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Controlling the Population

Adding and Subtracting Polynomials

Vocabulary

Match each definition with its corresponding term.

1. polynomial
   c.
2. term
   f.
3. coefficient
   e.
4. monomial
   a.
5. binomial
   g.
6. trinomial
   d.
7. degree of a term
   h.
8. degree of a polynomial
   b.

a. a polynomial with only 1 term

b. the degree of the term with the greatest exponent

c. a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients

d. a polynomial with exactly 3 terms

e. any number being multiplied by a power within a polynomial expression

f. each product in a polynomial expression

g. a polynomial with exactly 2 terms

h. the exponent of a term in a polynomial

Problem Set

Identify the terms and coefficients in each expression.

1. $5x + 8$

   The terms are $5x$ and 8. The coefficients are 5 and 8.

2. $2m^3$

   The term is $2m^3$. The coefficient is 2.

3. $x^2 - 4x$

   The terms are x^2 and $-4x$. The coefficients are 1 and -4.

4. $-3w^4 + w^2 - 9$

   The terms are $-3w^4$, w^2, and -9. The coefficients are -3, 1, and -9.

5. -18

   The term is -18. The coefficient is -18.

6. $10 - 3x^3 - 6x$

   The terms are 10, $-3x^3$, and $6x$. The coefficients are 10, -3, and 6.

Determine whether each expression is a polynomial. If the expression is not a polynomial, explain why it is not.

7. $9 + 12x$

   The expression is a polynomial.

8. $6m^{\frac{1}{2}}$

   The expression is not a polynomial, because the term has an exponent that is not a whole number.

9. $\frac{3}{x} - 8x$

   The expression is not a polynomial. The first term can be rewritten as $3x^{-1}$ which has an exponent that is not a whole number.

10. $-2w^3 + w^2 - 5$

    The expression is a polynomial.

11. $-2.5m$

    The expression is a polynomial.

12. $\frac{x}{7} + 10$

    The expression is a polynomial.

13. $\sqrt[3]{x} + 12$

    The expression is not a polynomial. The first term can be rewritten as $x^{1/3}$ which has an exponent that is not a whole number.

14. $\frac{4}{5}m - \frac{1}{5}$

    The expression is a polynomial.

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Determine whether each polynomial is a monomial, binomial, or trinomial. State the degree of the polynomial.

15. $8x + 3$

The polynomial is a binomial with a degree of 1.

16. $5m^2$

The polynomial is a monomial with a degree of 2.

17. $x^2 - 7x$

The polynomial is a binomial with a degree of 2.

18. $-9n^4 + 6n^2 - 1$

The polynomial is a trinomial with a degree of 4.

19. -12

The polynomial is a monomial with a degree of 0.

20. $4 - 10x^3 + 8x$

The polynomial is a trinomial with a degree of 3.

Write each polynomial in standard form. Classify the polynomial by its number of terms and by its degree.

21. $2x + 6x^2$

$6x^2 + 2x$

The polynomial is a binomial with a degree of 2.

22. $-9m^2 + 4m^3$

$4m^3 - 9m^2$

The polynomial is a binomial with a degree of 3.

23. $10 - 5x$

$-5x + 10$

The polynomial is a binomial with a degree of 1.

24. $7x - 3 + 12x^2$

$12x^2 + 7x - 3$

The polynomial is a trinomial with a degree of 2.

25. $15 + 4w - w^3$

$-w^3 + 4w + 15$

The polynomial is a trinomial with a degree of 3.

26. $5x^2 - 15 + 20x$

$5x^2 + 20x - 15$

The polynomial is a trinomial with a degree of 2.

27. $-1 - p^4$

$-p^4 - 1$

The polynomial is a binomial with a degree of 4.

28. $-6t^2 + 4t + 3t^3$

$3t^3 - 6t^2 + 4t$

The polynomial is a trinomial with a degree of 3.

29. $-18a^3 + 54a - 22a^2$

$-18a^3 - 22a^2 + 54a$

The polynomial is a trinomial with a degree of 3.

30. $x^3 - x^2 - x^5$

$-x^5 + x^3 - x^2$

The polynomial is a trinomial with a degree of 5.

Simplify each expression.

31. $(5x - 8) + (7x + 10)$

$5x - 8 + 7x + 10$

$(5x + 7x) + (-8 + 10)$

$12x + 2$

32. $(4m^2 + 9m) - (2m^2 + 6)$

$4m^2 + 9m - 2m^2 - 6$

$(4m^2 - 2m^2) + 9m - 6$

$2m^2 + 9m - 6$

33. $(-x^2 + 5x - 12) + (2x^2 - 6)$

$-x^2 + 5x - 12 + 2x^2 - 6$

$(-x^2 + 2x^2) + 5x + (-12 - 6)$

$x^2 + 5x - 18$

34. $(10t^2 - 3t + 9) - (6t^2 - 7t)$

$10t^2 - 3t + 9 - 6t^2 + 7t$

$(10t^2 - 6t^2) + (-3t + 7t) + 9$

$4t^2 + 4t + 9$

35. $(-5w^2 + 3w - 8) + (15w^2 - 4w + 11)$

$-5w^2 + 3w - 8 + 15w^2 - 4w + 11$

$(-5w^2 + 15w^2) + (3w - 4w) + (-8 + 11)$

$10w^2 - w + 3$

36. $(3x^3 + 10x - 1) - (5x^2 + 10x - 9)$

$3x^3 + 10x - 1 - 5x^2 - 10x + 9$

$3x^3 - 5x^2 + (10x - 10x) + (-1 + 9)$

$3x^3 - 5x^2 + 8$

37. $(-a^2 + 2a - 8) + (2a^2 - 9a + 15)$

$-a^2 + 2a - 8 + 2a^2 - 9a + 15$

$(-a^2 + 2a^2) + (2a - 9a) + (-8 + 15)$

$a^2 - 7a + 7$

38. $(14p^4 + 7p^2) + (8p^3 + 7p^2 - p)$

$14p^4 - 7p^2 + 8p^3 + 7p^2 - p$

$14p^4 + 8p^3 + (-7p^2 + 7p^2) - p$

$14p^4 + 8p^3 - p$

39. $(3x^4 + 3x^2 - 3) - (6x^5 - 9x^3 + 2)$

$3x^4 + 3x^2 - 3 - 6x^5 + 9x^3 - 2$

$-6x^5 + 3x^4 + 9x^3 + 3x^2 + (-3 - 2)$

$-6x^5 + 3x^4 + 9x^3 + 3x^2 - 5$

40. $(-7m^3 - m^2 - m) - (-10m^3 - m - 1)$

$-7m^3 - m^2 - m + 10m^3 + m + 1$

$(-7m^3 + 10m^3) - m^2 + (-m + m) + 1$

$3m^3 - m^2 + 1$

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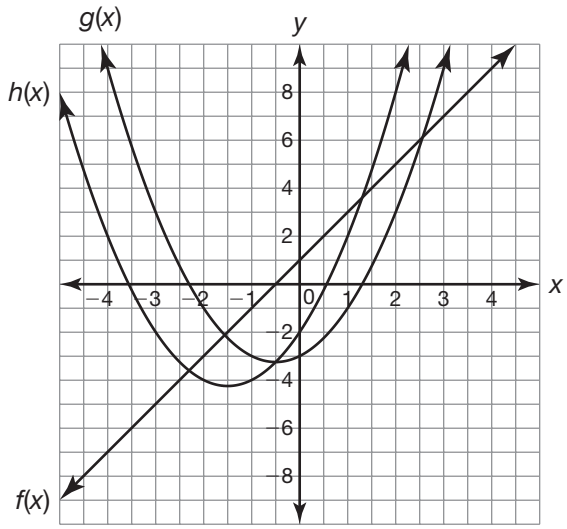
The graphs of the functions $f(x) = 2x + 1$, $g(x) = x^2 + x - 3$, and $h(x) = f(x) + g(x)$ are shown. Evaluate the function $h(x)$ for each given value of x. Use the graph of $h(x)$ to verify your answer.



