## Whirlygigs for Sale!

Rotating Two-Dimensional Figures through Space

## Vocabulary

Describe the term in your own words.

1. disc

A disc is the set of all points on a circle and in the interior of a circle.

## Problem Set

Write the name of the solid figure that would result from rotating the plane figure shown around the axis shown.
1.

sphere
2.

cylinder
3.

cylinder
4.

cone
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5.

6.

cone

Relate the dimensions of the plane figure to the solid figure that results from its rotation around the given axis.


The base of the rectangle is equal to the radius of the cylinder's base.


The radius of the circle is equal to the radius of the sphere.

Half of the length of the top side of the triangle is equal to the radius of the cone's base.
10.


The bottom side of the triangle is equal to the radius of the cone's base.
$\qquad$
11.


Half the width of the square is equal to the radius of the cylinder's base.
12.


The height of the rectangle is equal to the radius of the cylinder's base.
$\qquad$

## Cakes and Pancakes <br> Translating and Stacking Two-Dimensional Figures

## Vocabulary

Match each definition to its corresponding term.

1. oblique triangular prism

D
2. oblique rectangular prism

B
3. oblique cylinder

C
4. isometric paper
A
5. right triangular prism
F
6. right rectangular prism

G
7. right cylinder
E
A. dotted paper used to show three-dimensional diagrams
B. a prism with rectangles as bases whose lateral faces are not perpendicular to those bases
C. a 3-dimensional object with two parallel, congruent, circular bases, and a lateral face not perpendicular to those bases
D. a prism with triangles as bases whose lateral faces are not perpendicular to those bases
E. a 3-dimensional object with two parallel, congruent, circular bases, and a lateral face perpendicular to those bases
F. a prism with triangles as bases whose lateral faces are perpendicular to those bases
G. a prism with rectangles as bases whose lateral faces are perpendicular to those bases

## Problem Set

Connect the corresponding vertices of the figure and the translated figure. Name the shape that was translated and name the resulting solid figure.

right triangle; right triangular prism
3.

square; square prism

rectangle; rectangular prism

rectangle; rectangular prism
4.

triangle; triangular prism

square; square prism

Name
Date $\qquad$

Name the solid formed by stacking 1000 of the congruent shapes shown.

cylinder

triangular prism
11.

rectangular prism
8.

pentagonal prism
10.

square prism
12.

hexagonal prism

## Lesson 11.2 Skills Practice

Name the solid formed by stacking similar shapes so that each layer of the stack is composed of a slightly smaller shape than the previous layer.
13.

cone
15.

triangular pyramid
17.

rectangular pyramid
14.

pentagonal pyramid
16.

square pyramid
18.

hexagonal pyramid

Name Date

Relate the dimensions of the given plane shape to the related solid figure. Tell whether the shape was made by stacking congruent or similar shapes.
19.

20.

21.


The lengths of the sides of the triangle are the same as the lengths of the sides of the base of the triangular prism. The triangular prism was made by stacking congruent triangles.

The lengths of the sides of the square are the same as the lengths of the sides of the base of the square pyramid. The square pyramid was made by stacking similar squares.

The radius of the circle is the same as the radius of the base of the cone. The cone was made by stacking similar circles.
22.


The lengths of the sides of the triangle are the same as the lengths of the sides of the base of the triangular pyramid. The triangular pyramid was made by stacking similar triangles.
23.


The lengths of the sides of the rectangle are the same as the lengths of the sides of the base of the rectangular prism. The rectangular prism was made by stacking congruent rectangles.
24.



The radius of the circle is the same as the radius of the base of the cylinder. The cylinder was made by stacking congruent circles.
$\qquad$

## Cavalieri's Principles <br> Application of Cavalieri's Principles

## Vocabulary

Describe the term in your own words.

1. Cavalieri's principle

For two-dimensional figures, if the lengths of one-dimensional slices—just a line segment-of the two figures are the same, then the figures have the same area.

For three-dimensional figures between parallel planes, if every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.

## Problem Set

Estimate the approximate area or volume each irregular or oblique figure. Round your answers to the nearest tenth, if necessary.

