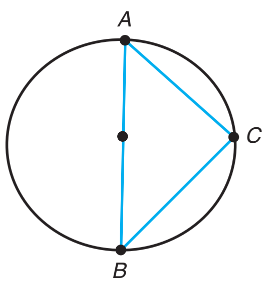
8.



No. The triangle is not a right triangle. None of the sides of the triangle is a diameter of the circle.

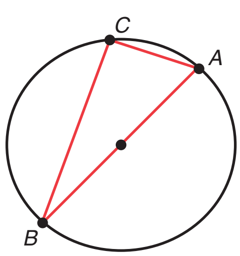
Draw a triangle inscribed in the circle through the given points. Then determine the measure of the indicated angle.

9. In $\triangle ABC$, $m\angle A = 55°$. Determine $m\angle B$.



$m\angle B = 180° - 90° - 55° = 35°$

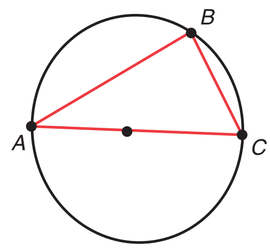
10. In $\triangle ABC$, $m\angle B = 38°$. Determine $m\angle A$.



$m\angle A = 180° - 90° - 38° = 52°$

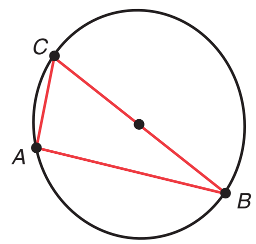
© Carnegie Learning

11. In $\triangle ABC$, $m\angle C = 62°$. Determine $m\angle A$.



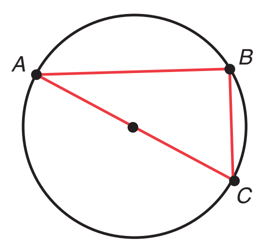
$m\angle A = 180° - 90° - 62° = 28°$

12. In $\triangle ABC$, $m\angle B = 26°$. Determine $m\angle C$.



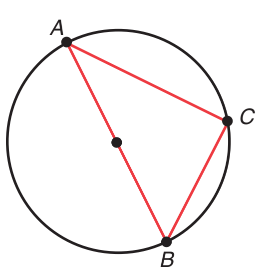
$m\angle C = 180° - 90° - 26° = 64°$

13. In $\triangle ABC$, $m\angle C = 49°$. Determine $m\angle A$.



$m\angle A = 180° - 90° - 49° = 41°$

14. In $\triangle ABC$, $m\angle B = 51°$. Determine $m\angle A$.



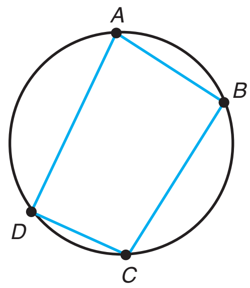
$m\angle A = 180° - 90° - 51° = 39°$

© Carnegie Learning

Name ______________________________ Date ____________

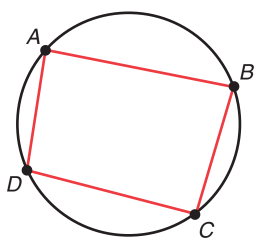
Draw a quadrilateral inscribed in the circle through the given four points. Then determine the measure of the indicated angle.

15. In quadrilateral *ABCD*, $m\angle B = 81°$. Determine $m\angle D$.



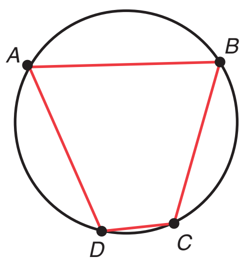
$m\angle D = 180° - 81° = 99°$

16. In quadrilateral *ABCD*, $m\angle C = 75°$. Determine $m\angle A$.



$m\angle A = 180° - 75° = 105°$

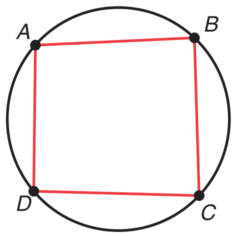
17. In quadrilateral *ABCD*, $m\angle B = 112°$. Determine $m\angle D$.



$m\angle D = 180° - 112° = 68°$

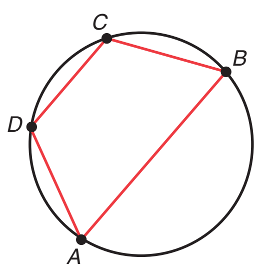
© Carnegie Learning

18. In quadrilateral $ABCD$, $m\angle D = 93°$. Determine $m\angle B$.



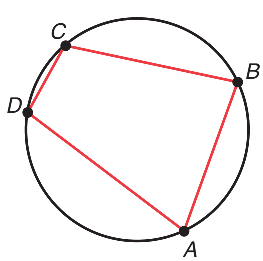
$m\angle B = 180° - 93° = 87°$

19. In quadrilateral $ABCD$, $m\angle A = 72°$. Determine $m\angle C$.



$m\angle C = 180° - 72° = 108°$

20. In quadrilateral $ABCD$, $m\angle B = 101°$. Determine $m\angle D$.



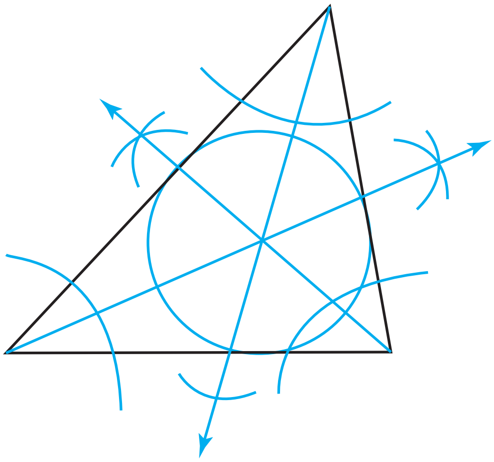
$m\angle D = 180° - 101° = 79°$

© Carnegie Learning

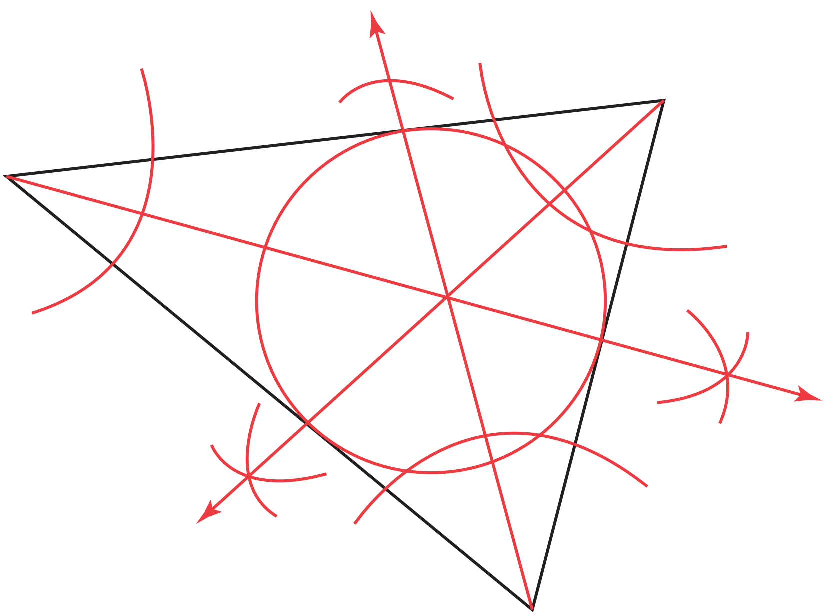
Name ______________________________ Date ____________

Construct a circle inscribed in each polygon.

21.



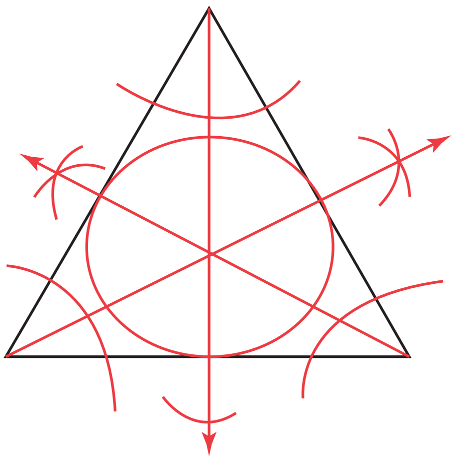
22.



