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Ladies and Gentlemen: Please Welcome the Quadratic Formula!

The Quadratic Formula

Vocabulary

Complete the Quadratic Formula. Then, identify the discriminant and explain what it indicates about the function.

The quadratic equation of the form $ax^2 + bx + c = 0$, can be written as the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant of the Quadratic Formula is the radicand, $b^2 - 4ac$. This portion of the formula indicates the number of zeros. If the discriminant is zero, there is 1 zero, if the discriminant is positive, there are 2 zeros, and if the discriminant is negative, there are no zeros.

Problem Set

Determine the approximate zeros or roots of each function or equation. Round your answers to the nearest thousandth, if necessary.

1. $f(x) = x^2 + 3x - 5$

$a = 1, b = 3, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 20}}{2}$$

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

$$x = \frac{-3 + 5.385}{2} \quad \text{or} \quad x = \frac{-3 - 5.385}{2}$$

$$x \approx 1.193 \quad \text{or} \quad x \approx -4.193$$

2. $f(x) = -3x^2 - x + 7$

$a = -3, b = -1, c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-3)(7)}}{2(-3)}$$

$$x = \frac{1 \pm \sqrt{1 + 84}}{-6}$$

$$x = \frac{1 \pm \sqrt{85}}{-6}$$

$$x = \frac{1 + 9.22}{-6} \quad \text{or} \quad x = \frac{1 - 9.22}{-6}$$

$$x \approx -1.703 \quad \text{or} \quad x \approx 1.370$$

3. $2x^2 + 6x - 7 = 2$

$2x^2 + 6x - 7 = 2$

$2x^2 + 6x - 9 = 0$

$a = 2, b = 6, c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-9)}}{2(2)}$

$x = \frac{-6 \pm \sqrt{36 + 72}}{4}$

$x = \frac{-6 \pm \sqrt{108}}{4}$

$x = \frac{-6 + 10.392}{4}$ or $x = \frac{-6 - 10.392}{4}$

$x \approx 1.098$ or $x \approx -4.098$

4. $4x^2 - x - 1 = 5$

$4x^2 - x - 1 = 5$

$4x^2 - x - 6 = 0$

$a = 4, b = -1, c = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-6)}}{2(4)}$

$x = \frac{1 \pm \sqrt{1 + 96}}{8}$

$x = \frac{1 \pm \sqrt{97}}{8}$

$x = \frac{1 + 9.849}{8}$ or $x = \frac{1 - 9.849}{8}$

$x \approx 1.356$ or $x \approx -1.106$

5. $f(x) = -8x^2 + 2x + 1$

$a = -8, b = 2, c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(-8)(1)}}{2(-8)}$

$x = \frac{-2 \pm \sqrt{4 + 32}}{-16}$

$x = \frac{-2 \pm \sqrt{36}}{-16}$

$x = \frac{-2 + 6}{-16}$ or $x = \frac{-2 - 6}{-16}$

$x = -0.25$ or $x = 0.5$

6. $3x^2 + x + 3 = 5$

$3x^2 + x + 3 = 5$

$3x^2 + x - 2 = 0$

$a = 3, b = 1, c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{1 + 24}}{6}$

$x = \frac{-1 \pm \sqrt{25}}{6}$

$x = \frac{-1 + 5}{6}$ or $x = \frac{-1 - 5}{6}$

$x \approx 0.667$ or $x = -1$

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Determine the exact zeros or roots of each function or equation.

7. $f(x) = -2x^2 - 8x + 1$

$a = -2, b = -8, c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-2)(1)}}{2(-2)}$

$x = \frac{8 \pm \sqrt{64 + 8}}{-4}$

$x = \frac{8 \pm \sqrt{72}}{-4}$

$x = \frac{8 \pm \sqrt{36 \cdot 2}}{-4}$

$x = \frac{8 \pm 6\sqrt{2}}{-4}$

$x = \frac{8 + 6\sqrt{2}}{-4}$ or $x = \frac{8 - 6\sqrt{2}}{-4}$

$x = -2 - \frac{3}{2}\sqrt{2}$ or $x = -2 + \frac{3}{2}\sqrt{2}$

8. $5x^2 + 8x - 3 = 1$

$5x^2 + 8x - 3 = 1$

$5x^2 + 8x - 4 = 0$

$a = 5, b = 8, c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(5)(-4)}}{2(5)}$

$x = \frac{-8 \pm \sqrt{64 + 80}}{10}$

$x = \frac{-8 \pm \sqrt{144}}{10}$

$x = \frac{-8 \pm \sqrt{144}}{10}$

$x = \frac{-8 \pm 12}{10}$

$x = \frac{-8 + 12}{10}$ or $x = \frac{-8 - 12}{10}$

$x = \frac{4}{10}$ or $x = \frac{-20}{10}$

$x = \frac{2}{5}$ or $x = -2$

9. $-3x^2 + 6x + 2 = -5$

$-3x^2 + 6x + 2 = -5$

$-3x^2 + 6x + 7 = 0$

$a = -3, b = 6, c = 7$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(-3)(7)}}{2(-3)}$

$x = \frac{-6 \pm \sqrt{36 + 84}}{-6}$

$x = \frac{-6 \pm \sqrt{120}}{-6}$

$x = \frac{-6 \pm \sqrt{4 \cdot 30}}{-6}$

$x = \frac{-6 \pm 2\sqrt{30}}{-6}$

$x = \frac{-6 + 2\sqrt{30}}{-6}$ or $x = \frac{-6 - 2\sqrt{30}}{-6}$

$x = 1 - \frac{\sqrt{30}}{3}$ or $x = 1 + \frac{\sqrt{30}}{3}$

10. $f(x) = x^2 + 6x + 5$

$a = 1, b = 6, c = 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36 - 20}}{2}$