41. Evaluate $h(x)$ at $x = 2$.

$h(x) = f(x) + g(x)$
$= 2x + 1 + x^2 + x - 3$
$= x^2 + 3x - 2$
$h(2) = (2)^2 + 3(2) - 2$
$= 4 + 6 - 2$
$= 8$

42. Evaluate $h(x)$ at $x = -4$.

$h(x) = f(x) + g(x)$
$= 2x + 1 + x^2 + x - 3$
$= x^2 + 3x - 2$
$h(-4) = (-4)^2 + 3(-4) - 2$
$= 16 - 12 - 2$
$= 2$

43. Evaluate $h(x)$ at $x = 0$.

$h(x) = f(x) + g(x)$
$= 2x + 1 + x^2 + x - 3$
$= x^2 + 3x - 2$
$h(0) = (0)^2 + 3(0) - 2$
$= 0 + 0 - 2$
$= -2$

44. Evaluate $h(x)$ at $x = 1$.

$h(x) = f(x) + g(x)$
$= 2x + 1 + x^2 + x - 3$
$= x^2 + 3x - 2$
$h(1) = (1)^2 + 3(1) - 2$
$= 1 + 3 - 2$
$= 2$

45. Evaluate $h(x)$ at $x = -2$.

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= 2x + 1 + x^2 + x - 3 \\ &= x^2 + 3x - 2 \\ h(-2) &= (-2)^2 + 3(-2) - 2 \\ &= 4 - 6 - 2 \\ &= -4 \end{aligned}$$

46. Evaluate $h(x)$ at $x = -1.5$.

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= 2x + 1 + x^2 + x - 3 \\ &= x^2 + 3x - 2 \\ h(-1.5) &= (-1.5)^2 + 3(-1.5) - 2 \\ &= 2.25 - 4.5 - 2 \\ &= -4.25 \end{aligned}$$

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They're Multiplying—Like Polynomials!

Multiplying Polynomials

Problem Set

Determine the product of the binomials using algebra tiles.

1. $x + 1$ and $x + 1$

•	x	1
x	x^2	x
1	x	1

$(x + 1)(x + 1) = x^2 + 2x + 1$

2. $x + 1$ and $x + 4$

•	x	1	1	1	1
x	x^2	x	x	x	x
1	x	1	1	1	1

$(x + 1)(x + 4) = x^2 + 5x + 4$

3. $x + 2$ and $x + 2$

•	x	1	1
x	x^2	x	x
1	x	1	1
1	x	1	1

$(x + 2)(x + 2) = x^2 + 4x + 4$

4. $x + 3$ and $x + 3$

•	x	1	1	1
x	x^2	x	x	x
1	x	1	1	1
1	x	1	1	1
1	x	1	1	1

$(x + 3)(x + 3) = x^2 + 6x + 9$

5. $2x + 1$ and $x + 3$

•	x	1	1	1
x	x^2	x	x	x
x	x^2	x	x	x
1	x	1	1	1

$(2x + 1)(x + 3) = 2x^2 + 7x + 3$

6. $2x + 3$ and $x + 2$

•	x	1	1
x	x^2	x	x
x	x^2	x	x
1	x	1	1
1	x	1	1
1	x	1	1

$(2x + 3)(x + 2) = 2x^2 + 7x + 6$

Determine the product of the binomials using multiplication tables.

7. $3x + 4$ and $2x + 2$

•	$2x$	2
$3x$	$6x^2$	$6x$
4	$8x$	8

$(3x + 4)(2x + 2) = 6x^2 + 6x + 8x + 8$
$= 6x^2 + 14x + 8$

8. $5m + 3$ and $4m + 6$

•	$4m$	6
$5m$	$20m^2$	$30m$
3	$12m$	18

$(5m + 3)(4m + 6) = 20m^2 + 30m + 12m + 18$
$= 20m^2 + 42m + 18$

9. $6t + 5$ and $7t - 5$

•	$7t$	-5
$6t$	$42t^2$	$-30t$
5	$35t$	-25

$(6t + 5)(7t - 5) = 42t^2 - 30t + 35t - 25$
$= 42t^2 + 5t - 25$

10. $4x + 2$ and $4x - 2$

•	$4x$	-2
$4x$	$16x^2$	$-8x$
2	$8x$	-4

$(4x + 2)(4x - 2) = 16x^2 - 8x + 8x - 4$
$= 16x^2 - 4$

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11. $10w - 1$ and $9w + 8$

·	$9w$	8
$10w$	$90w^2$	$80w$
-1	$-9w$	-8

$(10w - 1)(9w + 8) = 90w^2 + 80w - 9w - 8$
$= 90w^2 + 71w - 8$

12. $y + 12$ and $5y + 15$

·	$5y$	15
y	$5y^2$	$15y$
12	$60y$	180

$(y + 12)(5y + 15) = 5y^2 + 15y + 60y + 180$
$= 5y^2 + 75y + 180$

Determine the product of the polynomials using the Distributive Property.

13. $2x(x + 6)$

$2x(x + 6) = 2x(x) + 2x(6)$
$= 2x^2 + 12x$

14. $4x^2(x + 2)$

$4x^2(x + 2) = 4x^2(x) + 4x^2(2)$
$= 4x^3 + 8x^2$

15. $7x(x - 5)$

$7x(x - 5) = 7x(x) - 7x(5)$
$= 7x^2 - 35x$

16. $(2x + 1)(x + 8)$

$(2x + 1)(x + 8) = (2x + 1)(x) + (2x + 1)(8)$
$= 2x(x) + 1(x) + 2x(8) + 1(8)$
$= 2x^2 + x + 16x + 8$
$= 2x^2 + 17x + 8$

17. $(x + 3)(x^2 - 1)$

$(x + 3)(x^2 - 1) = (x + 3)(x^2) - (x + 3)(1)$
$= x(x^2) + 3(x^2) - x(1) - 3(1)$
$= x^3 + 3x^2 - x - 3$

18. $(4x + 4)(5x - 5)$

$(4x + 4)(5x - 5) = (4x + 4)(5x) - (4x + 4)(5)$
$= 4x(5x) + 4(5x) - 4x(5) - 4(5)$
$= 20x^2 + 20x - 20x - 20$
$= 20x^2 - 20$

19. $3x(x^2 + 5x - 1)$

$3x(x^2 + 5x - 1) = 3x(x^2) + 3x(5x) - 3x(1)$
$= 3x^3 + 15x^2 - 3x$

20. $9x(3x^2 - 4x + 2)$

$9x(3x^2 - 4x + 2) = 9x(3x^2) - 9x(4x) + 9x(2)$
$= 27x^3 - 36x^2 + 18x$

21. $(x + 2)(x^2 + 6x - 1)$

$$\begin{aligned}(x + 2)(x^2 + 6x - 1) &= (x + 2)(x^2) + (x + 2)(6x) - (x + 2)(1)\\ &= x(x^2) + 2(x^2) + x(6x) + 2(6x) - x(1) - 2(1)\\ &= x^3 + 2x^2 + 6x^2 + 12x - x - 2\\ &= x^3 + 8x^2 + 11x - 2\end{aligned}$$

22. $(x - 4)(x^2 + 2x - 3)$

$$\begin{aligned}(x - 4)(x^2 + 2x - 3) &= (x - 4)(x^2) + (x - 4)(2x) - (x - 4)(3)\\ &= x(x^2) - 4(x^2) + x(2x) - 4(2x) - x(3) - (-4)(3)\\ &= x^3 - 4x^2 + 2x^2 - 8x - 3x - (-12)\\ &= x^3 - 2x^2 - 11x + 12\end{aligned}$$

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What *Factored* Into It?

Factoring Polynomials

Vocabulary

State the given property.

1. Symmetric Property of Equality

   The Symmetric Property of Equality states that if $m = n$, then $n = m$.

Problem Set

Factor out the greatest common factor of each polynomial, if possible.

1. $x^2 + 9x$

   $x(x + 9)$

2. $m^2 - 4m$

   $m(m - 4)$

3. $5x^2 + 20x - 15$

   $5(x^2 + 4x - 3)$

4. $24w^2 - 16$

   $8(3w^2 - 2)$

5. $y^3 - 7y$

   $y(y^2 - 7)$

6. $2x^3 + 10x^2$

   $2x^2(x + 5)$

7. $3w + 10$

   There is no greatest common factor.

