## Up and Down or Down and Up Exploring Quadratic Functions

## Vocabulary

Write the given quadratic function in standard form. Then describe the shape of the graph and whether it has an absolute maximum or absolute minimum. Explain your reasoning.
$f(x)=-4+2 x^{2}-x$
Standard form: $f(x)=2 x^{2}-x-4$
Graph: The graph of this function is a parabola that opens up because the sign of the coefficient is positive. The graph has an absolute minimum because the a value is greater than 0 .

## Problem Set

Write each quadratic function in standard form.

1. $f(x)=x(x+3)$
$f(x)=x(x+3)$
$f(x)=x \cdot x+x \cdot 3$
$f(x)=x^{2}+3 x$
2. $f(x)=3 x(x-8)+5$
$f(x)=3 x(x-8)+5$
$f(x)=3 x \cdot x-3 x \cdot 8+5$
$f(x)=3 x^{2}-24 x+5$
3. $d(t)=(20+3 t) t$
$d(t)=(20+3 t) t$
$d(t)=20 \cdot t+3 t \cdot t$
$d(t)=20 t+3 t^{2}$
$d(t)=3 t^{2}+20 t$
4. $f(n)=\frac{2 n(3 n-6)}{3}$
5. $m(s)=\frac{s(s+3)}{4}$
$f(n)=\frac{2 n(3 n-6)}{3}$
$m(s)=\frac{s(s+3)}{4}$
$f(n)=\frac{2 n \cdot 3 n-2 n \cdot 6}{3}$
$m(s)=\frac{s \cdot s+s \cdot 3}{4}$
$f(n)=\frac{6 n^{2}-12 n}{3}$
$m(s)=\frac{s^{2}+3 s}{4}$
$f(n)=\frac{6 n^{2}}{3}-\frac{12 n}{3}$
$m(s)=\frac{1}{4} s^{2}+\frac{3}{4} s$
$f(n)=2 n^{2}-4 n$

Write a quadratic function in standard form that represents each area as a function of the width.
Remember to define your variables.
7. A builder is designing a rectangular parking lot. She has 300 feet of fencing to enclose the parking lot around three sides.
Let $x=$ the width of the parking lot
Area of a rectangle $=$ width $\times$ length
The length of the parking lot $=300-2 x$

$$
\begin{aligned}
A & =w \cdot I \\
A(x) & =x \cdot(300-2 x) \\
& =x \cdot 300-x \cdot 2 x \\
& =300 x-2 x^{2} \\
& =-2 x^{2}+300 x
\end{aligned}
$$

8. Aiko is enclosing a new rectangular flower garden with a rabbit garden fence. She has 40 feet of fencing.
Let $x=$ the width of the garden
Area of a rectangle $=$ width $\times$ length
The length of the garden $=\frac{40-2 x}{2}$

$$
=20-x
$$

Let $A=$ the area of the garden

$$
\begin{aligned}
A & =w \cdot l \\
A(x) & =x \cdot(20-x) \\
& =x \cdot 20-x \cdot x \\
& =20 x-x^{2} \\
& =-x^{2}+20 x
\end{aligned}
$$

9. Pedro is building a rectangular sandbox for the community park. The materials available limit the perimeter of the sandbox to at most 100 feet.
Let $x=$ the width of the sandbox

$$
\text { Area of a rectangle }=\text { width } \times \text { length }
$$

The length of the sandbox $=\frac{100-2 x}{2}$

$$
=50-x
$$

Let $A=$ the area of the sandbox

$$
\begin{aligned}
A & =w \cdot l \\
A(x) & =x \cdot(50-x) \\
& =x \cdot 50-x \cdot x \\
& =50 x-x^{2} \\
& =-x^{2}+50 x
\end{aligned}
$$

$\qquad$
10. Lea is designing a rectangular quilt. She has 16 feet of piping to finish the quilt around three sides.

Let $x=$ the width of the quilt
The length of the quilt $=16-2 x$
Let $A=$ the area of the quilt

Area of a rectangle $=$ width $\times$ length

$$
\begin{aligned}
A & =w \cdot l \\
A(x) & =x \cdot(16-2 x) \\
& =x \cdot 16-x \cdot 2 x \\
& =16 x-2 x^{2} \\
& =-2 x^{2}+16 x
\end{aligned}
$$

11. Kiana is making a rectangular vegetable garden alongside her home. She has 24 feet of fencing to enclose the garden around the three open sides.
Let $x=$ the width of the garden
Area of a rectangle $=$ width $\times$ length
The length of the garden $=24-2 x$

$$
\begin{aligned}
A & =w \cdot l \\
A(x) & =x \cdot(24-2 x) \\
& =x \cdot 24-x \cdot 2 x \\
& =24 x-2 x^{2} \\
& =-2 x^{2}+24 x
\end{aligned}
$$

12. Nelson is building a rectangular ice rink for the community park. The materials available limit the perimeter of the ice rink to at most 250 feet.

Let $x=$ the width of the ice rink
The length of the ice rink $=\frac{250-2 x}{2}$

$$
=125-x
$$

Let $A=$ the area of the ice rink

$$
\begin{aligned}
& \text { Area of a rectangle }=\text { width } \times \text { length } \\
& \begin{aligned}
A & =w \cdot I \\
A(x) & =x \cdot(125-x) \\
& =x \cdot 125-x \cdot x \\
& =125 x-x^{2} \\
& =-x^{2}+125 x
\end{aligned}
\end{aligned}
$$