1. The height of each rectangle is 10 yards and the base of each rectangle is 1.5 yards.


I determined that the area is approximately 300 square yards.

$$
\begin{aligned}
\text { area } & =\text { base } \times \text { height } \\
& =(1.5 \times 20)(10) \\
& =(30)(10) \\
& =300
\end{aligned}
$$

I determined the sum of the areas of the rectangles to estimate the area of the figure because Cavalieri's principle says that the area of the irregular figure is equal to the sum of the areas of the multiple rectangles when the base and height of all the rectangles are equal.

## Lesson 11.3 Skills Practice

2. The height of each rectangle is 0.6 inch and the base of each rectangle is 2 inches.


I determined that the area is approximately 18 square inches.

$$
\begin{aligned}
\text { area } & =\text { base } \times \text { height } \\
& =(2)(0.6 \times 15) \\
& =(2)(9) \\
& =18
\end{aligned}
$$

I determined the sum of the areas of the rectangles to estimate the area of the figure because Cavalieri's principle says that the area of the irregular figure is equal to the sum of the areas of the multiple rectangles when the base and height of all the rectangles are equal.
3.


$$
\begin{aligned}
\text { volume } & =\pi r^{2} h \\
& =\pi(1)^{2}(10) \\
& =10 \pi \\
& \approx 31.4 \text { cubic centimeters }
\end{aligned}
$$

I used the formula for the volume of a cylinder because Cavalieri's principle says that the volume of an oblique cylinder is equal to the volume of a right cylinder when the radii and heights are equal.


$$
\begin{aligned}
\text { volume } & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(12) \\
& =64 \pi \\
& \approx 201.1 \text { cubic feet }
\end{aligned}
$$

I used the formula for the volume of a cone because Cavalieri's principle says that the volume of an oblique cone is equal to the volume of a right cone, when the radii and heights are equal.

## Lesson 11.4 Skills Practice

Name
Date

## Spin to Win

Volume of Cones and Pyramids

## Problem Set

Calculate the volume of each cone. Use 3.14 for $\pi$.
1.


$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(5) \\
& \approx 83.73 \text { cubic centimeters }
\end{aligned}
$$

2. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(2)^{2}(7) \\
& \approx 29.31 \text { cubic centimeters }
\end{aligned}
$$

3. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(6)^{2}(3) \\
& \approx 113.04 \text { cubic inches }
\end{aligned}
$$

4. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(13) \\
& \approx 217.71 \text { cubic inches }
\end{aligned}
$$



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(15)^{2}(10) \\
& \approx 2355 \text { cubic meters }
\end{aligned}
$$

Name Date
6.


$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(5)^{2}(14) \\
& \approx 366.33 \text { cubic millimeters }
\end{aligned}
$$

7. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(5)^{2}(6.5) \\
& \approx 170.08 \text { cubic centimeters }
\end{aligned}
$$

8. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(1)^{2}(3.2) \\
& \approx 3.35 \text { cubic centimeters }
\end{aligned}
$$

9. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4.5)^{2}(7) \\
& \approx 148.37 \text { cubic feet }
\end{aligned}
$$

10. 



$$
\begin{aligned}
\text { volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(7)^{2}(16.4) \\
& \approx 841.10 \text { cubic feet }
\end{aligned}
$$

Name Date $\qquad$

Calculate the volume of the square pyramid.
11.


$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(10)^{2}(9) \\
& =300 \text { cubic inches }
\end{aligned}
$$

12. 



Volume $=\frac{1}{3} B h$
$=\frac{1}{3} s^{2} h$
$=\frac{1}{3}(12)^{2}(9)$
$=432$ cubic feet
13.


$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(7)^{2}(11) \\
& \approx 179.67 \text { cubic centimeters }
\end{aligned}
$$

14. 



$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(25)^{2}(20) \\
& \approx 4166.67 \text { cubic meters }
\end{aligned}
$$

Name $\qquad$ Date $\qquad$
15.


$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(30)^{2}(22) \\
& =6600 \text { cubic feet }
\end{aligned}
$$

16. 



$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(21)^{2}(28) \\
& =4116 \text { cubic millimeters }
\end{aligned}
$$

17. 



$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(42)^{2}(34.5) \\
& =20,286 \text { cubic inches }
\end{aligned}
$$

18. 



$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(90)^{2}(75) \\
& =202,500 \text { cubic centimeters }
\end{aligned}
$$

Name $\qquad$ Date $\qquad$
19.