8. $20x^3 + 16x^2 + 8x$

   $4x(5x^2 + 4x + 2)$

9. $7m^3 - 21$

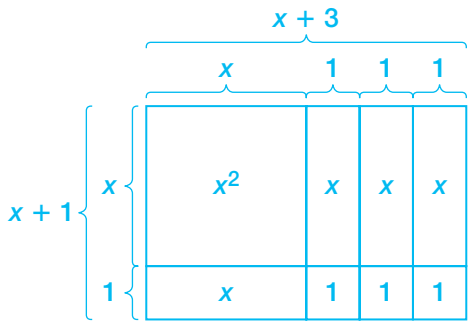
   $7(m^3 - 3)$

10. $15x^3 + 4$

    There is no greatest common factor.

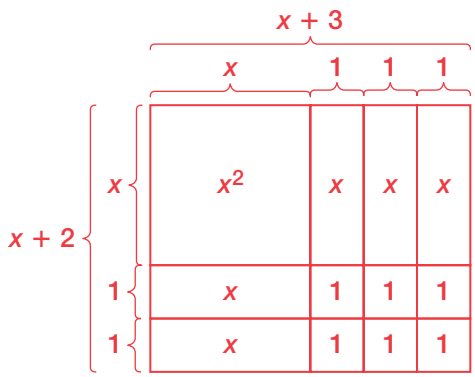
Factor each trinomial using an area model.

11. $x^2 + 4x + 3$



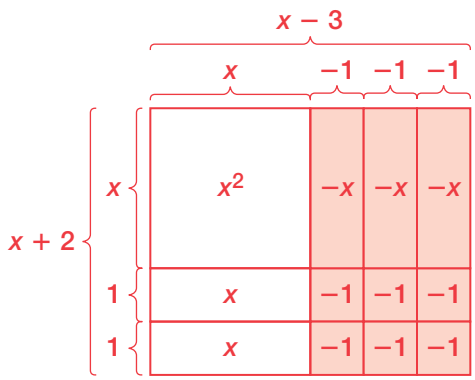
$x^2 + 4x + 3 = (x + 1)(x + 3)$

12. $x^2 + 5x + 6$



$x^2 + 5x + 6 = (x + 2)(x + 3)$

13. $x^2 - x - 6$



$x^2 - x - 6 = (x + 2)(x - 3)$

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14. $x^2 - x - 12$

		$x - 4$			
		x	-1	-1	-1	-1
$x + 3$	x	x^2	$-x$	$-x$	$-x$	$-x$
	1	x	-1	-1	-1	-1
	1	x	-1	-1	-1	-1
	1	x	-1	-1	-1	-1

$x^2 - x - 12 = (x + 3)(x - 4)$

15. $x^2 + 7x + 10$

		$x + 5$					
		x	1	1	1	1	1
$x + 2$	x	x^2	x	x	x	x	x
	1	x	1	1	1	1	1
	1	x	1	1	1	1	1

$x^2 + 7x + 10 = (x + 2)(x + 5)$

16. $x^2 + 3x - 4$

		$x + 4$				
		x	1	1	1	1
$x - 1$	x	x^2	x	x	x	x
	-1	$-x$	-1	-1	-1	-1

$x^2 + 3x - 4 = (x - 1)(x + 4)$

Factor each trinomial completely using multiplication tables. If possible, factor out the greatest common factor first.

17. $x^2 - 2x - 8$

·	x	2
x	x^2	$2x$
-4	$-4x$	-8

$x^2 - 2x - 8 = (x - 4)(x + 2)$

18. $y^2 + 13y + 42$

·	y	7
y	y^2	$7y$
6	$6y$	42

$y^2 + 13y + 42 = (y + 6)(y + 7)$

19. $m^2 + 6m - 7$

·	m	-1
m	m^2	$-m$
7	$7m$	-7

$m^2 + 6m - 7 = (m + 7)(m - 1)$

20. $x^2 - 9x + 18$

·	x	-6
x	x^2	$-6x$
-3	$-3x$	18

$x^2 - 9x + 18 = (x - 3)(x - 6)$

21. $4w^2 + 12w - 40$

$4w^2 + 12w - 40 = 4(w^2 + 3w - 10)$

Factor $w^2 + 3w - 10$ using a multiplication table.

·	w	-2
w	w^2	$-2w$
5	$5w$	-10

$4w^2 + 12w - 40 = 4(w + 5)(w - 2)$

22. $2t^3 - 14t^2 + 24t$

$2t^3 - 14t^2 + 24t = 2t(t^2 - 7t + 12)$

Factor $t^2 - 7t + 12$ using a multiplication table.

·	t	-4
t	t^2	$-4t$
-3	$-3t$	12

$2t^3 - 14t^2 + 24t = 2t(t - 3)(t - 4)$

23. $3m^3 + 36m^2 + 60m$

$3m^3 + 36m^2 + 60m = 3m(m^2 + 12m + 20)$

Factor $m^2 + 12m + 20$ using a multiplication table.

·	m	2
m	m^2	$2m$
10	$10m$	20

$3m^3 + 36m^2 + 60m = 3m(m + 10)(m + 2)$

24. $2x^2 - 8x - 42$

$2x^2 - 8x - 42 = 2(x^2 - 4x - 21)$

Factor $x^2 - 4x - 21$ using a multiplication table.

·	x	-7
x	x^2	$-7x$
3	$3x$	-21

$2x^2 - 8x - 42 = 2(x + 3)(x - 7)$

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Factor each polynomial using the trial and error method. If possible, factor out the greatest common factor first.

25. $x^2 + 11x + 10$

The factors of the constant term, 10, are:

$-1, -10$ $\quad$ $1, 10$

$-2, -5$ $\quad$ $2, 5$

$x^2 + 11x + 10 = (x + 1)(x + 10)$

26. $w^2 + 6w - 16$

The factors of the constant term, -16, are:

$-1, 16$ $\quad$ $1, -16$

$-2, 8$ $\quad$ $2, -8$

$-4, 4$ $\quad$ $4, -4$

$w^2 + 6w - 16 = (w + 8)(w - 2)$

27. $m^2 + 2m - 35$

The factors of the constant term, -35, are:

$-1, 35$ $\quad$ $1, -35$

$-5, 7$ $\quad$ $5, -7$

$m^2 + 2m - 35 = (m + 7)(m - 5)$

28. $x^2 + 4x - 12$

The factors of the constant term, -12, are:

$-1, 12$ $\quad$ $1, -12$

$-2, 6$ $\quad$ $2, -6$

$-3, 4$ $\quad$ $3, -4$

$x^2 + 4x - 12 = (x + 6)(x - 2)$

29. $3n^2 - 27n + 60$

$3n^2 - 27n + 60 = 3(n^2 - 9n + 20)$

Factor $n^2 - 9n + 20$ using trial and error.

The factors of the constant term, 20, are:

$-1, -20$ $\quad$ $1, 20$

$-2, -10$ $\quad$ $2, 10$

$-4, -5$ $\quad$ $4, 5$

$3n^2 - 27n + 60 = 3(n - 5)(n - 4)$

30. $2x^2 + 22x + 60$

$2x^2 + 22x + 60 = 2(x^2 + 11x + 30)$

Factor $x^2 + 11x + 30$ using trial and error.

The factors of the constant term, 30, are:

$-1, -30$ $\quad$ $1, 30$

$-2, -15$ $\quad$ $2, 15$

$-3, -10$ $\quad$ $3, 10$

$-5, -6$ $\quad$ $5, 6$

$2x^2 + 22x + 60 = 2(x + 5)(x + 6)$

Factor each polynomial.