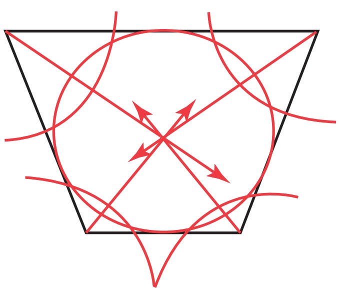
© Carnegie Learning

10

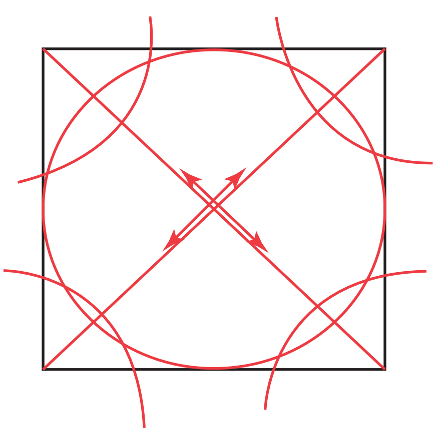
23.



24.



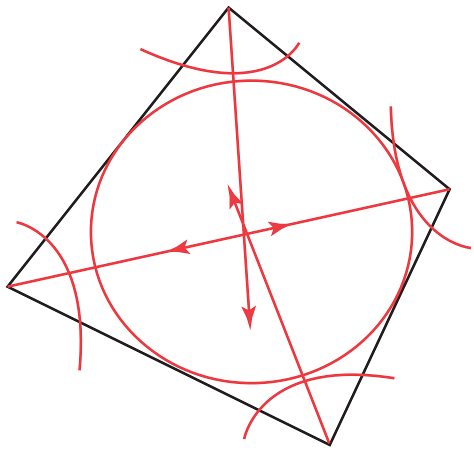
25.



© Carnegie Learning

Name ______________________________ Date ______________

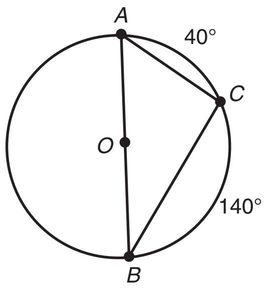
26.



Create a proof to prove each statement.

27. Given: Inscribed $\triangle ABC$ in circle O, $m\overset{\frown}{AC} = 40°$, and $m\overset{\frown}{BC} = 140°$

Prove: $\overline{AB}$ is a diameter of circle O.

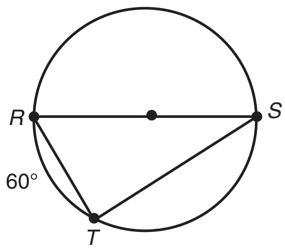


Statements	Reasons
1. $m\overset{\frown}{AC} = 40°$, $m\overset{\frown}{BC} = 140°$	1. Given
2. $m\overset{\frown}{AC} + m\overset{\frown}{BC} + m\overset{\frown}{AB} = 360°$	2. Arc Addition Postulate
3. $40° + 140° + m\overset{\frown}{AB} = 360°$	3. Substitution
4. $m\overset{\frown}{AB} = 180°$	4. Subtraction Property of Equality
5. $m\angle C = \frac{1}{2} m\overset{\frown}{AB}$	5. Definition of inscribed angle
6. $m\angle C = 90°$	6. Substitution
7. $\triangle ABC$ is a right triangle with right angle C.	7. Definition of right triangle
8. $\overline{AB}$ is the diameter of circle O.	8. Converse of Inscribed Right Triangle-Diameter Theorem

© Carnegie Learning

28. Given: Inscribed $\triangle RST$ in circle O with diameter $\overline{RS}$, and $m\overset{\frown}{RT} = 60°$

Prove: $m\overset{\frown}{ST} = 120°$



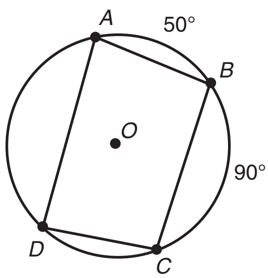
Statements	Reasons
1. Inscribed $\triangle RST$ in circle O with diameter $\overline{RS}$, $m\overset{\frown}{RT} = 60°$	1. Given
2. $\triangle RST$ is a right triangle with right angle $\angle T$.	2. Inscribed Right Triangle-Diameter Theorem
3. $m\angle T = \frac{1}{2} m\overset{\frown}{RS}$	3. Definition of inscribed angle
4. $90° = \frac{1}{2} m\overset{\frown}{RS}$	4. Substitution
5. $m\overset{\frown}{RS} = 180°$	5. Multiplication Property of Equality
6. $m\overset{\frown}{RS} + m\overset{\frown}{ST} + m\overset{\frown}{RT} = 360°$	6. Arc Addition Postulate
7. $180° + m\overset{\frown}{ST} + 60° = 360°$	7. Substitution
8. $m\overset{\frown}{ST} = 120°$	8. Subtraction Property of Equality

© Carnegie Learning

Name ____ Date ______________

29. Given: Inscribed quadrilateral $ABCD$ in circle O, $m\widehat{AB} = 50°$, and $m\widehat{BC} = 90°$

Prove: $m\angle B = 110°$

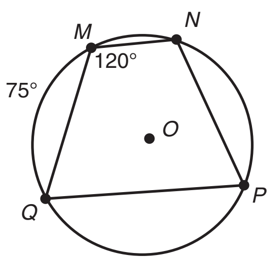


Statements	Reasons
1. Inscribed quad $ABCD$ in circle O, $m\widehat{AB} = 50°$, $m\widehat{BC} = 90°$	1. Given
2. $m\widehat{AB} + m\widehat{BC} = m\widehat{ABC}$	2. Angle Addition Postulate
3. $50° + 90° = m\widehat{ABC}$	3. Substitution
4. $140° = m\widehat{ABC}$	4. Addition Property of Equality
5. $m\angle D = \frac{1}{2} m\widehat{ABC}$	5. Inscribed angle
6. $m\angle D = \frac{1}{2}(140°) = 70°$	6. Substitution
7. $\angle D$ and $\angle B$ are supplementary	7. Inscribed Quadrilateral-Opposite Angles Theorem
8. $m\angle D + m\angle B = 180°$	8. Definition of supplementary
9. $70° + m\angle B = 180°$	9. Substitution
10. $m\angle B = 110°$	10. Subtraction Property of Equality

© Carnegie Learning

30. Given: Inscribed quadrilateral $MNPQ$ in circle O, $m\overset{\frown}{MQ} = 75°$, and $m\angle NMQ = 120°$

Prove: $m\overset{\frown}{MN} = 45°$



10

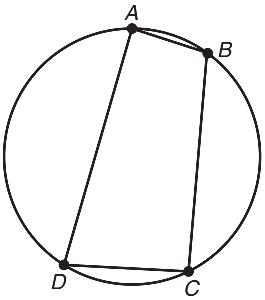
Statements	Reasons
1. Inscribed quad $MNPQ$ in circle O, $m\overset{\frown}{MQ} = 75°$, $m\angle NMQ = 120°$	1. Given
2. $\angle M$ and $\angle P$ are supplementary	2. Inscribed Quadrilateral-Opposite Angles Theorem
3. $m\angle M + m\angle P = 180°$	3. Definition of supplementary
4. $120° + m\angle P = 180°$	4. Substitution
5. $m\angle P = 60°$	5. Subtraction Property of Equality
6. $m\angle P = \frac{1}{2}m\overset{\frown}{QMN}$	6. Inscribed angle
7. $60° = \frac{1}{2}m\overset{\frown}{QMN}$	7. Substitution
8. $120° = m\overset{\frown}{QMN}$	8. Multiplication Property of Equality
9. $m\overset{\frown}{QMN} = m\overset{\frown}{QM} + n\overset{\frown}{MN}$	9. Arc Addition Postulate
10. $120° = 75° + m\overset{\frown}{MN}$	10. Substitution
11. $45° = m\overset{\frown}{MN}$	11. Subtraction Property of Equality