Use your graphing calculator to determine the absolute maximum of each function. Describe what the $x$ - and $y$-coordinates of this point represent in terms of the problem situation.
13. A builder is designing a rectangular parking lot. He has 400 feet of fencing to enclose the parking lot around three sides. Let $x=$ the width of the parking lot. Let $A=$ the area of the parking lot.
The function $A(x)=-2 x^{2}+400 x$ represents the area of the parking lot as a function of the width.
The absolute maximum of the function is at $(100,20,000)$.
The $x$-coordinate of 100 represents the width in feet that produces the maximum area.
The $y$-coordinate of 20,000 represents the maximum area in square feet of the parking lot.
14. Joelle is enclosing a portion of her yard to make a pen for her ferrets. She has 20 feet of fencing. Let $x=$ the width of the pen. Let $A=$ the area of the pen. The function $A(x)=-x^{2}+10 x$ represents the area of the pen as a function of the width.
The absolute maximum of the function is at $(5,25)$.
The $x$-coordinate of 5 represents the width in feet that produces the maximum area.
The $y$-coordinate of 25 represents the maximum area in square feet of the pen.
15. A baseball is thrown upward from a height of 5 feet with an initial velocity of 42 feet per second. Let $t=$ the time in seconds after the baseball is thrown. Let $h=$ the height of the baseball. The quadratic function $h(t)=-16 t^{2}+42 t+5$ represents the height of the baseball as a function of time. The absolute maximum of the function is at about $(1.31,32.56)$.
The $x$-coordinate of 1.31 represents the time in seconds after the baseball is thrown that produces the maximum height.
The $y$-coordinate of 32.56 represents the maximum height in feet of the baseball.
16. Hector is standing on top of a playground set at a park. He throws a water balloon upward from a height of 12 feet with an initial velocity of 25 feet per second. Let $t=$ the time in seconds after the balloon is thrown. Let $h=$ the height of the balloon. The quadratic function $h(t)=-16 t^{2}+25 t+12$ represents the height of the balloon as a function of time.
The absolute maximum of the function is at about ( $0.78,21.77$ ).
The $x$-coordinate of 0.78 represents the time in seconds after the balloon is thrown that produces the maximum height.
The $y$-coordinate of 21.77 represents the maximum height in feet of the balloon.
17. Franco is building a rectangular roller-skating rink at the community park. The materials available limit the perimeter of the skating rink to at most 180 feet. Let $x=$ the width of the skating rink. Let $A=$ the area of the skating rink. The function $A(x)=-x^{2}+90 x$ represents the area of the skating rink as a function of the width.
The absolute maximum of the function is at $(45,2025)$.
The $x$-coordinate of 45 represents the width in feet that produces the maximum area.
The $y$-coordinate of 2025 represents the maximum area in square feet of the skating rink.
18. A football is thrown upward from a height of 6 feet with an initial velocity of 65 feet per second. Let $t=$ the time in seconds after the football is thrown. Let $h=$ the height of the football. The quadratic function $h(t)=-16 t^{2}+65 t+6$ represents the height of the football as a function of time. The absolute maximum of the function is at about (2.03, 72.02).
The $x$-coordinate of 2.03 represents the time in seconds after the football is thrown that produces the maximum height.
The $y$-coordinate of 72.02 represents the maximum height in feet of the football.

## Lesson 12.2 Skills Practice

Name Date $\qquad$

## Just U and I

Comparing Linear and Quadratic Functions

## Vocabulary

Write a definition for each term in your own words.

1. leading coefficient

The leading coefficient of a function is the numerical coefficient of the term with the greatest power.
2. second differences

Second differences are the differences between consecutive values of the first differences.

## Problem Set

Graph each table of values. Describe the type of function represented by the graph.
1.

| $x$ | $y$ |
| :---: | :---: |
| -4 | 7 |
| -2 | 6 |
| 0 | 5 |
| 2 | 4 |
| 4 | 3 |

The function represented by the graph is a linear function.

2.

| $x$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -3 | -2 |
| -2 | 0 |
| -1 | 2 |
| 0 | 4 |
| 1 | 6 |

The function represented by the graph is a linear function.
3.

| $x$ | $y$ |
| :---: | :---: |
| -2 | -8 |
| 0 | 0 |
| 2 | 4 |
| 4 | 4 |
| 6 | 0 |

The function represented by the graph is a quadratic function.
4.

| $x$ | $y$ |
| :---: | :---: |
| -6 | 6 |
| -4 | 0 |
| -2 | -2 |
| 0 | 0 |
| 2 | 6 |

The function represented by the graph is a quadratic function.



$\qquad$
5.

| $x$ | $y$ |
| :---: | :---: |
| 1 | 6 |
| 2 | 3 |
| 3 | 0 |
| 4 | -3 |
| 5 | -6 |

The function represented by the graph is a linear function.
6.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -3 | -9 |
| 0 | 0 |
| 3 | 3 |
| 6 | 0 |
| 9 | -9 |

The function represented by the graph is a quadratic function.



## LESSON 12.2 Skills Practice

Calculate the first and second differences for each table of values. Describe the type of function represented by the table.
7.

| $x$ | $y$ | First Differences | Second Differences |
| :---: | :---: | :---: | :---: |
| -2 | -6 |  |  |
|  | -6 | 3 |  |
| -1 | -3 | 3 | 0 |
| 0 | 0 | 3 | 0 |
| 1 | 3 |  |  |
|  | 3 | 3 | 0 |
| 2 | 6 |  |  |

The function represented by the table is a linear function.
9.

| $x$ | $y$ | First Differences | Second Differences |
| :---: | :---: | :---: | :---: |
| - | 3 |  |  |
|  |  | 1 |  |
| -2 | 4 |  | 0 |
| -1 | 5 |  | 0 |
| 0 | 6 | 1 |  |
|  |  | 1 | 0 |
| 1 | 7 | 1 |  |
|  |  |  |  |

The function represented by the table is a linear function.
8.

| $x$ | $y$ | First Differences | Second Differences |
| :---: | :---: | :---: | :---: |
| -2 | 12 |  |  |
|  |  | -9 |  |
| -1 | 3 |  | 6 |
| 0 | 0 |  | 6 |
|  |  | 3 |  |
| 1 | 3 |  | 6 |
| 2 | 12 | 9 |  |
|  |  |  |  |

The function represented by the table is a quadratic function.
10.

| $x$ | $y$ |  |  |
| :---: | :---: | :---: | :---: |
| -1 | 1 | Differences | Second Differences |
|  |  | -1 |  |
| 0 | 0 | 3 | 4 |
| 1 | 3 |  | 4 |
|  |  | 7 |  |
| 2 | 10 | 11 | 4 |
| 3 | 21 |  |  |
| 3 |  |  |  |

The function represented by the table is a quadratic function.