$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(100)^{2}(125) \\
& \approx 416,666.67 \text { cubic yards }
\end{aligned}
$$

20. 



$$
\begin{aligned}
\text { Volume } & =\frac{1}{3} B h \\
& =\frac{1}{3} s^{2} h \\
& =\frac{1}{3}(200)^{2}(180) \\
& =2,400,000 \text { cubic feet }
\end{aligned}
$$

## Spheres à la Archimedes <br> Volume of a Sphere

## Vocabulary

Describe a similarity and a difference between each term.

1. radius of a sphere and diameter of a sphere

Similarity: Both measure a distance within a great circle of a sphere.
Difference: The diameter measures all the way across a great circle, and the radius only measures halfway across a great circle.
2. cross section of a sphere and great circle of a sphere

Similarity: Both are circles created by a plane intersecting a sphere.
Difference: The cross section can be anywhere on the sphere. The great circle is a cross section that contains the center of the sphere.
3. hemisphere and sphere

Similarity: Both are types of three-dimensional figures.
Difference: A hemisphere is half of a sphere.

Describe the term in your own words.
4. annulus

The area between two concentric circles is called the annulus.


## Problem Set

Calculate the volume of each sphere. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.

1. $r=7$ meters


$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(7)^{3} \\
& =\frac{1372}{3} \pi \\
& \approx 1436.0 \text { cubic meters }
\end{aligned}
$$

2. $r=6$ inches


$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(6)^{3} \\
& =288 \pi \\
& \approx 904.3 \text { cubic inches }
\end{aligned}
$$

4. $d=16$ meters


$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi\left(\frac{16}{2}\right)^{3} \\
& =\frac{2048}{3} \pi \\
& \approx 2143.6 \mathrm{~m}^{3}
\end{aligned}
$$

5. $r=2.5$ centimeters


$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(2.5)^{3} \\
& =\frac{125}{6} \pi \\
& \approx 65.4 \text { cubic centimeters }
\end{aligned}
$$

6. $r=11.25$ millimeters


$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(11.25)^{3} \\
& =\frac{30,375}{16} \pi \\
& \approx 5961.1 \text { cubic millimeters }
\end{aligned}
$$

7. The radius of the great circle of a sphere is 8 meters.

$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(8)^{3} \\
& =\frac{2048}{3} \pi \\
& \approx 2143.6 \text { cubic meters }
\end{aligned}
$$

8. The radius of the great circle of a sphere is 12 feet.

$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(12)^{3} \\
& =2304 \pi \\
& \approx 7234.6 \text { cubic feet }
\end{aligned}
$$

9. The diameter of the great circle of a sphere is 20 centimeters.

$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(10)^{3} \\
& =\frac{4000}{3} \pi \\
& \approx 4186.7 \text { cubic centimeters }
\end{aligned}
$$

10. The diameter of the great circle of a sphere is 15 yards.

$$
\begin{aligned}
\text { Volume } & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(7.5)^{3} \\
& =\frac{1125}{2} \pi \\
& \approx 1766.3 \text { cubic yards }
\end{aligned}
$$

## Lesson 11.6 Skills Practice

Name
Date $\qquad$

## Turn Up the . . . <br> Using Volume Formulas

## Problem Set

Calculate the volume of each pyramid.
1.


$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}(3)(3)(5) \\
& =15 \text { cubic meters }
\end{aligned}
$$

2. 



$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}(3)(10)(6) \\
& =60 \text { cubic feet }
\end{aligned}
$$

3. 



$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}\left(\frac{1}{2}\right)(4)(6)(10) \\
& =40 \text { cubic inches }
\end{aligned}
$$



$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}\left|\frac{1}{2}\right|(6+3)(4)(3) \\
& =18 \text { cubic feet }
\end{aligned}
$$

5. 


6.


$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}\left(\left.\frac{1}{2} \right\rvert\,(8)(6)(5)\right. \\
& =40 \text { cubic feet }
\end{aligned}
$$

$$
\begin{aligned}
V & =\frac{1}{3} B h \\
& =\frac{1}{3}(7)(4)(6) \\
& =56 \text { cubic meters }
\end{aligned}
$$

Calculate the volume of each cylinder. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.
7. 5.5 m


$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(5.5)^{2}(7) \\
& =211.75 \pi \\
& \approx 664.9 \text { cubic meters }
\end{aligned}
$$



$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi\left(\frac{20}{2}\right)^{2}(5) \\
& =500 \pi \\
& \approx 1570 \text { cubic meters }
\end{aligned}
$$

10. 