31. $x^2 + 11x + 28 =$ $(x + 4)(x + 7)$

$x^2 - 11x + 28 =$ $(x - 4)(x - 7)$

$x^2 + 3x - 28 =$ $(x - 4)(x + 7)$

$x^2 - 3x + 28 =$ $(x + 4)(x - 7)$

32. $x^2 + 10x + 9 =$ $(x + 1)(x + 9)$

$x^2 - 10x + 9 =$ $(x - 1)(x - 9)$

$x^2 + 8x - 9 =$ $(x - 1)(x + 9)$

$x^2 - 8x - 9 =$ $(x + 1)(x - 9)$

33. $x^2 + 12x + 27 =$ $(x + 3)(x + 9)$

$x^2 - 12x + 27 =$ $(x - 3)(x - 9)$

$x^2 + 6x - 27 =$ $(x - 3)(x + 9)$

$x^2 - 6x - 27 =$ $(x + 3)(x - 9)$

34. $x^2 + 13x + 40 =$ $(x + 5)(x + 8)$

$x^2 - 13x + 40 =$ $(x - 5)(x - 8)$

$x^2 + 3x - 40 =$ $(x - 5)(x + 8)$

$x^2 - 3x - 40 =$ $(x + 5)(x - 8)$

35. $x^2 + 12x + 11 =$ $(x + 1)(x + 11)$

$x^2 - 12x + 11 =$ $(x - 1)(x - 11)$

$x^2 + 10x - 11 =$ $(x - 1)(x + 11)$

$x^2 - 10x - 11 =$ $(x + 1)(x - 11)$

36. $x^2 + 13x + 36 =$ $(x + 4)(x + 9)$

$x^2 - 13x + 36 =$ $(x - 4)(x - 9)$

$x^2 + 5x - 36 =$ $(x - 4)(x + 9)$

$x^2 - 5x - 36 =$ $(x + 4)(x - 9)$

Factor each polynomial completely. If possible, factor out the greatest common factor first.

37. $x^2 + 4x + 4$

·	x	2
x	x^2	$2x$
2	$2x$	4

$x^2 + 4x + 4 = (x + 2)(x + 2)$

38. $x^2 - 10x + 25$

·	x	-5
x	x^2	$-5x$
-5	$-5x$	25

$x^2 - 10x + 25 = (x - 5)(x - 5)$

39. $-32 - 12m - m^2$

$-32 - 12m - m^2 = -(m^2 + 12m + 32)$

Factor $m^2 + 12m + 32$ using a multiplication table.

·	m	8
m	m^2	$8m$
4	$4m$	32

$-32 - 12m - m^2 = -(m + 4)(m + 8)$

40. $45 + 4w - w^2$

$45 + 4w - w^2 = -(w^2 - 4w - 45)$

Factor $w^2 - 4w - 45$ using a multiplication table.

·	w	-9
w	w^2	$-9w$
5	$5w$	-45

$45 + 4w - w^2 = -(w + 5)(w - 9)$

41. $5x^2 + 10x - 15$

$5x^2 + 10x - 15 = 5(x^2 + 2x - 3)$

Factor $x^2 + 2x - 3$ using a multiplication table.

·	x	-1
x	x^2	$-x$
3	$3x$	-3

$5x^2 + 10x - 15 = 5(x + 3)(x - 1)$

42. $4x^2 + 32x + 64$

$4x^2 + 32x + 64 = 4(x^2 + 8x + 16)$

Factor $x^2 + 8x + 16$ using a multiplication table.

·	x	4
x	x^2	$4x$
4	$4x$	16

$4x^2 + 32x + 64 = 4(x + 4)(x + 4)$

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Zeroing In
Solving Quadratics by Factoring

Vocabulary

Complete the definition of the Zero Product Property.

1. The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero.

   If $ab = 0$, then $a = 0$ or $b = 0$.
   This property is also known as the Converse of the Multiplication Property of Zero.

Define the term in your own words.

2. roots
   The solutions to a quadratic equation are called the roots. The roots indicate where the graph of a quadratic equation crosses the x-axis.

Problem Set

Factor and solve each quadratic equation. Check your answer.

1. $x^2 + 5x + 6 = 0$

$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x + 3 = 0$ or $x + 2 = 0$
$x = -3$ or $x = -2$
The roots are -3 and -2.

Check:
$(-3)^2 + 5(-3) + 6 = 0$
$9 - 15 + 6 = 0$
$0 = 0$

$(-2)^2 + 5(-2) + 6 = 0$
$4 - 10 + 6 = 0$
$0 = 0$

2. $x^2 - 3x - 4 = 0$

$x^2 - 3x - 4 = 0$
$(x - 4)(x + 1) = 0$
$x - 4 = 0$ or $x + 1 = 0$
$x = 4$ or $x = -1$
The roots are 4 and -1.

Check:
$(4)^2 - 3(4) - 4 = 0$
$16 - 12 - 4 = 0$
$0 = 0$

$(-1)^2 - 3(-1) - 4 = 0$
$1 + 3 - 4 = 0$
$0 = 0$

3. $m^2 + 2m - 35 = 0$

$m^2 + 2m - 35 = 0$

$(m + 7)(m - 5) = 0$

$m + 7 = 0$ or $m - 5 = 0$

$m = -7$ or $m = 5$

The roots are −7 and 5.

Check:

$(-7)^2 + 2(-7) - 35 = 0$

$49 - 14 - 35 = 0$

$0 = 0$

$(5)^2 + 2(5) - 35 = 0$

$25 + 10 - 35 = 0$

$0 = 0$

4. $-x^2 - 4x + 12 = 0$

$-x^2 - 4x + 12 = 0$

$-(x^2 + 4x - 12) = 0$

$-(x - 2)(x + 6) = 0$

$x - 2 = 0$ or $x + 6 = 0$

$x = 2$ or $x = -6$

The roots are 2 and −6.

Check:

$-(2)^2 - 4(2) + 12 = 0$

$-4 - 8 + 12 = 0$

$0 = 0$

$-(-6)^2 - 4(-6) + 12 = 0$

$-36 + 24 + 12 = 0$

$0 = 0$

5. $x^2 + 8x = 0$

$x^2 + 8x = 0$

$x(x + 8) = 0$

$x = 0$ or $x + 8 = 0$

$x = 0$ or $x = -8$

The roots are 0 and −8.

Check:

$(0)^2 + 8(0) = 0$

$0 + 0 = 0$

$0 = 0$

$(-8)^2 + 8(-8) = 0$

$64 - 64 = 0$

$0 = 0$

6. $w^2 + 50 = -15w$

$w^2 + 50 = -15w$

$w^2 + 50 + 15w = -15w + 15w$

$w^2 + 15w + 50 = 0$

$(w + 10)(w + 5) = 0$

$w + 10 = 0$ or $w + 5 = 0$

$w = -10$ or $w = -5$

The roots are −10 and −5.

Check:

$(-10)^2 + 50 = -15(-10)$

$100 + 50 = 150$

$150 = 150$

$(-5)^2 + 50 = -15(-5)$

$25 + 50 = 75$

$75 = 75$

7. $-t^2 + 12t = 32$

$-t^2 + 12t = 32$

$-t^2 + 12t - 32 = 32 - 32$

$-t^2 + 12t - 32 = 0$

$-(t^2 - 12t + 32) = 0$

$-(t - 4)(t - 8) = 0$

$t - 4 = 0$ or $t - 8 = 0$

$t = 4$ or $t = 8$

The roots are 4 and 8.

Check:

$-(4)^2 + 12(4) = 32$

$-16 + 48 = 32$

$32 = 32$

$-(8)^2 + 12(8) = 32$

$-64 + 96 = 32$

$32 = 32$

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8. $x^2 + 2x + 2 = 0$

The equation has no real roots.

9. $2t^2 + t - 3 = 0$

$2t^2 + t - 3 = 0$

$(2t + 3)(t - 1) = 0$

$2t + 3 = 0$ or $t - 1 = 0$

$t = -\frac{3}{2}$ or $t = 1$

The roots are $-\frac{3}{2}$ and 1.

Check:

$2\left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right) - 3 = 0$

$2\left(\frac{9}{4}\right) - \frac{3}{2} - 3 = 0$

$\frac{9}{2} - \frac{3}{2} - 3 = 0$

$\frac{6}{2} - 3 = 0$

$3 - 3 = 0$

$0 = 0$

$2(1)^2 + (1) - 3 = 0$

$2(1) + 1 - 3 = 0$

$2 + 1 - 3 = 0$

$0 = 0$

10. $w^2 + 5w - 32 = 2w - 4$

$w^2 + 5w - 32 = 2w - 4$

$w^2 + 5w - 32 - 2w + 4 = 2w - 4 - 2w + 4$

$w^2 + 3w - 28 = 0$

$(w + 7)(w - 4) = 0$

$w + 7 = 0$ or $w - 4 = 0$

$w = -7$ or $w = 4$

The roots are -7 and 4.