© Carnegie Learning

Name ____ Date ______________

31. Given: $m\overset{\frown}{AB} = 50°$, $m\overset{\frown}{BC} = 90°$, and $m\overset{\frown}{CD} = 90°$

Prove: $m\angle BCD = 90°$



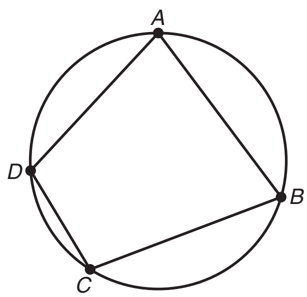
Statements	Reasons
1. $m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CD} + m\overset{\frown}{AD} = 360°$	1. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, and $\overset{\frown}{AD}$ form the circle
2. $50° + 90° + 90° + m\overset{\frown}{AD} = 360°$	2. Substitution
3. $230° + m\overset{\frown}{AD} = 360°$	3. Addition
4. $m\overset{\frown}{AD} = 130°$	4. Subtraction Property of Equality
5. $m\angle BCD = \frac{1}{2}m\overset{\frown}{BAD}$	5. Definition of measure of inscribed angle
6. $m\angle BCD = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{AD})$	6. Substitution Property of Equality
7. $m\angle BCD = \frac{1}{2}(50° + 130°)$	7. Substitution Property of Equality
8. $m\angle BCD = \frac{1}{2}(180°)$	8. Addition
9. $m\angle BCD = 90°$	9. Multiplication

10

© Carnegie Learning

32. Given: $\angle BAD$ and $\angle ADC$ are supplementary angles

Prove: $m\angle BAD = m\angle ABC$



Statements	Reasons
1. $m\angle BAD + m\angle ADC = 180°$	1. Given
2. $m\angle ADC = 180° - m\angle BAD$	2. Subtraction Property of Equality
3. $m\angle ADC + m\angle ABC = 180°$	3. Opposite angles of a quadrilateral inscribed in a circle are supplementary.
4. $180° - m\angle BAD + m\angle ABC = 180°$	4. Substitution Property of Equality
5. $-m\angle BAD + m\angle ABC = 0°$	5. Subtraction Property of Equality
6. $m\angle BAD = m\angle ABC$	6. Addition Property of Equality

© Carnegie Learning

Name ______________________ Date __________

Gears

Arc Length

Vocabulary

Define the key term in your own words.

1. arc length
   Arc length is a portion of the circumference of a circle.

2. radian
   A radian is the measure of a central angle whose arc length is the same as the radius of the circle.

Problem Set

Calculate the ratio of the length of each arc to the circle's circumference.

1. The measure of $\overset{\frown}{AB}$ is 40°.
   $\frac{40°}{360°} = \frac{1}{9}$
   The arc is $\frac{1}{9}$ of the circle's circumference.

2. The measure of $\overset{\frown}{CD}$ is 90°.
   $\frac{90°}{360°} = \frac{1}{4}$
   The arc is $\frac{1}{4}$ of the circle's circumference.

3. The measure of $\overset{\frown}{EF}$ is 120°.
   $\frac{120°}{360°} = \frac{1}{3}$
   The arc is $\frac{1}{3}$ of the circle's circumference.

4. The measure of $\overset{\frown}{GH}$ is 150°.
   $\frac{150°}{360°} = \frac{5}{12}$
   The arc is $\frac{5}{12}$ of the circle's circumference.

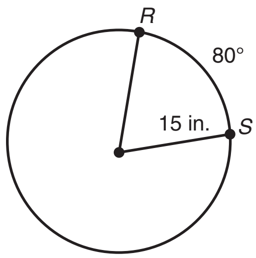
5. The measure of $\overset{\frown}{IJ}$ is 105°.
   $\frac{105°}{360°} = \frac{7}{24}$
   The arc is $\frac{7}{24}$ of the circle's circumference.

6. The measure of $\overset{\frown}{KL}$ is 75°.
   $\frac{75°}{360°} = \frac{5}{24}$
   The arc is $\frac{5}{24}$ of the circle's circumference.

© Carnegie Learning

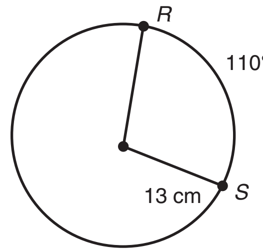
Write an expression that you can use to calculate the length of $\overset{\frown}{RS}$. You do not need to simplify the expression.

7.



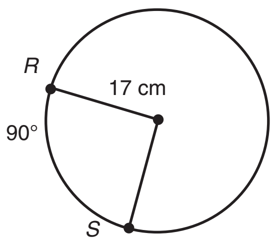
$\frac{80}{360} \cdot 2\pi(15)$

8.



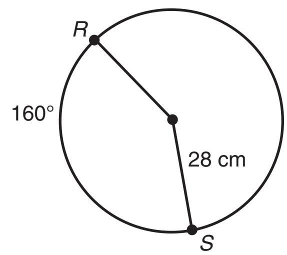
$\frac{110}{360} \cdot 2\pi(13)$

9.



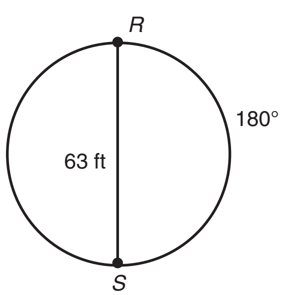
$\frac{90}{360} \cdot 2\pi(17)$

10.



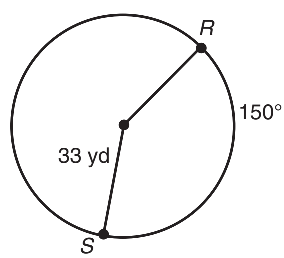
$\frac{160}{360} \cdot 2\pi(28)$

11.



$\frac{180}{360} \cdot 2\pi(31.5)$

12.



$\frac{150}{360} \cdot 2\pi(33)$

Calculate each arc length. Write your answer in terms of π.

13. If the measure of $\overset{\frown}{AB}$ is 45° and the radius is 12 meters, what is the arc length of $\overset{\frown}{AB}$?

$C = 2\pi(12) = 24\pi$

Fraction of C: $\frac{45°}{360°} = \frac{1}{8}$

Arc length of $\overset{\frown}{AB}$: $\frac{1}{8}(24\pi) = 3\pi$

The arc length of $\overset{\frown}{AB}$ is 3π meters.

© Carnegie Learning

Name ______________________________ Date ____________

14. If the measure of $\overset{\frown}{CD}$ is 120° and the radius is 15 centimeters, what is the arc length of $\overset{\frown}{CD}$?

$C = 2\pi(15) = 30\pi$

Fraction of C: $\frac{120°}{360°} = \frac{1}{3}$

Arc length of $\overset{\frown}{CD}$: $\frac{1}{3}(30\pi) = 10\pi$

The arc length of $\overset{\frown}{CD}$ is 10π cm.

15. If the measure of $\overset{\frown}{EF}$ is 60° and the radius is 8 inches, what is the arc length of $\overset{\frown}{EF}$?

$C = 2\pi(8) = 16\pi$

Fraction of C: $\frac{60°}{360°} = \frac{1}{6}$

Arc length of $\overset{\frown}{EF}$: $\frac{1}{6}(16\pi) = \frac{16}{6}\pi = \frac{8}{3}\pi$

The arc length of $\overset{\frown}{EF}$ is $\frac{8}{3}\pi$ inches.

16. If the measure of $\overset{\frown}{GH}$ is 30° and the radius is 6 meters, what is the arc length of $\overset{\frown}{GH}$?

$C = 2\pi(6) = 12\pi$

Fraction of C: $\frac{30°}{360°} = \frac{1}{12}$

Arc length of $\overset{\frown}{GH}$: $\frac{1}{12}(12\pi) = \frac{12}{12}\pi = \pi$

The arc length of $\overset{\frown}{GH}$ is π meters.