Name
Date
11.

| $x$ | $y$ | First <br> Differences |  |
| :---: | :---: | :---: | :---: |
| -4 | -48 |  | Second |
|  |  | 21 | Differences |
| -3 | -27 |  | -6 |
| -2 | -12 | 15 | -6 |
| -1 | -3 | 9 | -6 |
| 0 | 0 | 3 |  |

The function represented by the table is a quadratic function.
12.

| $x$ | $y$ | First |  |
| :---: | :---: | :---: | :---: |
| -1 | 10 | Differences | Second Differences |
|  |  | -2 |  |
| 0 | 8 |  | 0 |
|  |  | -2 | 0 |
| 1 | 6 | -2 |  |
| 2 | 4 |  | 0 |
|  |  | -2 |  |
| 3 | 2 |  |  |

The function represented by the table is a linear function.
$\qquad$

## Walking the . . . Curve? <br> Domain, Range, Zeros, and Intercepts

## Vocabulary

Choose the term that best completes each sentence.

| zeros <br> closed interval | vertical motion model <br> half-closed interval | interval <br> half-open interval | open interval |
| :--- | :--- | :--- | :--- |

1. $A n$ $\qquad$ is defined as the set of real numbers between two given numbers.
2. The $x$-intercepts of a graph of a quadratic function are also called the $\qquad$ of the quadratic function.
3. An open interval $(a, b)$ describes the set of all numbers between $a$ and $b$, but not including $a$ or $b$.
4. A half-closed interval or half-open interval ( $a, b$ ] describes the set of all numbers between $a$ and $b$, including $b$ but not including $a$. Or, $[a, b)$ describes the set of all numbers between $a$ and $b$, including $a$ but not including $b$.
5. A quadratic equation that models the height of an object at a given time is a vertical motion model
6. A $\qquad$ $[a, b]$ describes the set of all numbers between $a$ and $b$, including $a$ and $b$.

## Problem Set

Graph the function that represents each problem situation. Identify the absolute maximum, zeros, and the domain and range of the function in terms of both the graph and problem situation. Round your answers to the nearest hundredth, if necessary.

1. A model rocket is launched from the ground with an initial velocity of 120 feet per second. The function $g(t)=-16 t^{2}+120 t$ represents the height of the rocket, $g(t), t$ seconds after it was launched.


Time (seconds)

Absolute maximum: $(3.75,225)$
Zeros: (0, 0), (7.5, 0)
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.
Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 7.5 .
Range of graph: The range is all real numbers less than or equal to 225.
Range of the problem: The range is all real numbers less than or equal to 225 and greater than or equal to 0 .
2. A model rocket is launched from the ground with an initial velocity of 60 feet per second. The function $g(t)=-16 t^{2}+60 t$ represents the height of the rocket, $g(t), t$ seconds after it was launched.


Absolute maximum: $(1.875,56.25)$
Zeros: (0, 0), (3.75, 0)
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.
Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 3.75 .
Range of graph: The range is all real numbers less than or equal to 56.25 .
Range of the problem: The range is all real numbers less than or equal to 56.25 and greater than or equal to 0 .
$\qquad$
3. A baseball is thrown into the air from a height of 5 feet with an initial vertical velocity of 15 feet per second. The function $g(t)=-16 t^{2}+15 t+5$ represents the height of the baseball, $g(t), t$ seconds after it was thrown.


Absolute maximum: ( $0.47,8.52$ )
Zeros: ( $-0.26,0$ ), (1.20, 0 )
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.

Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 1.20.

Range of graph: The range is all real numbers less than or equal to 8.52 .

Range of the problem: The range is all real numbers less than or equal to 8.52 and greater than or equal to 0 .
4. A football is thrown into the air from a height of 6 feet with an initial vertical velocity of 50 feet per second. The function $g(t)=-16 t^{2}+50 t+6$ represents the height of the football, $g(t), t$ seconds after it was thrown.


Absolute maximum: $(1.56,45.06)$
Zeros: (-0.12, 0), (3.24, 0)
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.

Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 3.24.

Range of graph: The range is all real numbers less than or equal to 45.06 .

Range of the problem: The range is all real numbers less than or equal to 45.06 and greater than or equal to 0 .
5. A tennis ball is dropped from a height of 25 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t)=-16 t^{2}+25$ represents the height of the tennis ball, $g(t)$, $t$ seconds after it was dropped.


Time (seconds)

Absolute maximum: $(0,25)$
Zeros: ( $-1.25,0$ ), (1.25, 0 )
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.
Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 1.25 .
Range of graph: The range is all real numbers less than or equal to 25.
Range of the problem: The range is all real numbers less than or equal to 25 and greater than or equal to 0 .
6. A tennis ball is dropped from a height of 150 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t)=-16 t^{2}+150$ represents the height of the tennis ball, $g(t)$, $t$ seconds after it was dropped.