$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi\left(\frac{22}{2}\right)^{2}(30) \\
& =3630 \pi \\
& \approx 11398.2 \text { cubic yards }
\end{aligned}
$$

8. 



$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(4.5)^{2}(10) \\
& =202.5 \pi \\
& \approx 635.9 \text { cubic feet }
\end{aligned}
$$

Name Date
11. 4 mm


$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(4)^{2}(6) \\
& =96 \pi \\
& \approx 301.4 \text { cubic millimeters }
\end{aligned}
$$

13. 9 m


$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(4.5)^{2}(12) \\
& =243 \pi \\
& \approx 763.0 \text { cubic meters }
\end{aligned}
$$

12. 



$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(8)^{2}(5) \\
& =320 \pi \\
& \approx 1004.8 \text { cubic feet }
\end{aligned}
$$

$$
\begin{aligned}
V & =\pi r^{2} h \\
& =\pi(3.5)^{2}(13) \\
& =159.25 \pi \\
& \approx 500.0 \text { cubic centimeters }
\end{aligned}
$$

Calculate the volume of each cone. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.
15.


$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(5)^{2}(6) \\
& =50 \pi \\
& \approx 157 \text { cubic millimeters }
\end{aligned}
$$

11
17.


$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(7)^{2}(10) \\
& =\frac{490}{3} \pi \\
& \approx 512.9 \text { cubic feet }
\end{aligned}
$$

16. 



$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(20)^{2}(12) \\
& =1600 \pi \\
& \approx 5024 \text { cubic meters }
\end{aligned}
$$

18. 



$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(11) \\
& =\frac{176}{3} \pi \\
& \approx 184.2 \text { cubic yards }
\end{aligned}
$$

Name $\qquad$ Date $\qquad$


$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(6) \\
& =32 \pi \\
& \approx 100.5 \text { cubic meters }
\end{aligned}
$$

21. 



$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(3)^{2}(8) \\
& =24 \pi \\
& \approx 75.4 \text { cubic inches }
\end{aligned}
$$

20. [ $\left.-\cdots-\cdots 3 \mathrm{ft}-\cdots-]_{]}\right]$


$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(1.5)^{2}(3) \\
& =2.25 \pi \\
& \approx 7.1 \text { cubic feet }
\end{aligned}
$$

$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h \\
& =\frac{1}{3} \pi(2)^{2}(5) \\
& =\frac{20}{3} \pi \\
& \approx 20.9 \text { cubic millimeters }
\end{aligned}
$$

22. 



## LeSSON 11.6 Skills Practice

Calculate the volume of each sphere. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.
23.


$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(9)^{3} \\
& =972 \pi \\
& \approx 3052.08 \text { cubic inches }
\end{aligned}
$$

24. 



$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(5)^{3} \\
& =\frac{500}{3} \pi
\end{aligned}
$$

$$
\approx 523.33 \text { cubic centimeters }
$$

Name Date $\qquad$
25.


$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(7)^{3} \\
& =\frac{1372}{3} \pi \\
& \approx 1436.03 \text { cubic millimeters }
\end{aligned}
$$

26. 



$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
& =\frac{4}{3} \pi(2.5)^{3} \\
& =\frac{125}{6} \pi \\
& \approx 65.42 \text { cubic feet }
\end{aligned}
$$

$\qquad$

## Tree Rings

## Cross Sections

## Problem Set

Describe the shape of each cross section shown.


The cross section is a rectangle.
3.

The cross section is a rectangle.

The cross section is a trapezoid.


The cross section is a point.



The cross section is a great circle.


The cross section is a rectangle.


The cross section is a triangle.


The cross section is a circle.


The cross section is a hexagon.
10.


The cross section is a hexagon.

Name Date

Use the given information to sketch and describe the cross sections.
11. Consider a cylinder. Sketch and describe three different cross sections formed when a plane intersects a cylinder.

circle

rectangle

ellipse
12. Consider a rectangular prism. Sketch and describe three different cross sections formed when a plane intersects a rectangular prism.