17. If the measure of $\overset{\frown}{IJ}$ is 80° and the diameter is 10 centimeters, what is the arc length of $\overset{\frown}{IJ}$?

$C = \pi(10) = 10\pi$

Fraction of C: $\frac{80°}{360°} = \frac{2}{9}$

Arc length of $\overset{\frown}{IJ}$: $\frac{2}{9}(10\pi) = \frac{20\pi}{9}$

The arc length of $\overset{\frown}{IJ}$ is $\frac{20\pi}{9}$ cm.

© Carnegie Learning

18. If the measure of $\overset{\frown}{KL}$ is 15° and the diameter is 18 feet, what is the arc length of $\overset{\frown}{KL}$?

$C = \pi(18) = 18\pi$

Fraction of C: $\frac{15°}{360°} = \frac{1}{24}$

Arc length of $\overset{\frown}{KL}$: $\frac{1}{24}(18\pi) = \frac{18}{24}\pi = \frac{3}{4}\pi$

The arc length of $\overset{\frown}{KL}$ is $\frac{3}{4}\pi$ ft.

19. If the measure of $\overset{\frown}{MN}$ is 75° and the diameter is 20 millimeters, what is the arc length of $\overset{\frown}{MN}$?

$C = \pi(20) = 20\pi$

Fraction of C: $\frac{75°}{360°} = \frac{5}{24}$

Arc length of $\overset{\frown}{MN}$: $\frac{5}{24}(20\pi) = \frac{100}{24}\pi = \frac{25}{6}\pi$

The arc length of $\overset{\frown}{MN}$ is $\frac{25}{6}\pi$ mm.

20. If the measure of $\overset{\frown}{OP}$ is 165° and the diameter is 21 centimeters, what is the arc length of $\overset{\frown}{OP}$?

$C = \pi(21) = 21\pi$

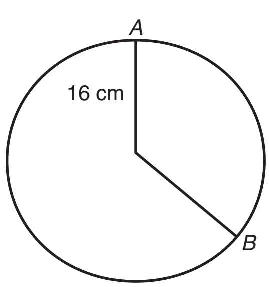
Fraction of C: $\frac{165°}{360°} = \frac{11}{24}$

Arc length of $\overset{\frown}{OP}$: $\frac{11}{24}(21\pi) = \frac{231}{24}\pi = \frac{77}{8}\pi$

The arc length of $\overset{\frown}{OP}$ is $\frac{77}{8}\pi$ meters.

Calculate each arc length. Write your answer in terms of π.

21. If the measure of $\overset{\frown}{AB}$ is 135°, what is the arc length of $\overset{\frown}{AB}$?



$C = 2\pi(16) = 32\pi$

Fraction of C: $\frac{135°}{360°} = \frac{3}{8}$

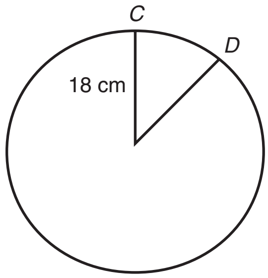
Arc length of $\overset{\frown}{AB}$: $\frac{3}{8}(32\pi) = \frac{96}{8}\pi = 12\pi$

The arc length of $\overset{\frown}{AB}$ is 12π cm.

© Carnegie Learning

Name ______________________________ Date ______________

22. If the measure of $\overset{\frown}{CD}$ is 45°, what is the arc length of $\overset{\frown}{CD}$?



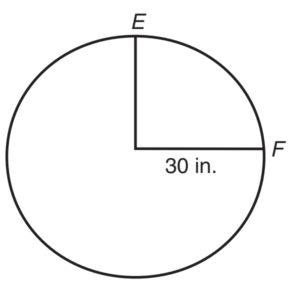
$C = 2\pi(18) = 36\pi$

Fraction of C: $\frac{45^\circ}{360^\circ} = \frac{1}{8}$

Arc length of $\overset{\frown}{CD}$: $\frac{1}{8}(36\pi) = \frac{36}{8}\pi = \frac{9}{2}\pi$

The arc length of $\overset{\frown}{CD}$ is $\frac{9}{2}\pi$ cm.

23. If the measure of $\overset{\frown}{EF}$ is 90°, what is the arc length of $\overset{\frown}{EF}$?



$C = 2\pi(30) = 60\pi$

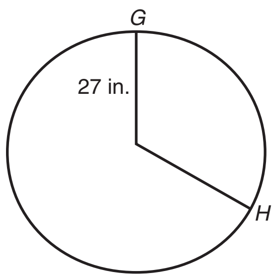
Fraction of C: $\frac{90^\circ}{360^\circ} = \frac{1}{4}$

Arc length of $\overset{\frown}{EF}$: $\frac{1}{4}(60\pi) = \frac{60}{4}\pi = 15\pi$

The arc length of $\overset{\frown}{EF}$ is 15π in.

© Carnegie Learning

24. If the measure of $\overset{\frown}{GH}$ is 120°, what is the arc length of $\overset{\frown}{GH}$?



$C = 2\pi(27) = 54\pi$

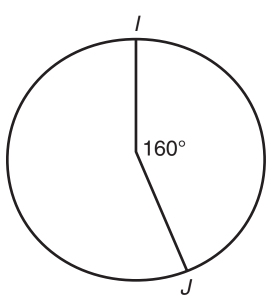
Fraction of C: $\frac{120°}{360°} = \frac{1}{3}$

Arc length of $\overset{\frown}{GH}$: $\frac{1}{3}(54\pi) = \frac{54}{3}\pi = 18\pi$

The arc length of $\overset{\frown}{GH}$ is 18π in.

10

25. If the length of the radius is 4 centimeters, what is the arc length of $\overset{\frown}{IJ}$?



$C = 2\pi(4) = 8\pi$

Fraction of C: $\frac{160°}{360°} = \frac{4}{9}$

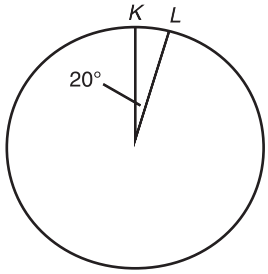
Arc length of $\overset{\frown}{IJ}$: $\frac{4}{9}(8\pi) = \frac{32}{9}\pi$

The arc length of $\overset{\frown}{IJ}$ is $\frac{32}{9}\pi$ cm.

© Carnegie Learning

Name ____ Date ______________

26. If the length of the radius is 7 centimeters, what is the arc length of $\overset{\frown}{KL}$?



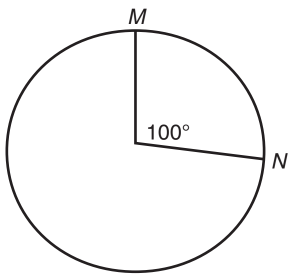
$C = 2\pi(7) = 14\pi$

Fraction of C: $\frac{20°}{360°} = \frac{1}{18}$

Arc length of $\overset{\frown}{KL}$: $\frac{1}{18}(14\pi) = \frac{14}{18}\pi = \frac{7}{9}\pi$

The arc length of $\overset{\frown}{KL}$ is $\frac{7}{9}\pi$ cm.

27. If the length of the radius is 11 centimeters, what is the arc length of $\overset{\frown}{MN}$?



$C = 2\pi(11) = 22\pi$

Fraction of C: $\frac{100°}{360°} = \frac{5}{18}$

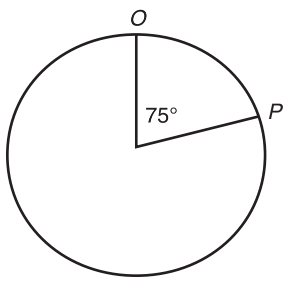
Arc length of $\overset{\frown}{MN}$: $\frac{5}{18}(22\pi) = \frac{110}{18}\pi = \frac{55}{9}\pi$

The arc length of $\overset{\frown}{MN}$ is $\frac{55}{9}\pi$ cm.