Absolute maximum: $(0,150)$
Zeros: ( $-3.06,0$ ), $(3.06,0)$
Domain of graph: The domain is all real numbers from negative infinity to positive infinity.
Domain of the problem: The domain is all real numbers greater than or equal to 0 and less than or equal to 3.06 .
Range of graph: The range is all real numbers less than or equal to 150.
Range of the problem: The range is all real numbers less than or equal to 150 and greater than or equal to 0 .
$\qquad$

Use interval notation to represent each interval described.
7. All real numbers greater than or equal to -3 but less than 5 .
$[-3,5)$
8. All real numbers greater than or equal to -100 .
$[-100, \infty)$
9. All real numbers greater than -36 and less than or equal to 14 .
$(-36,14]$
10. All real numbers less than or equal to $b$.
$(-\infty, b]$
11. All real numbers greater than or equal to $c$ and less than or equal to $d$.
$[c, d]$
12. All real numbers greater than or equal to $n$.
$[n, \infty)$

Identify the intervals of increase and decrease for each function.
13. $f(x)=x^{2}+6 x$
14. $f(x)=3 x^{2}-6 x$


Interval of increase: $(-3, \infty)$
Interval of decrease: $(-\infty,-3)$


Interval of increase: $(1, \infty)$
Interval of decrease: $(-\infty, 1)$
15. $f(x)=-x^{2}+2 x+8$


Interval of increase: $(-\infty, 1)$
Interval of decrease: $(1, \infty)$
17. $f(x)=x^{2}-9$


Interval of increase: ( $0, \infty$ )
Interval of decrease: $(-\infty, 0)$
16. $f(x)=-6 x^{2}+24 x$


Interval of increase: $(-\infty, 2)$
Interval of decrease: ( $2, \infty$ )
18. $f(x)=x^{2}-4 x+6$


Interval of increase: $(2, \infty)$
Interval of decrease: ( $-\infty$, 2)

## Are You Afraid of Ghosts?

## Factored Form of a Quadratic Function

## Vocabulary

Write a definition for each term in your own words.

1. factor an expression

To factor an expression means to use the Distributive Property in reverse to rewrite the expression as a product of factors.
2. factored form

A quadratic function written in factored form is in the form $f(x)=a\left(x-r_{1}\right)\left(x-r_{2}\right)$, where $a \neq 0$.

## Problem Set

Factor each expression.

1. $6 x-24$
$6 x-24=6(x)-6(4)$ $=6(x-4)$
2. $3 x+36$
$3 x+36=3(x)+3(12)$
$=3(x+12)$
3. $10 x+15$
$10 x+15=5(2 x)+5(3)$ $=5(2 x+3)$
4. $42 x-35$
$42 x-35=7(6 x)-7(5)$
$=7(6 x-5)$
5. $-x-9$
$-x-9=(-1)(x)+(-1)(9)$
$=-(x+9)$
6. $-2 x+14$
$-2 x+14=(-2)(x)+(-2)(-7)$
$=-2(x-7)$

Determine the $x$-intercepts of each quadratic function in factored form.
7. $f(x)=(x-2)(x-8)$
The $x$-intercepts are $(2,0)$ and $(8,0)$.
8. $f(x)=(x+1)(x-6)$
The $x$-intercepts are $(-1,0)$ and $(6,0)$.
9. $f(x)=3(x+4)(x-2)$

The $x$-intercepts are $(-4,0)$ and $(2,0)$.
10. $f(x)=0.25(x-1)(x-12)$

The $x$-intercepts are $(1,0)$ and $(12,0)$.
11. $f(x)=0.5(x+15)(x+5)$
The $x$-intercepts are $(-15,0)$ and $(-5,0)$.
12. $f(x)=4(x-1)(x-9)$

The $x$-intercepts are $(1,0)$ and $(9,0)$.

Write a quadratic function in factored form with each set of given characteristics.
13. Write a quadratic function that represents a parabola that opens downward and has $x$-intercepts $(-2,0)$ and $(5,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x+2)(x-5)$ for $a<0$
14. Write a quadratic function that represents a parabola that opens downward and has $x$-intercepts $(2,0)$ and $(14,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x-2)(x-14)$ for $a<0$
15. Write a quadratic function that represents a parabola that opens upward and has $x$-intercepts $(-8,0)$ and $(-1,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x+8)(x+1)$ for $a>0$
16. Write a quadratic function that represents a parabola that opens upward and has $x$-intercepts $(3,0)$ and $(7,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x-3)(x-7)$ for $a>0$
$\qquad$
17. Write a quadratic function that represents a parabola that opens downward and has $x$-intercepts $(-5,0)$ and $(2,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x+5)(x-2)$ for $a<0$
18. Write a quadratic function that represents a parabola that opens upward and has $x$-intercepts $(-12,0)$ and $(-4,0)$.
Answers will vary but functions should be in the form:
$f(x)=a(x+12)(x+4)$ for $a>0$

Determine the $x$-intercepts for each function using your graphing calculator. Write the function in factored form.
19. $f(x)=x^{2}-8 x+7$
$x$-intercepts: $(1,0)$ and $(7,0)$
factored form: $f(x)=(x-1)(x-7)$
21. $f(x)=-x^{2}-20 x-75$
$x$-intercepts: $(-5,0)$ and $(-15,0)$
factored form: $f(x)=-(x+5)(x+15)$
23. $f(x)=-3 x^{2}-9 x+12$
$x$-intercepts: $(-4,0)$ and $(1,0)$
factored form: $f(x)=-3(x+4)(x-1)$
20. $f(x)=2 x^{2}-10 x-48$
$x$-intercepts: $(-3,0)$ and $(8,0)$
factored form: $f(x)=2(x+3)(x-8)$
22. $f(x)=x^{2}+8 x+12$
$x$-intercepts: $(-2,0)$ and $(-6,0)$
factored form: $f(x)=(x+2)(x+6)$
24. $f(x)=x^{2}-6 x$
$x$-intercepts: $(6,0)$ and $(0,0)$
factored form: $f(x)=(x-0)(x-6)$

$$
=x(x-6)
$$

Determine the $x$-intercepts for each function. If necessary, rewrite the function in factored form.
25. $f(x)=(3 x+18)(x-2)$
factored form: $f(x)=3(x+6)(x-2)$
$x$-intercepts: $(-6,0)$ and $(2,0)$
27. $f(x)=(-2 x+8)(x-14)$
factored form: $f(x)=-2(x-4)(x-14)$
$x$-intercepts: $(4,0)$ and $(14,0)$
29. $f(x)=x(x+7)$
factored form: $f(x)=(x-0)(x+7)$
$x$-intercepts: $(0,0)$ and $(-7,0)$
26. $f(x)=(x+8)(3-x)$
factored form: $f(x)=-(x+8)(x-3)$
$x$-intercepts: $(-8,0)$ and $(3,0)$
28. $f(x)=(x+16)(2 x+16)$
factored form: $f(x)=2(x+16)(x+8)$ $x$-intercepts: $(-16,0)$ and $(-8,0)$
30. $f(x)=(-3 x+9)(x+3)$
factored form: $f(x)=-3(x-3)(x+3)$
$x$-intercepts: $(-3,0)$ and $(3,0)$