13. Consider a pentagonal prism. Sketch and describe three different cross sections formed when a plane intersects a pentagonal prism.


pentagon

line segment
14. Consider a triangular prism. Sketch and describe three different cross sections formed when a plane intersects a triangular prism.

triangle

15. Consider a triangular pyramid. Sketch and describe three different cross sections formed when a plane intersects a triangular pyramid.


triangle

triangle
16. Consider a hexagonal pyramid. Sketch and describe three different cross sections formed when a plane intersects a hexagonal pyramid.

hexagon

triangle

hexagon

Name
Date $\qquad$

Consider two cross sections of the given solid. One cross section is parallel to the base of the solid, and the other cross section is perpendicular to the base of the solid. Determine the shape of each of these cross sections.
17.


A cross section that is parallel to the base is a hexagon congruent to the hexagonal bases. A cross section that is perpendicular to the base is a rectangle.
18.


A cross section that is parallel to the base is a rectangle congruent to the rectangular bases.
A cross section that is perpendicular to the base is a rectangle.
19.


A cross section that is parallel to the base is a triangle similar to the triangle base.
A cross section that is perpendicular to the base, but not parallel to any of the sides of the base is a triangle.
A cross section that is perpendicular to the base and parallel to one of the sides of the base is a trapezoid.

## LeSSON 11.7 Skills Practice

20. 



A cross section that is parallel to the base is a pentagon similar to the pentagonal base.
A cross section that is perpendicular to the base and passes through the apex is a triangle.
A cross section that is perpendicular to the base and parallel to one of the sides of the base is a trapezoid.
21.


A cross section that is parallel to the base is a circle congruent to the circular bases.
A cross section that is perpendicular to the base is a rectangle.
22.


A cross section that is parallel to the base is a circle similar to the circular base.
A cross section that is perpendicular to the base and passes through the apex is a triangle.

Name $\qquad$ Date
23.


A cross section that is parallel to the base is a pentagon congruent to the pentagonal bases.
A cross section that is perpendicular to the base is a rectangle.
24.


A cross section that is parallel to the base is a triangle congruent to the triangular bases.
A cross section that is perpendicular to the base is a rectangle.

Draw a solid that could have each cross section described.
25. cross section parallel to the base


The solid is a cone. (The solid could also be a cylinder.)

## LeSSON 11.7 Skills Practice

26. cross section perpendicular to the base


The solid is a rectangular prism. (The solid could also be a cylinder.)
27. cross section parallel to the base


The solid is a pentagonal prism. (The solid could also be a pentagonal pyramid.)
28. cross section parallel to the base


The solid is a triangular prism. (The solid could also be a triangular pyramid.)

## Lesson 11.7 Skills Practice

Name $\qquad$ Date $\qquad$
29. cross section perpendicular to the base


The solid is a rectangular pyramid. (The solid could also be a cone.)
30. cross section parallel to the base


The solid is a hexagonal prism. (The solid could also be a hexagonal pyramid.)

## Two Dimensions Meet Three Dimensions

## Diagonals in Three Dimensions

## Problem Set

Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then, draw a three-dimensional diagonal using a solid line.
1.

2.

3.

4.

5.

6.


Determine the length of the diagonal of each rectangular solid.
7.


The length of the diagonal of the rectangular solid is about 12.33 inches.
The length of the first leg is 10 inches.
Length of Second Leg:

$$
\begin{aligned}
& \text { Length of Diagonal: } \\
& \begin{aligned}
d & =7.21^{2}+10^{2} \\
& =51.98+100 \\
d & =\sqrt{151.98} \approx 12.33
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
d^{2} & =6^{2}+4^{2} \\
& =36+16 \\
d & =\sqrt{52} \approx 7.21
\end{aligned}
$$



The length of the diagonal of the rectangular solid is about 11.35 meters.
The length of the first leg is 7 meters.
Length of Second Leg: Length of Diagonal:

$$
\begin{aligned}
d^{2} & =8^{2}+4^{2} \\
& =64+16 \\
d & =\sqrt{80} \approx 8.94
\end{aligned}
$$

$$
\begin{aligned}
d & =8.94^{2}+7^{2} \\
& =79.92+49 \\
d & =\sqrt{128.92} \approx 11.35
\end{aligned}
$$

$\qquad$
9.


The length of the diagonal of the rectangular solid is about 19.0 centimeters.
The length of the first leg is 15 centimeters.
Length of Second Leg: Length of Diagonal:

$$
\begin{aligned}
d^{2} & =10^{2}+6^{2} \\
& =100+36 \\
d & =\sqrt{136} \approx 11.66
\end{aligned}
$$

$$
\begin{aligned}
d^{2} & =11.66^{2}+15^{2} \\
& =135.96+225 \\
d & =\sqrt{360.96} \approx 19.0
\end{aligned}
$$

10. 