Check:

$(-7)^2 + 5(-7) - 32 = 2(-7) - 4$

$49 - 35 - 32 = -14 - 4$

$-18 = -18$

$(4)^2 + 5(4) - 32 = 2(4) - 4$

$16 + 20 - 32 = 8 - 4$

$4 = 4$

Determine the zeros of each quadratic function, if possible. Check your answer.

11. $f(x) = x^2 - 5x$

$f(x) = x^2 - 5x$

$0 = x^2 - 5x$

$0 = x(x - 5)$

$x = 0$ or $x - 5 = 0$

$x = 0$ or $x = 5$

The zeros are 0 and 5.

Check:

$(0)^2 - 5(0) \stackrel{?}{=} 0$

$0 - 0 \stackrel{?}{=} 0$

$0 = 0$

$(5)^2 - 5(5) \stackrel{?}{=} 0$

$25 - 25 \stackrel{?}{=} 0$

$0 = 0$

12. $f(x) = 3x^2 + 6x$

$f(x) = 3x^2 + 6x$

$0 = 3x^2 + 6x$

$0 = 3x(x + 2)$

$3x = 0$ or $x + 2 = 0$

$x = 0$ or $x = -2$

The zeros are 0 and −2.

Check:

$3(0)^2 + 6(0) \stackrel{?}{=} 0$

$3(0) + 0 \stackrel{?}{=} 0$

$0 + 0 \stackrel{?}{=} 0$

$0 = 0$

$3(-2)^2 + 6(-2) \stackrel{?}{=} 0$

$3(4) - 12 \stackrel{?}{=} 0$

$12 - 12 \stackrel{?}{=} 0$

$0 = 0$

13. $f(x) = x^2 + 11x + 30$

$f(x) = x^2 + 11x + 30$

$0 = x^2 + 11x + 30$

$0 = (x + 6)(x + 5)$

$x + 6 = 0$ or $x + 5 = 0$

$x = -6$ or $x = -5$

The zeros are −6 and −5.

Check:

$(-6)^2 + 11(-6) + 30 \stackrel{?}{=} 0$

$36 - 66 + 30 \stackrel{?}{=} 0$

$0 = 0$

$(-5)^2 + 11(-5) + 30 \stackrel{?}{=} 0$

$25 - 55 + 30 \stackrel{?}{=} 0$

$0 = 0$

14. $f(x) = x^2 - 9x - 36$

$f(x) = x^2 - 9x - 36$

$0 = x^2 - 9x - 36$

$0 = (x - 12)(x + 3)$

$x - 12 = 0$ or $x + 3 = 0$

$x = 12$ or $x = -3$

The zeros are 12 and −3.

Check:

$(12)^2 - 9(12) - 36 \stackrel{?}{=} 0$

$144 - 108 - 36 \stackrel{?}{=} 0$

$0 = 0$

$(-3)^2 - 9(-3) - 36 \stackrel{?}{=} 0$

$9 + 27 - 36 \stackrel{?}{=} 0$

$0 = 0$

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15. $f(x) = 2x^2 + 9x + 10$

$f(x) = 2x^2 + 9 + 10$

$0 = 2x^2 + 9x + 10$

$0 = (2x + 5)(x + 2)$

$2x + 5 = 0$ or $x + 2 = 0$

$x = -\frac{5}{2}$ or $x = -2$

The zeros are $-\frac{5}{2}$ and -2.

Check:

$2\left(-\frac{5}{2}\right)^2 + 9\left(-\frac{5}{2}\right) + 10 \stackrel{?}{=} 0$

$2\left(\frac{25}{4}\right) - \frac{45}{2} + 10 \stackrel{?}{=} 0$

$\frac{25}{2} - \frac{45}{2} + 10 \stackrel{?}{=} 0$

$-\frac{20}{2} + 10 \stackrel{?}{=} 0$

$-10 + 10 \stackrel{?}{=} 0$

$0 = 0$

$2(-2)^2 + 9(-2) + 10 \stackrel{?}{=} 0$

$2(4) - 18 + 10 \stackrel{?}{=} 0$

$8 - 18 + 10 \stackrel{?}{=} 0$

$0 = 0$

16. $f(x) = x^2 + 5x + 14$

The function has no real zeros.

17. $f(x) = 3x^2 + 3x - 6$

$f(x) = 3x^2 + 3x - 6$

$0 = 3x^2 + 3x - 6$

$0 = 3(x^2 + x - 2)$

$0 = 3(x + 2)(x - 1)$

$x + 2 = 0$ or $x - 1 = 0$

$x = -2$ or $x = 1$

The zeros are -2 and 1.

Check:

$3(-2)^2 + 3(-2) - 6 \stackrel{?}{=} 0$

$3(4) - 6 - 6 \stackrel{?}{=} 0$

$12 - 6 - 6 \stackrel{?}{=} 0$

$0 = 0$

$3(1)^2 + 3(1) - 6 \stackrel{?}{=} 0$

$3(1) + 3 - 6 \stackrel{?}{=} 0$

$3 + 3 - 6 \stackrel{?}{=} 0$

$0 = 0$

18. $f(x) = \frac{1}{2}x^2 - \frac{3}{4}x$

$f(x) = \frac{1}{2}x^2 - \frac{3}{4}x$

$0 = \frac{1}{2}x^2 - \frac{3}{4}x$

$4(0) = 4\left(\frac{1}{2}x^2 - \frac{3}{4}x\right)$

$0 = 2x^2 - 3x$

$0 = x(2x - 3)$

$x = 0$ or $2x - 3 = 0$

$x = 0$ or $x = \frac{3}{2}$

The zeros are 0 and $\frac{3}{2}$.

Check:

$\frac{1}{2}(0)^2 - \frac{3}{4}(0) \stackrel{?}{=} 0$

$\frac{1}{2}(0) - 0 \stackrel{?}{=} 0$

$0 - 0 \stackrel{?}{=} 0$

$0 = 0$

$\frac{1}{2}\left(\frac{3}{2}\right)^2 - \frac{3}{4}\left(\frac{3}{2}\right) \stackrel{?}{=} 0$

$\frac{1}{2}\left(\frac{9}{4}\right) - \frac{9}{8} \stackrel{?}{=} 0$

$\frac{9}{8} - \frac{9}{8} \stackrel{?}{=} 0$

$0 = 0$

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What Makes You So Special?
Special Products

Vocabulary

Give an example of each term. Then, factor the expression.

1. perfect square trinomial

   Answers will vary.

   The trinomial $x^2 + 12x + 36$ is an example of a perfect square trinomial.

   $x^2 + 12x + 36 = (x + 6)(x + 6)$

2. difference of two squares

   Answers will vary.

   The binomial $x^2 - 400$ is an example of the difference of two squares.

   $x^2 - 400 = (x - 20)(x + 20)$

3. sum of two cubes

   Answers will vary.

   The binomial $x^3 + 1$ is an example of the sum of two cubes.

   $x^3 + 1 = (x + 1)(x^2 - x + 1)$

4. difference of two cubes

   Answers will vary.

   The binomial $x^3 - 1$ is an example of the difference of two cubes.

   $x^3 - 1 = (x - 1)(x^2 + x + 1)$

Problem Set

Factor each binomial completely.

1. $x^2 - 25$

   $x^2 - 25 = (x + 5)(x - 5)$

2. $x^3 - 64$

   $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$

3. $x^3 + 27$

   $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$

4. $m^2 - 100$

   $m^2 - 100 = (m + 10)(m - 10)$

5. $5x^3 + 40$

$5x^3 + 40 = 5(x^3 + 8)$

$= 5(x + 2)(x^2 - 2x + 4)$

6. $t^3 - 125$

$t^3 - 125 = (t - 5)(t^2 + 5t + 25)$

7. $8a^3 - 27$

$8a^3 - 27 = (2a - 3)(4a^2 + 6a + 9)$

8. $x^8 - y^8$

$x^8 - y^8 = (x^4 + y^4)(x^4 - y^4)$

$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$

$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

Factor the trinomial completely.