Name Date $\qquad$

## Just Watch that Pumpkin Fly! Investigating the Vertex of a Quadratic Function

## Vocabulary

Graph the quadratic function. Plot and label the vertex. Then draw and label the axis of symmetry. Explain how you determine each location.
$h(t)=t^{2}+2 t-3$


The vertex is at $(-1,-4)$ because it is the lowest point on the curve. The axis of symmetry is -1 because the axis of symmetry is equal to the $x$-coordinate of the vertex.

## Problem Set

Write a function that represents the vertical motion described in each problem situation.

1. A catapult hurls a watermelon from a height of 36 feet at an initial velocity of 82 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+82 t+36$
2. A catapult hurls a cantaloupe from a height of 12 feet at an initial velocity of 47 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+47 t+12$
3. A catapult hurls a pineapple from a height of 49 feet at an initial velocity of 110 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+110 t+49$
4. A basketball is thrown from a height of 7 feet at an initial velocity of 58 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+58 t+7$
5. A soccer ball is thrown from a height of 25 feet at an initial velocity of 46 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+46 t+25$
6. A football is thrown from a height of 6 feet at an initial velocity of 74 feet per second.
$h(t)=-16 t^{2}+v_{0} t+h_{0}$
$h(t)=-16 t^{2}+74 t+6$
Identify the vertex and the equation of the axis of symmetry for each vertical motion model.
7. A catapult hurls a grapefruit from a height of 24 feet at an initial velocity of 80 feet per second.

The function $h(t)=-16 t^{2}+80 t+24$ represents the height of the grapefruit $h(t)$ in terms of time $t$.
The vertex of the graph is $(2.5,124)$.
The axis of symmetry is $x=2.5$.
8. A catapult hurls a pumpkin from a height of 32 feet at an initial velocity of 96 feet per second.

The function $h(t)=-16 t^{2}+96 t+32$ represents the height of the pumpkin $h(t)$ in terms of time $t$.
The vertex of the graph is $(3,176)$.
The axis of symmetry is $x=3$.
9. A catapult hurls a watermelon from a height of 40 feet at an initial velocity of 64 feet per second. The function $h(t)=-16 t^{2}+64 t+40$ represents the height of the watermelon $h(t)$ in terms of time $t$.
The vertex of the graph is $(2,104)$.
The axis of symmetry is $x=2$.
10. A baseball is thrown from a height of 6 feet at an initial velocity of 32 feet per second. The function $h(t)=-16 t^{2}+32 t+6$ represents the height of the baseball $h(t)$ in terms of time $t$.
The vertex of the graph is $(1,22)$.
The axis of symmetry is $x=1$.
$\qquad$
11. A softball is thrown from a height of 20 feet at an initial velocity of 48 feet per second. The function $h(t)=-16 t^{2}+48 t+20$ represents the height of the softball $h(t)$ in terms of time $t$.
The vertex of the graph is $(1.5,56)$.
The axis of symmetry is $x=1.5$.
12. A rocket is launched from the ground at an initial velocity of 112 feet per second. The function $h(t)=-16 t^{2}+112 t$ represents the height of the rocket $h(t)$ in terms of time $t$.
The vertex of the graph is $(3.5,196)$.
The axis of symmetry is $x=3.5$.

Determine the axis of symmetry of each parabola.
13. The $x$-intercepts of a parabola are $(3,0)$ and $(9,0)$.
$\frac{3+9}{2}=\frac{12}{2}=6$
The axis of symmetry is $x=6$.
14. The $x$-intercepts of a parabola are $(-3,0)$ and $(1,0)$.
$\frac{-3+1}{2}=\frac{-2}{2}=-1$
The axis of symmetry is $x=-1$.
15. The $x$-intercepts of a parabola are $(-12,0)$ and $(-2,0)$.
$\frac{-12+(-2)}{2}=\frac{-14}{2}=-7$
The axis of symmetry is $x=-7$.
16. Two symmetric points on a parabola are $(-1,4)$ and $(5,4)$.
$\frac{-1+5}{2}=\frac{4}{2}=2$
The axis of symmetry is $x=2$.
17. Two symmetric points on a parabola are $(-4,8)$ and $(2,8)$.
$\frac{-4+2}{2}=\frac{-2}{2}=-1$
The axis of symmetry is $x=-1$.
18. Two symmetric points on a parabola are $(3,1)$ and $(15,1)$.
$\frac{3+15}{2}=\frac{18}{2}=9$
The axis of symmetry is $x=9$.