10

© Carnegie Learning

28. If the length of the radius is 17 centimeters, what is the arc length of $\overset{\frown}{OP}$?



$C = 2\pi(17) = 34\pi$

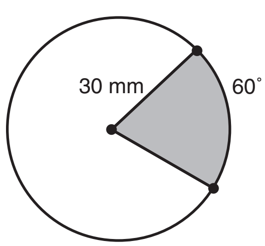
Fraction of C: $\frac{75°}{360°} = \frac{5}{24}$

Arc length of $\overset{\frown}{OP}$: $\frac{5}{24}(34\pi) = \frac{170}{24}\pi = \frac{85}{12}\pi$

The arc length of $\overset{\frown}{OP}$ is $\frac{85}{12}\pi$ cm.

Use the given information to answer each question. Where necessary, use 3.14 to approximate π.

29. Determine the perimeter of the shaded region.



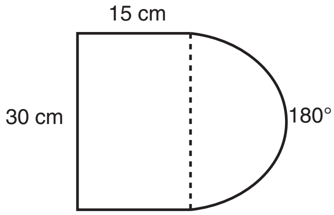
Arc length: $\frac{60}{360} \times 2(3.14)(30) = 31.4$ mm

Perimeter of shaded region: $31.4 + 30 + 30 = 91.4$ mm

© Carnegie Learning

Name ______________________________ Date ______________

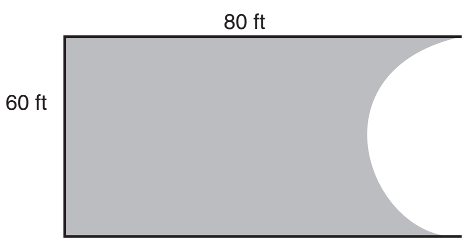
30. Determine the perimeter of the figure below.



Arc length: $\frac{180}{360} \times 2(3.14)(15) = 47.1$ cm

Perimeter of figure: $2(15) + 30 + 47.1 = 107.1$ cm

31. A semicircular cut was taken from the rectangle shown. Determine the perimeter of the shaded region.



Arc length: $\frac{180}{360} \times 2(3.14)(30) = 94.2$ ft

Perimeter of shaded region: $2(80) + 60 + 94.2 = 314.2$ ft

32. A circle has a circumference of 81.2 inches. What is the radius of the circle?

$C = 2\pi r$

$81.2 = 2\pi r$

$\frac{81.2}{2\pi} = r$

$12.9 \approx r$

The radius of the circle is about 12.9 inches.

© Carnegie Learning

33. Bella used a tape measure and determined the circumference of a flagpole to be 6.2 inches. What is the radius of the flagpole?

$$C = 2\pi r$$
$$6.2 = 2\pi r$$
$$\frac{6.2}{2\pi} = r$$
$$1 \approx r$$

The radius of the flagpole is about 1 inch.

34. Carla used a string and a tape measure and determine the circumference of a circular cup to be 12.56 inches. What is the radius of the cup?

$$C = 2\pi r$$
$$12.56 = 2\pi r$$
$$\frac{12.56}{2\pi} = r$$
$$2 \approx r$$

The radius of the cup is about 2 inches.

Solve for each measure given the information.

35. If $\theta = \frac{\pi}{3}$ and $r = 3$, what is the length of the intercepted arc?

$$\theta = \frac{s}{r}$$
$$\frac{\pi}{3} = \frac{s}{3}$$
$$s = \pi$$

36. If $r = 8$ and the intercepted arc length is 6π, what is the measure of the central angle?

$$\theta = \frac{s}{r}$$
$$\theta = \frac{6\pi}{8}$$
$$\theta = \frac{3}{4}\pi$$

37. The measure of a central angle is 80°. The length of the radius is 40 mm. Determine the arc length using the formula $\frac{\text{measure of angle}}{360°} \cdot 2\pi r$.

$$\theta = \frac{s}{r}$$
$$\frac{2\pi}{3} = \frac{s}{6}$$
$$s = 4\pi$$

© Carnegie Learning

Name ____ Date ____________

38. If $r = 6$ and the intercepted arc length is 4π, what is the measure of the central angle?

$\theta = \frac{s}{r}$

$\theta = \frac{4\pi}{6}$

$\theta = \frac{2}{3}\pi$

39. The measure of a central angle is 80°. The length of the radius is 40 mm. Determine the arc length using the formula $\frac{\text{measure of angle}}{360°} \cdot 2\pi r$.

$$\text{Arc length} = \frac{80°}{360°} \cdot 2\pi(40)$$
$$= \frac{2}{9} \cdot 80\pi$$
$$= \frac{160\pi}{9} \text{ mm}$$

40. The measure of a central angle is 110°. The length of the radius is 15 ft. Determine the arc length using the formula $\frac{\text{measure of angle}}{360°} \cdot 2\pi r$.

$$\text{Arc length} = \frac{110°}{360°} \cdot 2\pi(15)$$
$$= \frac{11}{36} \cdot 30\pi$$
$$= \frac{55\pi}{6} \text{ ft}$$

© Carnegie Learning

10

© Carnegie Learning

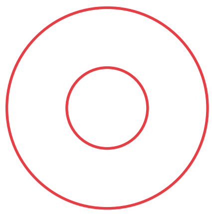
Name ____ Date ____________

Playing Darts
Sectors and Segments of a Circle

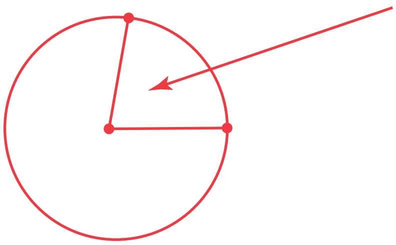
Vocabulary

Draw an example of each term.

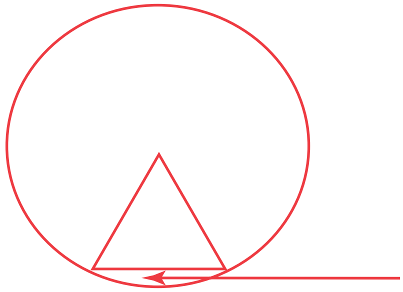
1. concentric circles



2. sector of a circle



3. segment of a circle

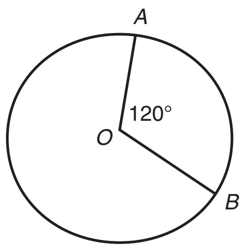


© Carnegie Learning

Problem Set

Calculate the area of each sector. Write your answer in terms of π.

1. If the radius of the circle is 9 centimeters, what is the area of sector *AOB*?



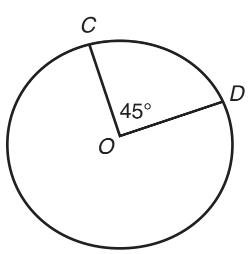
Total area of the circle $= \pi(9^2) = 81\pi$ cm^2

Sector *AOB*'s fraction of the circle $= \frac{120°}{360°} = \frac{1}{3}$

Area of sector *AOB* $= \frac{1}{3}(81\pi) = 27\pi$

The area of sector *AOB* is 27π cm^2.

2. If the radius of the circle is 16 meters, what is the area of sector *COD*?



Total area of the circle $= \pi(16^2) = 256\pi$ m^2

Sector *COD*'s fraction of the circle $= \frac{45°}{360°} = \frac{1}{8}$

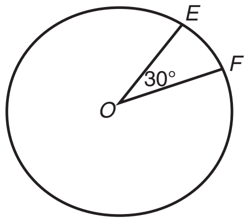
Area of sector *COD* $= \frac{1}{8}(256\pi) = 32\pi$

The area of sector *COD* is 32π m^2.

© Carnegie Learning

Name ______________________________ Date ____________

3. If the radius of the circle is 15 feet, what is the area of sector *EOF*?



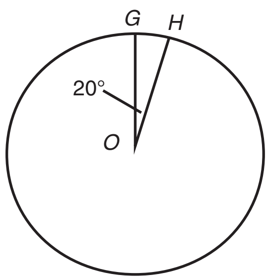
Total area of the circle $= \pi(15^2) = 225\pi$ ft^2

Sector *EOF*'s fraction of the circle $= \frac{30°}{360°} = \frac{1}{12}$

Area of sector *EOF* $= \frac{1}{12}(225\pi) = \frac{225}{12}\pi = \frac{75}{4}\pi$

The area of sector *EOF* is $\frac{75}{4}\pi$ ft^2.