9. $x^2 + 16x + 64$

$x^2 + 16x + 64 = (x + 8)(x + 8)$

10. $k^2 - 20k + 100$

$k^2 - 20k + 100 = (k - 10)(k - 10)$

11. $2x^2 - 28x + 98$

$2x^2 - 28x + 98 = 2(x^2 - 14x + 49)$

$= 2(x - 7)(x - 7)$

12. $5x^2 + 10x + 5$

$5x^2 + 10x + 5 = 5(x^2 + 2x + 1)$

$= 5(x + 1)(x + 1)$

13. $z^3 + 18z^2 + 81z$

$z^3 + 18z^2 + 81z = z(z^2 + 18z + 81)$

$= z(z + 9)(z + 9)$

14. $3x^3 - 30x^2 + 75x$

$3x^3 - 30x^2 + 75x = 3x(x^2 - 10x + 25)$

$= 3x(x - 5)(x - 5)$

Determine the root(s) of each quadratic equation. Check your answer(s).

15. $x^2 - 100 = 0$

$x^2 - 100 = 0$

$(x + 10)(x - 10) = 0$

$x + 10 = 0$ or $x - 10 = 0$

$x = -10$ or $x = 10$

The roots are -10 and 10.

Check:

$(-10)^2 - 100 \stackrel{?}{=} 0$

$100 - 100 \stackrel{?}{=} 0$

$0 = 0$

$(10)^2 - 100 \stackrel{?}{=} 0$

$100 - 100 \stackrel{?}{=} 0$

$0 = 0$

16. $m^2 - 16m + 64 = 0$

$m^2 - 16m + 64 = 0$

$(m - 8)(m - 8) = 0$

$m - 8 = 0$

$m = 8$

The root is 8.

Check:

$(8)^2 - 16(8) + 64 \stackrel{?}{=} 0$

$64 - 128 + 64 \stackrel{?}{=} 0$

$0 = 0$

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17. $6x^2 + 24x + 24 = 0$

$6x^2 + 24x + 24 = 0$

$6(x^2 + 4x + 4) = 0$

$6(x + 2)(x + 2) = 0$

$x + 2 = 0$

$x = -2$

The root is -2.

Check:

$6(-2)^2 + 24(-2) + 24 \stackrel{?}{=} 0$

$6(4) - 48 + 24 \stackrel{?}{=} 0$

$24 - 48 + 24 \stackrel{?}{=} 0$

$0 = 0$

18. $4x^2 - 9 = 0$

$4x^2 - 9 = 0$

$(2x + 3)(2x - 3) = 0$

$2x + 3 = 0$ or $2x - 3 = 0$

$x = -\frac{3}{2}$ or $x = \frac{3}{2}$

The roots are $-\frac{3}{2}$ and $\frac{3}{2}$.

Check:

$4\left(-\frac{3}{2}\right)^2 - 9 \stackrel{?}{=} 0$

$4\left(\frac{9}{4}\right) - 9 \stackrel{?}{=} 0$

$9 - 9 \stackrel{?}{=} 0$

$0 = 0$

$4\left(\frac{3}{2}\right)^2 - 9 \stackrel{?}{=} 0$

$4\left(\frac{9}{4}\right) - 9 \stackrel{?}{=} 0$

$9 - 9 \stackrel{?}{=} 0$

$0 = 0$

19. $t^2 + 22t + 121 = 0$

$t^2 + 22t + 121 = 0$

$(t + 11)(t + 11) = 0$

$t + 11 = 0$

$t = -11$

The root is -11.

Check:

$(-11)^2 + 22(-11) + 121 \stackrel{?}{=} 0$

$121 - 242 + 121 \stackrel{?}{=} 0$

$0 = 0$

20. $12w^2 - 48w + 48 = 0$

$12w^2 - 48w + 48 = 0$

$12(w^2 - 4w + 4) = 0$

$12(w - 2)(w - 2) = 0$

$w - 2 = 0$

$w = 2$

The root is 2.

Check:

$12(2)^2 - 48(2) + 48 \stackrel{?}{=} 0$

$12(4) - 96 + 48 \stackrel{?}{=} 0$

$48 - 96 + 48 \stackrel{?}{=} 0$

$0 = 0$

Determine the zero(s) of each quadratic function. Check your answer(s).

21. $f(x) = x^2 - 225$

$f(x) = x^2 - 225$

$0 = x^2 - 225$

$0 = (x + 15)(x - 15)$

$x + 15 = 0$ or $x - 15 = 0$

$x = -15$ or $x = 15$

The zeros are -15 and 15.

Check:

$(-15)^2 - 225 \stackrel{?}{=} 0$

$225 - 225 \stackrel{?}{=} 0$

$0 = 0$

$(15)^2 - 225 \stackrel{?}{=} 0$

$225 - 225 \stackrel{?}{=} 0$

$0 = 0$

22. $f(x) = x^2 + x + \frac{1}{4}$

$f(x) = x^2 + x + \frac{1}{4}$

$0 = x^2 + x + \frac{1}{4}$

$0 = \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$

$x + \frac{1}{2} = 0$

$x = -\frac{1}{2}$

The zero is $-\frac{1}{2}$.

Check:

$\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + \frac{1}{4} \stackrel{?}{=} 0$

$\frac{1}{4} - \frac{1}{2} + \frac{1}{4} \stackrel{?}{=} 0$

$0 = 0$

23. $f(x) = 9x^2 - 1$

$f(x) = 9x^2 - 1$

$0 = 9x^2 - 1$

$0 = (3x + 1)(3x - 1)$

$3x + 1 = 0$ or $3x - 1 = 0$

$x = -\frac{1}{3}$ or $x = \frac{1}{3}$

The zeros are $-\frac{1}{3}$ and $\frac{1}{3}$.

Check:

$9\left(-\frac{1}{3}\right)^2 - 1 \stackrel{?}{=} 0$

$9\left(\frac{1}{9}\right) - 1 \stackrel{?}{=} 0$

$1 - 1 \stackrel{?}{=} 0$

$0 = 0$

$9\left(\frac{1}{3}\right)^2 - 1 \stackrel{?}{=} 0$

$9\left(\frac{1}{9}\right) - 1 \stackrel{?}{=} 0$

$1 - 1 \stackrel{?}{=} 0$

$0 = 0$

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24. $f(x) = 8x^2 - 48x + 72$

$f(x) = 8x^2 - 48x + 72$

$0 = 8x^2 - 48x + 72$

$0 = 8(x^2 - 6x + 9)$

$0 = 8(x - 3)(x - 3)$

$x - 3 = 0$

$x = 3$

The zero is 3.

Check:

$8(3)^2 - 48(3) + 72 \stackrel{?}{=} 0$

$8(9) - 144 + 72 \stackrel{?}{=} 0$

$72 - 144 + 72 \stackrel{?}{=} 0$

$0 = 0$

25. $f(x) = 8x^2 - 50$

$f(x) = 8x^2 - 50$

$0 = 8x^2 - 50$

$0 = 2(4x^2 - 25)$

$0 = 2(2x + 5)(2x - 5)$

$2x + 5 = 0$ or $2x - 5 = 0$

$x = -\frac{5}{2}$ or $x = \frac{5}{2}$

The zeros are $-\frac{5}{2}$ and $\frac{5}{2}$.

Check:

$8\left(-\frac{5}{2}\right)^2 - 50 \stackrel{?}{=} 0$

$8\left(\frac{25}{4}\right) - 50 \stackrel{?}{=} 0$

$50 - 50 \stackrel{?}{=} 0$

$0 = 0$

$8\left(\frac{5}{2}\right)^2 - 50 \stackrel{?}{=} 0$

$8\left(\frac{25}{4}\right) - 50 \stackrel{?}{=} 0$

$50 - 50 \stackrel{?}{=} 0$

$0 = 0$

26. $f(x) = 2x^2 + 52x + 338$

$f(x) = 2x^2 + 52x + 338$

$0 = 2x^2 + 52x + 338$

$0 = 2(x^2 + 26x + 169)$

$0 = 2(x + 13)(x + 13)$

$x + 13 = 0$

$x = -13$

The zero is -13.