The length of the diagonal of the rectangular solid is about 11.09 yards.
The length of the first leg is 7 yards.
Length of Second Leg: Length of Diagonal:

$$
\begin{aligned}
d^{2} & =5^{2}+7^{2} \\
& =25+49 \\
d & =\sqrt{74} \approx 8.60
\end{aligned}
$$

$$
\begin{aligned}
d^{2} & =8.60^{2}+7^{2} \\
& =73.96+49 \\
d & =\sqrt{122.96} \approx 11.09
\end{aligned}
$$

11. 



The length of the diagonal of the rectangular solid is about 16.83 inches.
The length of the first leg is 5 inches.

Length of Second Leg:

$$
\begin{aligned}
d^{2} & =3^{2}+15^{2} \\
& =9+225 \\
d & =\sqrt{234} \approx 15.30
\end{aligned}
$$

Length of Diagonal:

$$
\begin{aligned}
d^{2} & =15.30^{2}+7^{2} \\
& =234.09+49 \\
d & =\sqrt{283.09} \approx 16.83
\end{aligned}
$$

12. 



The length of the diagonal of the rectangular solid is about 12.33 feet.
The length of the first leg is 12 feet.
Length of Second Leg:
Length of Diagonal:

$$
\begin{aligned}
d^{2} & =2^{2}+2^{2} \\
& =4+4 \\
d & =\sqrt{8} \approx 2.83
\end{aligned}
$$

$$
d^{2}=2.83^{2}+12^{2}
$$

$$
=8.01+144
$$

Name Date $\qquad$

Diagonals are shown on the front panel, side panel, and top panel of each rectangular solid. Sketch three triangles using the diagonals from each of the three panels and some combination of the length, width, and height of the solid.
13.

14.

15.

16.

17.



Name $\qquad$ Date $\qquad$
18.




A rectangular solid is shown. Use the diagonals across the front panel, the side panel, and the top panel of each solid to determine the length of the three-dimensional diagonal.
19.


$$
\begin{aligned}
& S D^{2}=\frac{1}{2}\left(8^{2}+6^{2}+3^{2}\right) \\
& S D^{2}=\frac{1}{2}(64+36+9) \\
& S D^{2}=\frac{1}{2}(109) \\
& S D^{2}=54.5 \\
& S D=\sqrt{54.5} \approx 7.4
\end{aligned}
$$

The length of the three-dimensional diagonal is $\sqrt{54.5}$ or approximately 7.4 inches.
20.

$S D^{2}=\frac{1}{2}\left(3^{2}+9^{2}+10^{2}\right)$
$S D^{2}=\frac{1}{2}(9+81+100)$
$S D^{2}=\frac{1}{2}(190)$
$S D^{2}=95$

$$
S D=\sqrt{95} \approx 9.7
$$

The length of the three-dimensional diagonal is $\sqrt{95}$ or approximately 9.7 meters.
21.

$S D^{2}=\frac{1}{2}\left(8^{2}+10^{2}+12^{2}\right)$
$S D^{2}=\frac{1}{2}(64+100+144)$
$S D^{2}=\frac{1}{2}(308)$
$S D^{2}=154$
$S D=\sqrt{154} \approx 12.4$
The length of the three-dimensional diagonal is $\sqrt{154}$ or approximately 12.4, feet.

Name $\qquad$ Date $\qquad$
22.

$S D^{2}=\frac{1}{2}\left(6^{2}+6^{2}+5^{2}\right)$
$S D^{2}=\frac{1}{2}(36+36+25)$
$S D^{2}=\frac{1}{2}(97)$
$S D^{2}=48.5$
$S D=\sqrt{48.5} \approx 7.0$
The length of the three-dimensional diagonal is $\sqrt{48.5}$ or approximately 7.0 meters.
23.


$$
\begin{aligned}
& S D^{2}=\frac{1}{2}\left(8^{2}+10^{2}+4^{2}\right) \\
& S D^{2}=\frac{1}{2}(64+100+16) \\
& S D^{2}=\frac{1}{2}(180) \\
& S D^{2}=90 \\
& S D=\sqrt{90} \approx 9.5
\end{aligned}
$$

The length of the three-dimensional diagonal is $\sqrt{90}$ or approximately 9.5 yards.
24.


$$
S D^{2}=\frac{1}{2}\left(15^{2}+13^{2}+3^{2}\right)
$$

$$
S D^{2}=\frac{1}{2}(225+169+9)
$$

$$
S D^{2}=\frac{1}{2}(403)
$$

$$
S D^{2}=201.5
$$

$$
S D=\sqrt{201.5} \approx 14.2
$$

The length of the three-dimensional diagonal is $\sqrt{201.5}$ or approximately 14.2 , inches.