4. If the radius of the circle is 10 inches, what is the area of sector *GOH*?



Total area of the circle $= \pi(10^2) = 100\pi$ inches^2

Sector *GOH*'s fraction of the circle $= \frac{20°}{360°} = \frac{1}{18}$

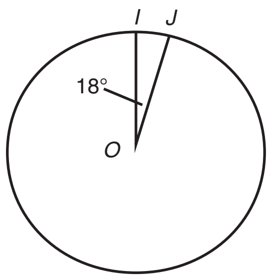
Area of sector *GOH* $= \frac{1}{18}(100\pi) = \frac{100}{18}\pi = \frac{50}{9}\pi$

The area of sector *GOH* is $\frac{50}{9}\pi$ inches^2.

© Carnegie Learning

10

5. If the radius of the circle is 32 centimeters, what is the area of sector *IOJ*?



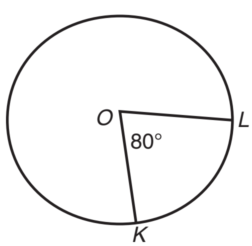
Total area of the circle $= \pi(32^2) = 1024\pi$ cm^2

Sector *IOJ*'s fraction of the circle $= \frac{18°}{360°} = \frac{1}{20}$

Area of sector *IOJ* $= \frac{1}{20}(1024\pi) = \frac{1024}{20}\pi = \frac{256}{5}\pi$

The area of sector *IOJ* is $\frac{256}{5}\pi$ cm^2.

6. If the radius of the circle is 20 millimeters, what is the area of sector *KOL*?



Total area of the circle $= \pi(20^2) = 400\pi$ mm^2

Sector *KOL*'s fraction of the circle $= \frac{80°}{360°} = \frac{2}{9}$

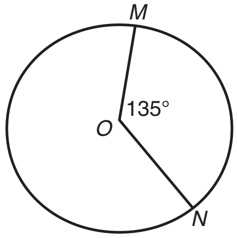
Area of sector *KOL* $= \frac{2}{9}(400\pi) = \frac{800}{9}\pi$

The area of sector *KOL* is $\frac{800}{9}\pi$ mm^2.

© Carnegie Learning

Name ____ Date ______________

7. If the radius of the circle is 24 centimeters, what is the area of sector *MON*?



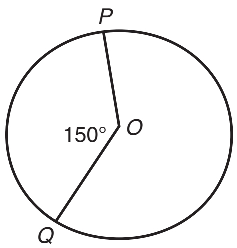
Total area of the circle $= \pi(24^2) = 576\pi$ cm^2

Sector *MON*'s fraction of the circle $= \frac{135°}{360°} = \frac{3}{8}$

Area of sector *MON* $= \frac{3}{8}(576\pi) = \frac{1728}{8}\pi = 216\pi$

The area of sector *MON* is 216π cm^2.

8. If the radius of the circle is 21 meters, what is the area of sector *POQ*?



Total area of the circle $= \pi(21^2) = 441\pi$ m^2

Sector *POQ*'s fraction of the circle $= \frac{150°}{360°} = \frac{5}{12}$

Area of sector *POQ* $= \frac{5}{12}(441\pi) = \frac{2205}{12}\pi = \frac{735}{4}\pi$

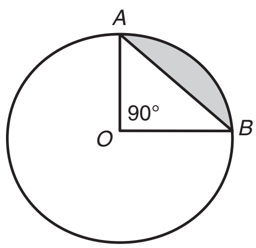
The area of sector *POQ* is $\frac{735}{4}\pi$ m^2.

10

© Carnegie Learning

Calculate the area of each segment. Round your answer to the nearest tenth, if necessary. Use 3.14 to estimate π.

9. If the radius of the circle is 6 centimeters, what is the area of the shaded segment?



Total area of the circle $= \pi(6^2) = 36\pi$ cm^2

Sector AOB's fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

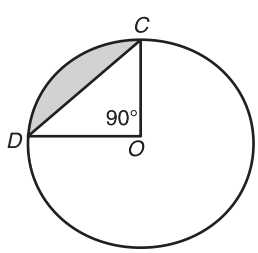
Area of sector $AOB = \frac{1}{4}(36\pi) = 9\pi$ cm^2

Area of $\triangle AOB = \frac{1}{2}(6 \cdot 6) = 18$ cm^2

Area of the segment: $9\pi - 18 \approx 28.3 - 18 = 10.3$

The area of the shaded segment is approximately 10.3 cm^2.

10. If the radius of the circle is 14 inches, what is the area of the shaded segment?



Total area of the circle $= \pi(14^2) = 196\pi$ inches^2

Sector COD's fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

Area of sector $COD = \frac{1}{4}(196\pi) = 49\pi$ inches^2

Area of $\triangle COD = \frac{1}{2}(14 \cdot 14) = 98$ inches^2

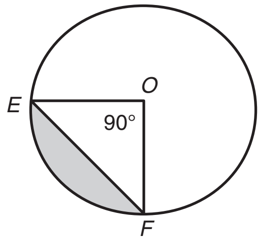
Area of the segment: $49\pi - 98 \approx 153.9 - 98 = 55.9$

The area of the shaded segment is approximately 55.9 inches^2.

© Carnegie Learning

Name ______________________________ Date ____________

11. If the radius of the circle is 17 feet, what is the area of the shaded segment?



Total area of the circle $= \pi(17^2) = 289\pi$ ft^2

Sector *EOF*'s fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

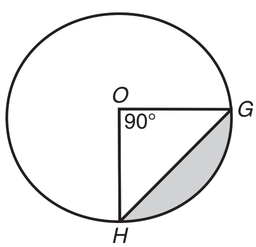
Area of sector *EOF* $= \frac{1}{4}(289\pi) = 72.5\pi$ ft^2

Area of $\triangle EOF = \frac{1}{2}(17 \cdot 17) = 144.5$ ft^2

Area of the segment: $72.25\pi - 144.5 \approx 226.9 - 144.5 = 82.4$ ft^2

The area of the shaded segment is approximately 82.4 ft^2.

12. If the radius of the circle is 22 centimeters, what is the area of the shaded segment?



Total area of the circle $= \pi(22^2) = 484\pi$ cm^2

Sector *GOH*'s fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

Area of sector *GOH* $= \frac{1}{4}(484\pi) = 121\pi$ cm^2

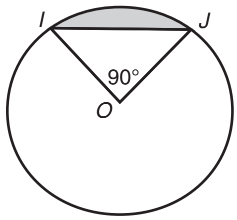
Area of $\triangle GOH = \frac{1}{2}(22 \cdot 22) = 242$ cm^2

Area of the segment: $121\pi - 242 \approx 379.9 - 242 = 137.9$

The area of the shaded segment is approximately 137.9 cm^2.

© Carnegie Learning

13. If the radius of the circle is 25 meters, what is the area of the shaded segment?



Total area of the circle $= \pi(25^2) = 625\pi$ m^2

Sector *IOJ*'s fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

Area of sector *IOJ* $= \frac{1}{4}(625\pi) = 156.25\pi$ m^2

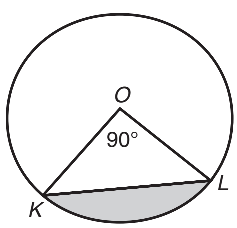
Area of $\triangle IOJ = \frac{1}{2}(25 \cdot 25) = 312.5$ m^2

Area of the segment: $156.25\pi - 312.5 \approx 490.6 - 312.5 = 178.1$

The area of the shaded segment is approximately 178.1 m^2.