Check:

$2(-13)^2 + 52(-13) + 338 \stackrel{?}{=} 0$

$2(169) - 676 + 338 \stackrel{?}{=} 0$

$338 - 676 + 338 \stackrel{?}{=} 0$

$0 = 0$

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Could It Be Groovy to Be a Square?
Approximating and Rewriting Radicals

Vocabulary

Choose the word that best completes each statement.

square root	positive (principal) square root	radicand
negative square root	extract the square root	radical expression

1. When solving certain quadratic equations, it is necessary to extract the square root from both sides of the equation.
2. Every positive number has both a(n) positive (or principal) square root and a(n) negative square root.
3. The radicand is the expression enclosed within a radical symbol.
4. A number b is a(n) square root of a if $b^2 = a$.
5. An expression involving a radical symbol is called a(n) radical expression.

Problem Set

Rewrite each radical by extracting all perfect squares.

1. $\sqrt{25}$

   $\sqrt{25} = \pm 5$

2. $\sqrt{144}$

   $\sqrt{144} = \pm 12$

3. $\sqrt{400}$

   $\sqrt{400} = \pm 20$

4. $\sqrt{12}$

   $\sqrt{12} = \sqrt{4 \cdot 3}$
   $= \sqrt{4} \cdot \sqrt{3}$
   $= \pm 2\sqrt{3}$

5. $\sqrt{32}$

   $\sqrt{32} = \sqrt{16 \cdot 2}$
   $= \sqrt{16} \cdot \sqrt{2}$
   $= \pm 4\sqrt{2}$

6. $\sqrt{45}$

   $\sqrt{45} = \sqrt{9 \cdot 5}$
   $= \sqrt{9} \cdot \sqrt{5}$
   $= \pm 3\sqrt{5}$

7. $\sqrt{300}$

$\sqrt{300} = \sqrt{100 \cdot 3}$

$= \sqrt{100} \cdot \sqrt{3}$

$= \pm 10\sqrt{3}$

8. $5\sqrt{54}$

$5\sqrt{54} = 5\sqrt{9 \cdot 6}$

$= 5\sqrt{9} \cdot \sqrt{6}$

$= 5(\pm 3)\sqrt{6}$

$= \pm 15\sqrt{6}$

Determine the approximate value of each radical expression to the nearest tenth.

9. $\sqrt{7}$

$2.6^2 = 6.76$

$2.7^2 = 7.29$

$\sqrt{7} \approx 2.6$

10. $\sqrt{37}$

$6.0^2 = 36.00$

$6.1^2 = 37.21$

$\sqrt{37} \approx 6.1$

11. $\sqrt{96}$

$9.7^2 = 94.09$

$9.8^2 = 96.04$

$\sqrt{96} \approx 9.8$

12. $\sqrt{27}$

$5.1^2 = 26.01$

$5.2^2 = 27.04$

$\sqrt{27} \approx 5.2$

13. $\sqrt{109}$

$10.4^2 = 108.16$

$10.5^2 = 110.25$

$\sqrt{109} \approx 10.4$

14. $\sqrt{405}$

$20.1^2 = 404.01$

$20.2^2 = 408.04$

$\sqrt{405} \approx 20.1$

Solve each quadratic equation. Approximate the roots to the nearest tenth.

15. $x^2 = 40$

$x^2 = 40$

$\sqrt{x^2} = \pm\sqrt{40}$

$x = \pm\sqrt{40}$

$6.3^2 = 39.69$

$6.4^2 = 40.96$

$\sqrt{40} \approx \pm 6.3$

$x \approx \pm 6.3$

The roots are approximately 6.3 and −6.3.

16. $m^2 = 68$

$m^2 = 68$

$\sqrt{m^2} = \pm\sqrt{68}$

$m = \pm\sqrt{68}$

$8.2^2 = 67.24$

$8.3^2 = 68.89$

$\sqrt{68} \approx \pm 8.2$

$m \approx \pm 8.2$

The roots are approximately 8.2 and −8.2.

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17. $t^2 = 15$

$t^2 = 15$

$\sqrt{t^2} = \pm\sqrt{15}$

$t = \pm\sqrt{15}$

$3.8^2 = 14.44$

$3.9^2 = 15.21$

$\sqrt{15} \approx \pm 3.9$

$t \approx \pm 3.9$

The roots are approximately 3.9 and −3.9.

18. $x^2 = 83$

$x^2 = 83$

$\sqrt{x^2} = \pm\sqrt{83}$

$x = \pm\sqrt{83}$

$9.1^2 = 82.81$

$9.2^2 = 84.64$

$\sqrt{83} \approx \pm 9.1$

$x \approx \pm 9.1$

The roots are approximately 9.1 and −9.1.

19. $(x - 5)^2 = 22$

$(x - 5)^2 = 22$

$\sqrt{(x - 5)^2} = \pm\sqrt{22}$

$x - 5 = \pm\sqrt{22}$

$x = 5 \pm \sqrt{22}$

$4.6^2 = 21.16$

$4.7^2 = 22.09$

$\sqrt{22} \approx \pm 4.7$

$x \approx 5 \pm 4.7$

$x \approx 0.3$ or $x \approx 9.7$

The roots are approximately 0.3 and 9.7.

20. $(x + 8)^2 = 29$

$(x + 8)^2 = 29$

$\sqrt{(x + 8)^2} = \pm\sqrt{29}$

$x + 8 = \pm\sqrt{29}$

$x = -8 \pm \sqrt{29}$

$5.3^2 = 28.09$

$5.4^2 = 29.16$

$\sqrt{29} \approx \pm 5.4$

$x \approx -8 \pm 5.4$

$x \approx -2.6$ or $x \approx -13.4$

The roots are approximately −2.6 and −13.4.

Solve each quadratic equation. Rewrite the roots in radical form.

21. $x^2 = 48$

$x^2 = 48$

$\sqrt{x^2} = \pm\sqrt{48}$

$x = \pm\sqrt{48}$

$x = \pm\sqrt{16 \cdot 3}$

$x = \pm\sqrt{16} \cdot \sqrt{3}$

$x = \pm 4\sqrt{3}$

The roots are $4\sqrt{3}$ and $-4\sqrt{3}$.

22. $x^2 = 52$

$x^2 = 52$

$\sqrt{x^2} = \pm\sqrt{52}$

$x = \pm\sqrt{52}$

$x = \pm\sqrt{4 \cdot 13}$

$x = \pm\sqrt{4} \cdot \sqrt{13}$

$x = \pm 2\sqrt{13}$

The roots are $2\sqrt{13}$ and $-2\sqrt{13}$.

23. $x^2 = 27$

$x^2 = 27$

$\sqrt{x^2} = \pm\sqrt{27}$

$x = \pm\sqrt{27}$

$x = \pm\sqrt{9 \cdot 3}$

$x = \pm\sqrt{9} \cdot \sqrt{3}$

$x = \pm 3\sqrt{3}$

The roots are $3\sqrt{3}$ and $-3\sqrt{3}$.

24. $x^2 = 175$

$x^2 = 175$

$\sqrt{x^2} = \pm\sqrt{175}$

$x = \pm\sqrt{175}$

$x = \pm\sqrt{25 \cdot 7}$

$x = \pm\sqrt{25} \cdot \sqrt{7}$

$x = \pm 5\sqrt{7}$

The roots are $5\sqrt{7}$ and $-5\sqrt{7}$.

25. $(12 - x)^2 = 8$

$(12 - x)^2 = 8$

$\sqrt{(12 - x)^2} = \pm\sqrt{8}$

$12 - x = \pm\sqrt{8}$

$-x = -12 \pm \sqrt{8}$

$x = 12 \pm \sqrt{8}$

$x = 12 \pm \sqrt{4 \cdot 2}$

$x = 12 \pm \sqrt{4} \cdot \sqrt{2}$

$x = 12 \pm 2\sqrt{2}$

The roots are $12 + 2\sqrt{2}$ and $12 - 2\sqrt{2}$.

26. $(x + 20)^2 = 80$

$(x + 20)^2 = 80$

$\sqrt{(x + 20)^2} = \pm\sqrt{80}$

$x + 20 = \pm\sqrt{80}$

$x = -20 \pm \sqrt{80}$

$x = -20 \pm \sqrt{16 \cdot 5}$

$x = -20 \pm \sqrt{16} \cdot \sqrt{5}$

$x = -20 \pm 4\sqrt{5}$

The roots are $-20 + 4\sqrt{5}$ and $-20 - 4\sqrt{5}$.

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Another Method
Completing the Square

Vocabulary

Define the term in your own words.