Use a formula to answer each question. Show your work and explain your reasoning.
25. A packing company is in the planning stages of creating a box that includes a diagonal support. The box has a width of 5 feet, a length of 6 feet, and a height of 8 feet. How long will the diagonal support need to be?
The diagonal support should be approximately 11.18 feet. I determined the answer by calculating the length of the box's diagonal.

$$
\begin{aligned}
d^{2} & =5^{2}+6^{2}+8^{2} \\
d^{2} & =25+36+64 \\
d^{2} & =125 \\
d & \approx 11.18
\end{aligned}
$$

26. A plumber needs to transport a 12-foot pipe to a jobsite. The interior of his van is 90 inches in length, 40 inches in width, and 40 inches in height. Will the pipe fit inside the plumber's van?
The pipe will not fit in the van. The diagonal length of the van is about 106 inches, but the length of the pipe is 12 feet or 144 inches.
$d^{2}=90^{2}+40^{2}+40^{2}$
$d^{2}=8100+1600+1600$
$d^{2}=11,300$
$d \approx 106.3$
27. You are landscaping the flower beds in your front yard. You choose to plant a tree that measures 5 feet from the root ball to the top. The interior of your car is 60 inches in length, 45 inches in width, and 40 inches in height. Will the tree fit inside your car?

The tree will fit in the car. The diagonal length of the car is 85 inches and the height of the tree is 5 feet or 60 inches.
$d^{2}=60^{2}+45^{2}+40^{2}$
$d^{2}=3600+2025+1600$
$d^{2}=7225$
$d=85$
28. Julian is constructing a box for actors to stand on during a school play. To make the box stronger, he decides to include diagonals on all sides of the box and a three-dimensional diagonal through the center of the box. The diagonals across the front and back of the box are each 2 feet, the diagonals across the sides of the box are each 3 feet, and the diagonals across the top and bottom of the box are each 7 feet. How long is the diagonal through the center of the box?
The diagonal through the center of the box is about 5.6 feet long.
$d^{2}=\frac{1}{2}\left(2^{2}+3^{2}+7^{2}\right)$
$d^{2}=\frac{1}{2}(4+9+49)$
$d^{2}=\frac{1}{2}(62)$
$d^{2}=31$
$d \approx 5.6$
29. Carmen has a cardboard box. The length of the diagonal across the front of the box is 9 inches. The length of the diagonal across the side of the box is 7 inches. The length of the diagonal across the top of the box is 5 inches. Carmen wants to place a 10-inch stick into the box and be able to close the lid. Will the stick fit inside the box?
A 10-inch stick will not fit inside the box because the diagonal's length is only about 8.8 inches.

$$
\begin{aligned}
d^{2} & =\frac{1}{2}\left(9^{2}+7^{2}+5^{2}\right) \\
d^{2} & =\frac{1}{2}(81+49+25) \\
d^{2} & =\frac{1}{2}(155) \\
d^{2} & =77.5 \\
d & \approx 8.8
\end{aligned}
$$

30. A technician needs to pack a television in a cardboard box. The length of the diagonal across the front of the box is 17 inches. The length of the diagonal across the side of the box is 19 inches. The length of the diagonal across the top of the box is 20 inches. The three-dimensional diagonal of the television is 24 inches. Will the television fit in the box?

A television with a 24 -inch diagonal will not fit inside the box because the diagonal's length is only about 22.9 inches.

$$
\begin{aligned}
& d^{2}=\frac{1}{2}\left(17^{2}+19^{2}+20^{2}\right) \\
& d^{2}=\frac{1}{2}(289+361+400) \\
& d^{2}=\frac{1}{2}(1050) \\
& d^{2}=525 \\
& d \approx 22.9
\end{aligned}
$$