10

14. If the radius of the circle is 30 centimeters, what is the area of the shaded segment?



Total area of the circle $= \pi(30^2) = 900\pi$ cm^2

Sector *KOL*'s fraction of the circle $= \frac{90°}{360°} = \frac{1}{4}$

Area of sector *KOL* $= \frac{1}{4}(900\pi) = 225\pi$ cm^2

Area of $\triangle KOL = \frac{1}{2}(30 \cdot 30) = 450$ cm^2

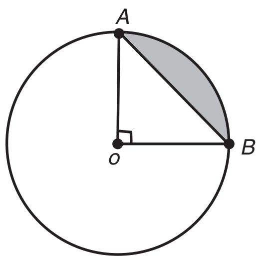
Area of the segment: $225\pi - 450 \approx 706.5 - 450 = 256.5$

The area of the shaded segment is approximately 256.5 cm^2.

© Carnegie Learning

Name ______________________________ Date ____________

In circle O below, $m\widehat{AB} = 90°$. Use the given information to determine the length of the radius of circle O.



10

15. If the area of the segment is $16\pi - 32$ square feet, what is the length of the radius of circle O?

$16\pi = \frac{90°}{360°}(\pi r^2)$

$16\pi = \frac{1}{4}(\pi r^2)$

$64 = r^2$

$8 = r$

The length of the radius is 8 feet.

16. If the area of the segment is $25\pi - 50$ square inches, what is the length of the radius of circle O?

$25\pi = \frac{90°}{360°}(\pi r^2)$

$25\pi = \frac{1}{4}(\pi r^2)$

$100 = r^2$

$10 = r$

The length of the radius is 10 inches.

17. If the area of the segment is $\pi - 2$ square meters, what is the length of the radius of circle O?

$\pi = \frac{90°}{360°}(\pi r^2)$

$\pi = \frac{1}{4}(\pi r^2)$

$4 = r^2$

$2 = r$

The length of the radius is 2 meters.

© Carnegie Learning

18. If the area of the segment is $56.25\pi - 112.5$ square yards, what is the length of the radius of circle O?

$$56.25\pi = \frac{90°}{360°}(\pi r^2)$$

$$56.25\pi = \frac{1}{4}(\pi r^2)$$

$$225 = r^2$$

$$15 = r$$

The length of the radius is 15 yards.

19. If the area of the segment is $121\pi - 242$ square feet, what is the length of the radius of circle O?

$$121\pi = \frac{90°}{360°}(\pi r^2)$$

$$121\pi = \frac{1}{4}(\pi r^2)$$

$$484 = r^2$$

$$22 = r$$

The length of the radius is 22 feet.

20. If the area of the segment is $90.25\pi - 180.5$ square millimeters, what is the length of the radius of circle O?

$$90.25\pi = \frac{90°}{360°}(\pi r^2)$$

$$90.25\pi = \frac{1}{4}(\pi r^2)$$

$$361 = r^2$$

$$19 = r$$

The length of the radius is 19 millimeters.

© Carnegie Learning

Name ____________________ Date ____________

Circle K. Excellent!
Circle Problems

Vocabulary

Define the key term in your own words.

1. linear velocity
   Linear velocity is an amount of distance over a specified amount of time.

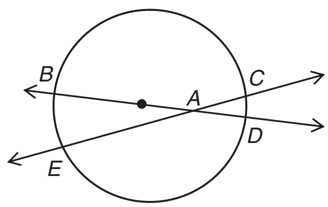
2. angular velocity
   Angular velocity is an amount of angle movement (in radians) over a specified amount of time.

© Carnegie Learning

Problem Set

Use the given arc measures to determine the measures of the indicated angles.

1.



$m\widehat{ED} = 140°$

$m\widehat{CD} = 10°$

$m\angle EAD =$ 155°

$m\angle CAD =$ 25°

Because arc *BCD* is a semicircle, its measure is 180°.

$m\widehat{BCD} = m\widehat{BC} + m\widehat{CD}$

$180° = m\widehat{BC} + 10°$

$m\widehat{BC} = 170°$

$m\angle EAD = \frac{1}{2}(m\widehat{ED} + m\widehat{BC})$

$m\angle EAD = \frac{1}{2}(140° + 170°)$

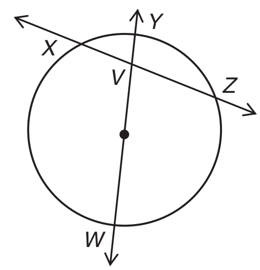
$m\angle EAD = \frac{1}{2}(310°)$

$m\angle EAD = 155°$

$m\angle CAD = 180° - 155°$

$= 25°$

2.



$m\widehat{XY} = 20°$

$m\widehat{YZ} = 50°$

$m\angle XVY =$ 75°

$m\angle YVZ =$ 105°

Because arc *YZW* is a semicircle, its measure is 180°.

$m\widehat{YZW} = m\angle\widehat{YZ} + m\angle\widehat{ZW}$

$180° = 50° + m\widehat{ZW}$

$m\widehat{ZW} = 130°$

$m\angle XVY = \frac{1}{2}(m\widehat{XY} + m\widehat{ZW})$

$m\angle XVY = \frac{1}{2}(20° + 130°)$

$m\angle XVY = \frac{1}{2}(150°)$

$m\angle XVY = 75°$

$m\angle YVZ = 180° - 75°$

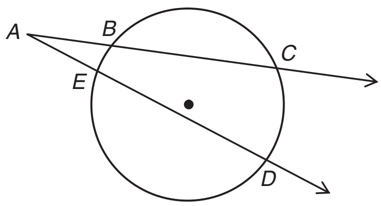
$= 105°$

10

© Carnegie Learning

Name ______________________________ Date ____________

3.



$m\overset{\frown}{BE} = 20°$

$m\overset{\frown}{CD} = 70°$

$m\angle A =$ 25°

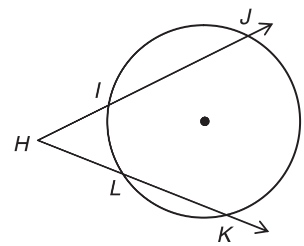
$m\angle A = \frac{1}{2}(m\overset{\frown}{CD} - m\overset{\frown}{BE})$

$m\angle A = \frac{1}{2}(70° - 20°)$

$m\angle A = \frac{1}{2}(50°)$

$m\angle A = 25°$

4.



$m\overset{\frown}{JK} = 164°$

$m\overset{\frown}{IL} = 42°$

$m\angle H =$ 61°

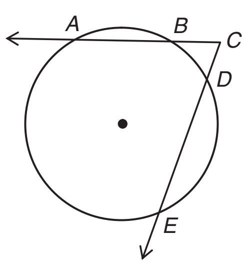
$m\angle H = \frac{1}{2}(m\overset{\frown}{JK} - m\overset{\frown}{IL})$

$m\angle H = \frac{1}{2}(164° - 42°)$

$m\angle H = \frac{1}{2}(122°)$

$m\angle H = 61°$

5.



$m\overset{\frown}{AE} = 170°$

$m\overset{\frown}{BD} = 20°$

$m\angle C =$ 75°

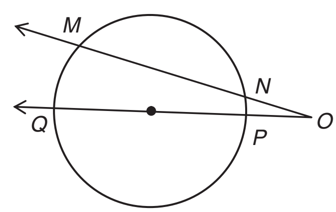
$m\angle C = \frac{1}{2}(m\overset{\frown}{AE} - m\overset{\frown}{BD})$

$m\angle C = \frac{1}{2}(170° - 20°)$

$m\angle C = \frac{1}{2}(150°)$

$m\angle C = 75°$

6.



$m\overset{\frown}{MQ} = 50°$

$m\overset{\frown}{NP} = 12°$

$m\angle O =$ 19°

$m\angle O = \frac{1}{2}(m\overset{\frown}{MQ} - m\overset{\frown}{NP})$

$m\angle O = \frac{1}{2}(50° - 12°)$

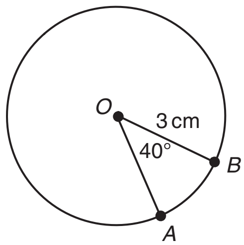
$m\angle O = \frac{1}{2}(38°)$

$m\angle O = 19°$

© Carnegie Learning

Calculate the area of each sector. Use 3.14 for π. Round to the nearest hundredth, if necessary.