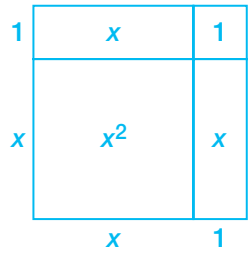
1. completing the square

   Completing the square is a process for writing a quadratic expression in vertex form which then allows you to solve for the zeros.

Problem Set

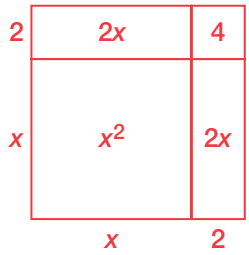
Use a geometric figure to complete the square for each expression. Factor the resulting trinomial.

1. $x^2 + 2x$



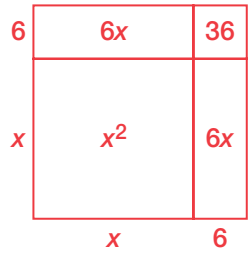
$x^2 + 2x + 1 = (x + 1)^2$

2. $x^2 + 4x$



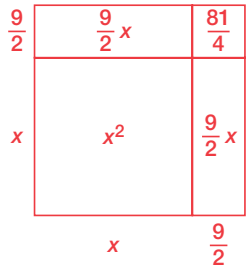
$x^2 + 4x + 4 = (x + 2)^2$

3. $x^2 + 12x$



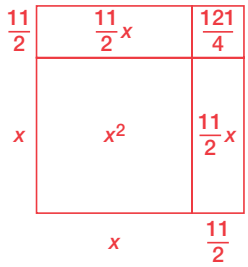
$x^2 + 12x + 36 = (x + 6)^2$

4. $x^2 + 9x$



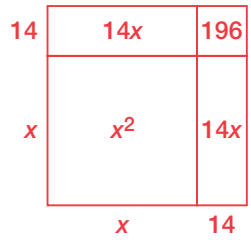
$x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$

5. $x^2 + 11x$



$x^2 + 11x + \frac{121}{4} = \left(x + \frac{11}{2}\right)^2$

6. $x^2 + 28x$



$x^2 + 28x + 196 = (x + 14)^2$

Determine the unknown value that would make each trinomial a perfect square.

7. $x^2 - 10x +$ 25

8. $x^2 + 14x +$ 49

9. $x^2 +$ 6 $x + 9$

10. $x^2 -$ 18 $x + 81$

11. $x^2 + 7x +$ $\frac{49}{4}$

12. $x^2 - 15x +$ $\frac{225}{4}$

13. $x^2 -$ 26 $x + 169$

14. $x^2 +$ 3 $x + \frac{9}{4}$

Determine the roots of each quadratic equation by completing the square. Round your answer to the nearest hundredth. Check your answer.

15. $x^2 + 4x - 6 = 0$

$x^2 + 4x - 6 = 0$

$x^2 + 4x = 6$

$x^2 + 4x + 4 = 6 + 4$

$(x + 2)^2 = 10$

$\sqrt{(x + 2)^2} = \pm\sqrt{10}$

$x + 2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

$x \approx 1.16$ or $x \approx -5.16$

The roots are approximately 1.16 and −5.16.

Check:

$(1.16)^2 + 4(1.16) - 6 \stackrel{?}{=} 0$

$1.3456 + 4.64 - 6 \stackrel{?}{=} 0$

$-0.0144 \approx 0$

$(-5.16)^2 - 4(-5.16) - 6 \stackrel{?}{=} 0$

$26.6256 - 20.64 - 6 \stackrel{?}{=} 0$

$-0.0144 \approx 0$

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16. $x^2 - 2x - 4 = 0$

$x^2 - 2x - 4 = 0$

$x^2 - 2x = 4$

$x^2 - 2x + 1 = 4 + 1$

$(x - 1)^2 = 5$

$\sqrt{(x - 1)^2} = \pm\sqrt{5}$

$x - 1 = \pm\sqrt{5}$

$x = 1 \pm \sqrt{5}$

$x \approx 3.24$ or $x \approx -1.24$

The roots are approximately 3.24 and −1.24.

Check:

$(3.24)^2 - 2(3.24) - 4 \stackrel{?}{=} 0$

$10.4976 - 6.48 - 4 \stackrel{?}{=} 0$

$0.0176 \approx 0$

$(-1.24)^2 - 2(-1.24) - 4 \stackrel{?}{=} 0$

$1.5376 + 2.48 - 4 \stackrel{?}{=} 0$

$0.0176 \approx 0$

17. $x^2 + 10x + 2 = 0$

$x^2 + 10x + 2 = 0$

$x^2 + 10x = -2$

$x^2 + 10x + 25 = -2 + 25$

$(x + 5)^2 = 23$

$\sqrt{(x + 5)^2} = \pm\sqrt{23}$

$x + 5 = \pm\sqrt{23}$

$x = -5 \pm \sqrt{23}$

$x \approx -0.20$ or $x \approx -9.80$

The roots are approximately −0.20 and −9.80.

Check:

$(-0.20)^2 + 10(-0.20) + 2 \stackrel{?}{=} 0$

$0.04 - 2 + 2 \stackrel{?}{=} 0$

$0.04 \approx 0$

$(-9.80)^2 + 10(-9.80) + 2 \stackrel{?}{=} 0$

$96.04 - 98 + 2 \stackrel{?}{=} 0$

$0.04 \approx 0$

18. $x^2 - 12x + 25 = 0$

$x^2 - 12x + 25 = 0$

$x^2 - 12x = -25$

$x^2 - 12x + 36 = -25 + 36$

$(x - 6)^2 = 11$

$\sqrt{(x - 6)^2} = \pm\sqrt{11}$

$x - 6 = \pm\sqrt{11}$

$x = 6 \pm \sqrt{11}$

$x \approx 9.32$ or $x \approx 2.68$

The roots are approximately 9.32 and 2.68.

Check:

$(9.32)^2 - 12(9.32) + 25 \stackrel{?}{=} 0$

$86.8624 - 111.84 + 25 \stackrel{?}{=} 0$

$0.0224 \approx 0$

$(2.68)^2 - 12(2.68) + 25 \stackrel{?}{=} 0$

$7.1824 - 32.16 + 25 \stackrel{?}{=} 0$

$0.0224 \approx 0$

19. $x^2 + 3x - 1 = 0$

$x^2 + 3x - 1 = 0$

$x^2 + 3x = 1$

$x^2 + 3x + \frac{9}{4} = 1 + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{13}{4}$

$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm\sqrt{\frac{13}{4}}$

$x + \frac{3}{2} = \pm\frac{\sqrt{13}}{2}$

$x = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$

$x = \frac{-3 \pm \sqrt{13}}{2}$

$x \approx 0.30$ or $x \approx -3.30$

The roots are approximately 0.30 and −3.30.

Check:

$(0.30)^2 + 3(0.30) - 1 \stackrel{?}{=} 0$

$0.09 + 0.90 - 1 \stackrel{?}{=} 0$

$-0.01 \approx 0$

$(-3.30)^2 + 3(-3.30) - 1 \stackrel{?}{=} 0$

$10.89 - 9.90 - 1 \stackrel{?}{=} 0$

$-0.01 \approx 0$

20. $x^2 + x - 10 = 0$

$x^2 + x - 10 = 0$

$x^2 + x = 10$

$x^2 + x + \frac{1}{4} = 10 + \frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 = \frac{41}{4}$

$\sqrt{\left(x + \frac{1}{2}\right)^2} = \pm\sqrt{\frac{41}{4}}$

$x + \frac{1}{2} = \pm\frac{\sqrt{41}}{2}$

$x = -\frac{1}{2} \pm \frac{\sqrt{41}}{2}$

$x = \frac{-1 \pm \sqrt{41}}{2}$

$x \approx 2.70$ or $x \approx -3.70$

The roots are approximately 2.70 and −3.70.

Check:

$(2.70)^2 + (2.70) - 10 \stackrel{?}{=} 0$

$7.29 + 2.70 - 10 \stackrel{?}{=} 0$

$-0.01 \approx 0$

$(-3.70)^2 + (-3.70) - 10 \stackrel{?}{=} 0$

$13.69 - 3.70 - 10 \stackrel{?}{=} 0$

$-0.01 \approx 0$