Determine the vertex of each parabola.
19. $f(x)=x^{2}+2 x-15$
axis of symmetry: $x=-1$
The axis of symmetry is $x=-1$.
The $x$-coordinate of the vertex is -1 .
The $y$-coordinate when $x=-1$ is:

$$
\begin{aligned}
f(-1) & =(-1)^{2}+2(-1)-15 \\
& =1-2-15 \\
& =-16
\end{aligned}
$$

The vertex is $(-1,-16)$.
21. $f(x)=x^{2}+4 x-12$
$x$-intercepts: $(2,0)$ and $(-6,0)$
$\frac{-6+2}{2}=\frac{-4}{2}=-2$
The axis of symmetry is $x=-2$, so
the $x$-coordinate of the vertex is -2 .
The $y$-coordinate when $x=-2$ is:

$$
\begin{aligned}
f(-2) & =(-2)^{2}+4(-2)-12 \\
& =4-8-12 \\
& =-16
\end{aligned}
$$

The vertex is $(-2,-16)$.
23. $f(x)=-x^{2}+8 x+20$
two symmetric points on the parabola:
$(-1,11)$ and $(9,11)$
$\frac{-1+9}{2}=\frac{8}{2}=4$
The axis of symmetry is $x=4$, so
the $x$-coordinate of the vertex is 4 .
The $y$-coordinate when $x=4$ is:

$$
\begin{aligned}
f(4) & =-(4)^{2}+8(4)+20 \\
& =-16+32+20 \\
& =36
\end{aligned}
$$

The vertex is $(4,36)$.
20. $f(x)=x^{2}-8 x+7$
axis of symmetry: $x=4$
The axis of symmetry is $x=4$.
The $x$-coordinate of the vertex is 4 .
The $y$-coordinate when $x=4$ is:

$$
\begin{aligned}
f(4) & =(4)^{2}-8(4)+7 \\
& =16-32+7 \\
& =-9
\end{aligned}
$$

The vertex is (4, -9 ).
22. $f(x)=-x^{2}-14 x-45$
$x$-intercepts: $(-9,0)$ and $(-5,0)$
$\frac{-9+(-5)}{2}=\frac{-14}{2}=-7$
The axis of symmetry is $x=-7$, so
the $x$-coordinate of the vertex is -7 .
The $y$-coordinate when $x=-7$ is:

$$
\begin{aligned}
f(-7) & =-(-7)^{2}-14(-7)-45 \\
& =-49+98-45 \\
& =4
\end{aligned}
$$

The vertex is ( $-7,4$ ).
24. $f(x)=-x^{2}+16$
two symmetric points on the parabola:
$(-3,7)$ and $(3,7)$
$\frac{-3+3}{2}=\frac{0}{2}=0$
The axis of symmetry is $x=0$, so
the $x$-coordinate of the vertex is 0 .
The $y$-coordinate when $x=0$ is:

$$
\begin{aligned}
f(0) & =-(0)^{2}+16 \\
& =0+16 \\
& =16
\end{aligned}
$$

The vertex is $(0,16)$.
$\qquad$

Determine another point on each parabola.
25. The axis of symmetry is $x=3$.

A point on the parabola is $(1,4)$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as $(1,4)$. The $x$-coordinate is:
$\frac{1+a}{2}=3$
$1+a=6$
$a=5$
Another point on the parabola is $(5,4)$.
27. The axis of symmetry is $x=1$.

A point on the parabola is $(-3,2)$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as $(-3,2)$. The $x$-coordinate is:

$$
\begin{array}{r}
\frac{-3+a}{2}=1 \\
-3+a=2 \\
a=5
\end{array}
$$

Another point on the parabola is $(5,2)$.
29. The vertex is $(-1,6)$.

A point on the parabola is $(2,3)$.
The axis of symmetry is $x=-1$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as $(2,3)$. The $x$-coordinate is:

$$
\begin{aligned}
\frac{2+a}{2} & =-1 \\
2+a & =-2 \\
a & =-4
\end{aligned}
$$

Another point on the parabola is $(-4,3)$.
26. The axis of symmetry is $x=-4$.

A point on the parabola is $(0,6)$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as $(0,6)$.
The $x$-coordinate is:
$\frac{0+a}{2}=-4$
$0+a=-8$

$$
a=-8
$$

Another point on the parabola is $(-8,6)$.
28. The vertex is $(5,2)$.

A point on the parabola is $(3,-1)$.
The axis of symmetry is $x=5$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as
$(3,-1)$. The $x$-coordinate is:
$\frac{3+a}{2}=5$
$3+a=10$
$a=7$
Another point on the parabola is $(7,-1)$.
30. The vertex is $(3,-1)$.

A point on the parabola is $(4,1)$.
The axis of symmetry is $x=3$.
Another point on the parabola is a symmetric point that has the same $y$-coordinate as $(4,1)$.
The $x$-coordinate is:
$\begin{aligned} \frac{4+a}{2} & =3 \\ 4+a & =6 \\ a & =2\end{aligned}$
Another point on the parabola is $(2,1)$.
$\qquad$

## The Form Is "Key" <br> Vertex Form of a Quadratic Function

## Vocabulary

Write a definition for the term in your own words.

1. vertex form

A quadratic function written in vertex form is in the form $f(x)=a(x-h)^{2}+k$, where $a \neq 0$.

## Problem Set

Determine the vertex of each quadratic function given in vertex form.

1. $f(x)=(x-3)^{2}+8$
The vertex is $(3,8)$.
2. $f(x)=(x+4)^{2}+2$
The vertex is $(-4,2)$.
3. $f(x)=-2(x-1)^{2}-8$
The vertex is $(1,-8)$.
4. $f(x)=\frac{1}{2}(x-2)^{2}+6$
The vertex is $(2,6)$.
5. $f(x)=-(x+9)^{2}-1$
The vertex is $(-9,-1)$.
6. $f(x)=(x-5)^{2}$
The vertex is $(5,0)$.