7.



$A = \frac{40}{360}\pi r^2$

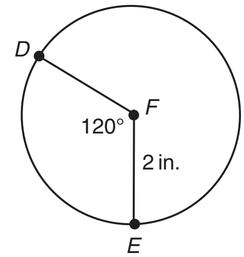
$A = \frac{1}{9}\pi(3)^2$

$A = \frac{1}{9}\pi(9)$

$A = \pi$

$A \approx 3.14$ square centimeters

8.



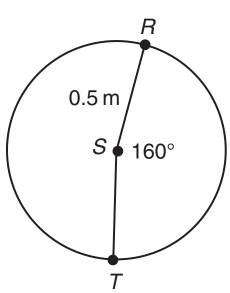
$A = \frac{120}{360}\pi r^2$

$A = \frac{1}{3}\pi(2)^2$

$A \approx \frac{1}{3}(3.14)(4)$

$A \approx 4.19$ square inches

9.



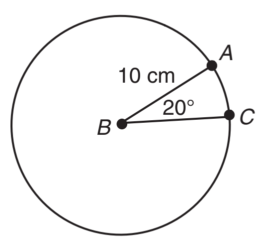
$A = \frac{160}{360}\pi r^2$

$A = \frac{4}{9}\pi(0.5)^2$

$A \approx \frac{4}{9}(3.14)(0.25)$

$A \approx 0.35$ square meters

10.



$A = \frac{20}{360}\pi r^2$

$A = \frac{1}{18}\pi(10)^2$

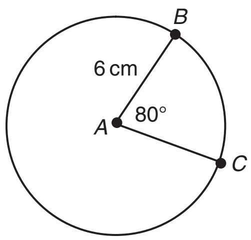
$A \approx \frac{1}{18}(3.14)(100)$

$A \approx 17.44$ square centimeters

© Carnegie Learning

Name ____________________ Date __________

11.



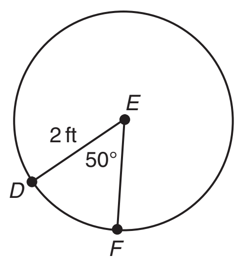
$A = \frac{80}{360}\pi r^2$

$A = \frac{2}{9}\pi(6)^2$

$A \approx \frac{2}{9}(3.14)(36)$

$A \approx 25.12$ square centimeters

12.



$A = \frac{50}{360}\pi r^2$

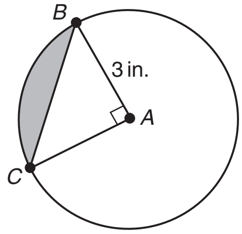
$A = \frac{5}{36}\pi(2)^2$

$A \approx \frac{5}{36}(3.14)(4)$

$A \approx 1.74$ square feet

Calculate the area of the shaded segment of the circle.

13.



The area of the shaded segment = the area of sector *ABC* − the area of triangle *ABC*.

Area of sector *ABC* $= \frac{90}{360}\pi r^2$

$= \frac{1}{4}\pi(3)^2$

$\approx \frac{1}{4}(3.14)(9)$

≈ 7.07 square inches

Area of $\triangle ABC = \frac{1}{2}bh$

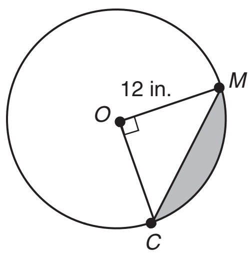
$= \frac{1}{2}(3)(3)$

$= \frac{9}{2} = 4.5$ square inches

Area of segment $\approx 7.07 - 4.5 \approx 2.57$ square inches

© Carnegie Learning

14.



The area of the shaded segment = the area of sector *MOC* − the area of triangle *MOC*.

$$\text{Area of sector } MOC = \frac{90}{360}\pi r^2$$
$$= \frac{1}{4}\pi(12)^2$$
$$\approx \frac{1}{4}(3.14)(144)$$
$$\approx 113.04 \text{ square inches}$$

$$\text{Area of } \triangle MOC = \frac{1}{2}bh$$
$$= \frac{1}{2}(12)(12)$$
$$= 72 \text{ square inches}$$

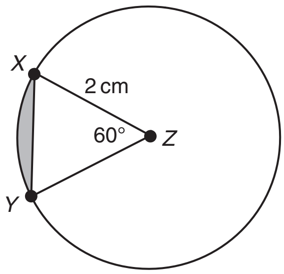
Area of segment ≈ 113.04 − 72 ≈ 41.04 square inches

10

© Carnegie Learning

Name ____________________ Date ____________

15.



The area of the shaded segment = the area of sector *XYZ* − the area of triangle *XYZ*.

Area of sector $XYZ = \pi r^2$

$$= \frac{60}{360}\pi(2)^2$$

$$\approx \frac{1}{6}(3.14)(4)$$

$$\approx 2.09 \text{ square centimeters}$$

$\triangle XYZ$ is an equilateral triangle, so the base is 2 centimeters and the height is $\sqrt{3}$ centimeters.

Area of $\triangle XYZ = \frac{1}{2}bh$

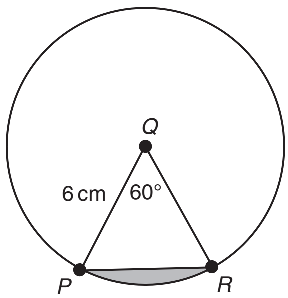
$$= \frac{1}{2}(2)(\sqrt{3})$$

$$= \sqrt{3} \approx 1.73 \text{ square centimeters}$$

Area of segment $\approx 2.09 - 1.73 \approx 0.36$ square centimeters

© Carnegie Learning

16.



The area of the shaded segment = the area of sector *PQR* − the area of triangle *PQR*.

$$\text{Area of sector } PQR = \frac{60}{360}\pi r^2$$
$$= \frac{60}{360}\pi(6)^2$$
$$\approx \frac{1}{6}(3.14)(36)$$
$$\approx 18.84 \text{ square centimeters}$$

$\triangle PQR$ is an equilateral triangle, so the base is 6 centimeters and the height is $3\sqrt{3}$ centimeters.

$$\text{Area of } \triangle XYZ = \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(3\sqrt{3})$$
$$\approx 15.59 \text{ square centimeters}$$

Area of segment ≈ 18.84 − 15.59 ≈ 3.25 square centimeters

Determine each linear or angular velocity.

17. Determine the linear velocity if $s = 12$ cm and $t = 2$ sec.

$$v = \frac{s}{t}$$
$$= \frac{12 \text{ cm}}{2 \text{ sec}}$$
$$= 6 \text{ cm/sec}$$

18. Determine the angular velocity if $\theta = 12\pi$ and $t = 5\pi$ seconds.

$$\omega = \frac{\theta}{t}$$
$$= \frac{12\pi}{5\pi}$$
$$= 2.4 \text{ radians/sec}$$

© Carnegie Learning

Name ______________________________ Date ____________

19. Determine the linear velocity if $s = 4.2$ in. and $t = 12$ s.

$$v = \frac{s}{t}$$
$$= \frac{4.2 \text{ in.}}{12\ s}$$
$$= 0.35 \text{ in./sec}$$

20. Determine the angular velocity if $\theta = 9\pi$ and $t = 16$ seconds.

$$\omega = \frac{\theta}{t}$$
$$= \frac{9\pi}{16}$$
$$= \frac{9}{16}\pi \text{ radians/sec}$$

21. Determine the linear velocity if $s = 25$ ft and $t = 120$ s.

$$v = \frac{s}{t}$$
$$= \frac{25 \text{ ft}}{120\ s}$$
$$= \frac{5}{24} \text{ ft/sec}$$

22. Determine the angular velocity if $\theta = \frac{3}{4}\pi$ and $t = 10\pi$ seconds.

$$\omega = \frac{\theta}{t}$$
$$= \frac{0.75\pi}{10\pi}$$
$$= 0.075 \text{ radians/sec}$$

© Carnegie Learning

© Carnegie Learning