Determine the vertex of each quadratic function given in standard form. Use your graphing calculator.
Rewrite the function in vertex form.
7. $f(x)=x^{2}-6 x-27$

The vertex is $(3,-36)$.
The function in vertex form is
$f(x)=(x-3)^{2}-36$.
9. $f(x)=2 x^{2}-4 x-6$

The vertex is $(1,-8)$.
The function in vertex form is
$f(x)=2(x-1)^{2}-8$.
8. $f(x)=-x^{2}-2 x+15$

The vertex is $(-1,16)$.
The function in vertex form is
$f(x)=-(x+1)^{2}+16$.
10. $f(x)=x^{2}-10 x+24$

The vertex is $(5,-1)$.
The function in vertex form is
$f(x)=(x-5)^{2}-1$.
11. $f(x)=-x^{2}+15 x-54$

The vertex is $(7.5,2.25)$.
The function in vertex form is $f(x)=-(x-7.5)^{2}+2.25$.
12. $f(x)=-2 x^{2}-14 x-12$

The vertex is $(-3.5,12.5)$.
The function in vertex form is $f(x)=-2(x+3.5)^{2}-12.5$.

Determine the $x$-intercepts of each quadratic function given in standard form. Use your graphing calculator. Rewrite the function in factored form.
13. $f(x)=x^{2}+2 x-8$

The $x$-intercepts are $(2,0)$ and $(-4,0)$.
The function in factored form is
$f(x)=(x-2)(x+4)$.
15. $f(x)=-4 x^{2}+12 x-8$

The $x$-intercepts are $(1,0)$ and $(2,0)$.
The function in factored form is
$f(x)=-4(x-1)(x-2)$.
17. $f(x)=\frac{1}{2} x^{2}-\frac{1}{2} x-3$

The $x$-intercepts are $(-2,0)$ and $(3,0)$.
The function in factored form is
$f(x)=\frac{1}{2}(x+2)(x-3)$.
14. $f(x)=-x^{2}-x+12$

The $x$-intercepts are $(-4,0)$ and $(3,0)$.
The function in factored form is
$f(x)=-(x+4)(x-3)$.
16. $f(x)=2 x^{2}+18 x+16$

The $x$-intercepts are $(-8,0)$ and $(-1,0)$.
The function in factored form is
$f(x)=2(x+8)(x+1)$.
18. $f(x)=\frac{1}{3} x^{2}-2 x$

The $x$-intercepts are $(0,0)$ and $(6,0)$.
The function in factored form is $f(x)=\frac{1}{3} x(x-6)$.

Identify the form of each quadratic function as either standard form, factored form, or vertex form. Then state all you know about the quadratic function's key characteristics, based only on the given equation of the function.
19. $f(x)=5(x-3)^{2}+12$

The function is in vertex form.
The parabola opens up and the vertex is $(3,12)$.
21. $f(x)=-3 x^{2}+5 x$

The function is in standard form.
The parabola opens down and the $y$-intercept is $(0,0)$.
23. $f(x)=-(x+2)^{2}-7$

The function is in vertex form.
The parabola opens down and the vertex is $(-2,-7)$.
20. $f(x)=-(x-8)(x-4)$

The function is in factored form.
The parabola opens down and the $x$-intercepts are $(8,0)$ and $(4,0)$.
22. $f(x)=\frac{2}{3}(x+6)(x-1)$

The function is in factored form.
The parabola opens up and the $x$-intercepts are $(-6,0)$ and $(1,0)$.
24. $f(x)=2 x^{2}-1$

The function is in standard form.
The parabola opens up, the $y$-intercept is $(0,-1)$ and the vertex is $(0,-1)$.
$\qquad$

Write an equation for a quadratic function with each set of given characteristics.
25. The vertex is $(-1,4)$ and the parabola opens down.

Answers will vary but functions should be in the form:
$f(x)=a(x-h)^{2}+k$
$f(x)=a(x+1)^{2}+4$, for $a<0$
26. The $x$-intercepts are -3 and 4 and the parabola opens down.

Answers will vary but functions should be in the form:
$f(x)=a\left(x-r_{1}\right)\left(x-r_{2}\right)$
$f(x)=a(x+3)(x-4)$, for $a<0$
27. The vertex is $(3,-2)$ and the parabola opens up.

Answers will vary but functions should be in the form:
$f(x)=a(x-h)^{2}+k$
$f(x)=a(x-3)^{2}-2$, for $a>0$
28. The vertex is $(0,8)$ and the parabola opens up.

Answers will vary but functions should be in the form:

$$
\begin{aligned}
f(x) & =a(x-h)^{2}+k \\
f(x) & =a(x-0)^{2}+8 \\
& =a x^{2}+8, \text { for } a>0
\end{aligned}
$$

29. The $x$-intercepts are 5 and 12 and the parabola opens up.

Answers will vary but functions should be in the form:
$f(x)=a\left(x-r_{1}\right)\left(x-r_{2}\right)$
$f(x)=a(x-5)(x-12)$, for $a>0$
30. The $x$-intercepts are 0 and 7 and the parabola opens down.

Answers will vary but functions should be in the form:

$$
\begin{aligned}
f(x) & =a\left(x-r_{1}\right)\left(x-r_{2}\right) \\
f(x) & =a(x-0)(x-7) \\
& =a x(x-7), \text { for } a<0
\end{aligned}
$$

$\qquad$

## More Than Meets the Eye <br> Transformations of Quadratic Functions

## Vocabulary

Write a definition for each term in your own words.

1. vertical dilation

A vertical dilation of a function is a transformation in which the $y$-coordinate of every point on the graph of the function is multiplied by a common factor.
2. dilation factor

The dilation factor is the common factor by which each $y$-coordinate is multiplied when a function is transformed by a vertical dilation.

## Problem Set

Describe the transformation performed on each function $g(x)$ to result in $d(x)$.

1. $g(x)=x^{2}$
$d(x)=x^{2}-5$
2. $g(x)=x^{2}$
The graph of $g(x)$ is translated down 5 units.
$d(x)=x^{2}+2$
The graph of $g(x)$ is translated up 2 units.

## 3. $g(x)=3 x^{2}$ <br> $d(x)=3 x^{2}+6$

The graph of $g(x)$ is translated up 6 units.
5. $g(x)=(x+2)^{2}$
$d(x)=(x+2)^{2}-3$
The graph of $g(x)$ is translated down 3 units.
4. $g(x)=\frac{1}{2} x^{2}$
$d(x)=\frac{1}{2} x^{2}-1$
The graph of $g(x)$ is translated down 1 unit.

Describe the transformation performed on each function $g(x)$ to result in $m(x)$.
7. $g(x)=x^{2}$
$m(x)=(x+4)^{2}$
8. $g(x)=x^{2}$
$m(x)=(x-8)^{2}$

The graph of $g(x)$ is translated left 4 units.
The graph of $g(x)$ is translated right 8 units.
9. $g(x)=x^{2}$
$m(x)=(x+1)^{2}$
The graph of $g(x)$ is translated left 1 unit.
10. $g(x)=x^{2}-7$
$m(x)=(x+2)^{2}-7$

The graph of $g(x)$ is translated left 2 units.
11. $g(x)=x^{2}+8$
$m(x)=(x+3)^{2}+8$

The graph of $g(x)$ is translated left 3 units.
12. $g(x)=x^{2}-6$
$m(x)=(x-5)^{2}-6$
The graph of $g(x)$ is translated right 5 units.

Describe the transformation performed on each function $g(x)$ to result in $p(x)$.
13. $g(x)=x^{2}$
$p(x)=-x^{2}$
The graph of $p(x)$ is a horizontal reflection of the graph of $g(x)$.
14. $g(x)=x^{2}$
$p(x)=(-x)^{2}$

The graph of $p(x)$ is a vertical reflection of the graph of $g(x)$.
15. $g(x)=x^{2}+2$
$p(x)=-\left(x^{2}+2\right)$

The graph of $p(x)$ is a horizontal reflection of the graph of $g(x)$.
16. $g(x)=x^{2}-5$
$p(x)=(-x)^{2}-5$

The graph of $p(x)$ is a vertical reflection of the graph of $g(x)$.
17. $g(x)=\frac{2}{3} x^{2}+4$
$p(x)=\frac{2}{3}(-x)^{2}+4$
The graph of $p(x)$ is a vertical reflection of the graph of $g(x)$.
18. $g(x)=5 x^{2}-7$
$p(x)=-\left(5 x^{2}-7\right)$
The graph of $p(x)$ is a horizontal reflection of the graph of $g(x)$.

Represent each function $n(x)$ as a vertical dilation of $g(x)$ using coordinate notation.
19. $g(x)=x^{2}$
$n(x)=4 x^{2}$
$(x, y) \rightarrow(x, 4 y)$
20. $g(x)=x^{2}$
$n(x)=\frac{1}{2} x^{2}$
$(x, y) \rightarrow\left|x, \frac{1}{2} y\right|$

Name Date
21. $g(x)=-x^{2}$
$n(x)=-5 x^{2}$
$(x, y) \rightarrow(x, 5 y)$
22. $g(x)=-x^{2}$

$$
\begin{aligned}
& n(x)=-\frac{3}{4} x^{2} \\
& (x, y) \rightarrow\left|x, \frac{3}{4} y\right|
\end{aligned}
$$

24. $g(x)=(x-3)^{2}$
$n(x)=\frac{1}{2}(x-3)^{2}$
$(x, y) \rightarrow\left|x, \frac{1}{2} y\right|$

Write an equation in vertex form for a function $g(x)$ with the given characteristics. Sketch a graph of each function $g(x)$.
25. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x)=x^{2}$.
The function $g(x)$ is translated 3 units up from $f(x)=-x^{2}$.
$g(x)=-(x-0)^{2}+3$

26. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x)=x^{2}$.
The function $g(x)$ is translated 2 units down and 5 units left from $f(x)=-x^{2}$.

$$
g(x)=-(x+5)^{2}-2
$$


27. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of 6 .
The function $g(x)$ is translated 1 unit up and 4 units right from $f(x)=6 x^{2}$.
$g(x)=6(x-4)^{2}+1$


Name Date
28. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of $\frac{1}{2}$.
The function $g(x)$ is translated 2 units down and 6 units left from $f(x)=\frac{1}{2} x^{2}$.
$g(x)=\frac{1}{2}(x+6)^{2}-2$

29. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x)=x^{2}$.
The function $g(x)$ is vertically dilated with a dilation factor of 3 .
The function $g(x)$ is translated 2 units down and 4 units right from $f(x)=-3 x^{2}$.
$g(x)=-3(x-4)^{2}-2$


## Lesson 12.7 Skills Practice

30. The function $g(x)$ is quadratic.

The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of $\frac{1}{4}$.
The function $g(x)$ is translated 3 units up and 2 units left from $f(x)=\frac{1}{4} x^{2}$.
$g(x)=\frac{1}{4}(x+2)^{2}+3$


Describe the transformation(s) necessary to translate the graph of the function $f(x)=x^{2}$ into the graph of each function $g(x)$.
31. $g(x)=x^{2}+7$

The function $g(x)$ is translated 7 units up from $f(x)=x^{2}$.
32. $g(x)=-x^{2}-4$

The graph of $g(x)$ is a horizontal reflection of the graph of $f(x)=x^{2}$ about the line $y=0$ that is translated 4 units down.
33. $g(x)=(x-2)^{2}+8$

The function $g(x)$ is translated 8 units up and 2 units right from $f(x)=x^{2}$.
34. $g(x)=4 x^{2}+1$

The function $g(x)$ is vertically dilated with a dilation factor of 4 and then translated 1 unit up from $f(x)=x^{2}$.
35. $g(x)=\frac{2}{3}(x+4)^{2}-9$

The function $g(x)$ is vertically dilated with a dilation factor of $\frac{2}{3}$ and then translated 9 units down and 4 units left from $f(x)=x^{2}$.
36. $g(x)=-(x-6)^{2}+3$

The graph of $g(x)$ is a horizontal reflection of the graph of $f(x)=x^{2}$ about the line $y=0$ that is translated 3 units up and 6 units right.