$x = \frac{-6 \pm \sqrt{16}}{2}$

$x = \frac{-6 \pm 4}{2}$

$x = \frac{-6 + 4}{2}$ or $x = \frac{-6 - 4}{2}$

$x = \frac{-2}{2}$ or $x = \frac{-10}{2}$

$x = -1$ or $x = -5$

11. $f(x) = -2x^2 + 5x - 1$

$a = -2, b = 5, c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(-2)(-1)}}{2(-2)}$

$x = \frac{-5 \pm \sqrt{25 - 8}}{-4}$

$x = \frac{-5 \pm \sqrt{17}}{-4}$

$x = \frac{-5 + \sqrt{17}}{-4}$ or $x = \frac{-5 - \sqrt{17}}{-4}$

12. $-3x^2 + 8x - 2 = -6$

$-3x^2 + 8x - 2 = -6$

$-3x^2 + 8x + 4 = 0$

$a = -3, b = 8, c = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(-3)(4)}}{2(-3)}$

$x = \frac{-8 \pm \sqrt{64 + 48}}{-6}$

$x = \frac{-8 \pm \sqrt{112}}{-6}$

$x = \frac{-8 \pm \sqrt{16 \cdot 7}}{-6}$

$x = \frac{-8 \pm 4\sqrt{7}}{-6}$

$x = \frac{4 \pm 2\sqrt{7}}{3}$

$x = \frac{4 + 2\sqrt{7}}{3}$ or $x = \frac{4 - 2\sqrt{7}}{3}$

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Use the discriminant to determine the number of zeros or roots each function or equation has. Then solve for the zeros or roots.

13. $f(x) = -x^2 + 6x + 7$

$a = -1, b = 6, c = 7$

$b^2 - 4ac = (6)^2 - 4(-1)(7)$

$= 36 + 28$

$= 64$

Because $b^2 - 4ac > 0$ the function has two zeros.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(6) \pm \sqrt{64}}{2(-1)}$

$x = \frac{-6 \pm 8}{-2}$

$x = \frac{-6 + 8}{-2}$ or $x = \frac{-6 - 8}{-2}$

$x = \frac{2}{-2}$ or $x = \frac{-14}{-2}$

$x = -1$ or $x = 7$

14. $2x^2 + 8x + 3 = -5$

$2x^2 + 8x + 3 = -5$

$2x^2 + 8x + 8 = 0$

$a = 2, b = 8, c = 8$

$b^2 - 4ac = (8)^2 - 4(2)(8)$

$= 64 - 64$

$= 0$

Because $b^2 - 4ac = 0$ the equation has one double root.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{0}}{2(2)}$

$x = \frac{-8 \pm 0}{4}$

$x = \frac{-8}{4}$

$x = -2$

15. $f(x) = 9x^2 + 5x + 1$

$a = 9, b = 5, c = 1$

$b^2 - 4ac = (5)^2 - 4(9)(1)$

$= 25 - 36$

$= -11$

Because $b^2 - 4ac < 0$ the function has no zeros.

16. $6x^2 + 3x - 5 = 2$

$6x^2 + 3x - 5 = 2$

$6x^2 + 3x - 7 = 0$

$a = 6, b = 3, c = -7$

$b^2 - 4ac = (3)^2 - 4(6)(-7)$

$= 9 + 168$

$= 177$

Because $b^2 + 4ac > 0$ the equation has two roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{177}}{2(6)}$$

$$x = \frac{-3 \pm \sqrt{177}}{12}$$

$$x = \frac{-3 + \sqrt{177}}{12} \quad \text{or} \quad x = \frac{-3 - \sqrt{177}}{12}$$

17. $f(x) = 5x^2 + 10x + 5$

$a = 5, b = 10, c = 5$

$b^2 - 4ac = (10)^2 - 4(5)(5)$

$= 100 - 100$

$= 0$

Because $b^2 - 4ac = 0$ the function has one zero.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(10) \pm \sqrt{0}}{2(5)}$$

$$x = \frac{-10 \pm 0}{10}$$

$$x = \frac{-10}{10}$$

$$x = -1$$

18. $f(x) = 7x^2 + 9x + 5$

$a = 7, b = 9, c = 5$

$b^2 - 4ac = (9)^2 - 4(7)(5)$

$= 81 - 140$

$= -59$

Because $b^2 - 4ac < 0$ the function has no zeros.

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It's Watching and Tracking!

Using a Calculator-Based Ranger to Model Quadratic Motion

Vocabulary

Define each term in your own words.

1. quadratic regression

   Quadratic regression is a mathematical method to determine the equation of a "parabola of best fit" for a data set.

2. coefficient of determination

   The coefficient of determination measures the "strength" of the relationship between the original data and the regression equation.

Problem Set

Use your graphing calculator to determine the quadratic regression equation and coefficient of determination for the line of best fit of each given data set. Determine if the equation is a good fit for the data. Round your answers to the nearest hundredth.

1.

x	y
0	0
1	4.05
2	5.50
3	6.25
4	3.50
5	0

$y = -0.97x^2 + 4.84x + 0.03$

$r^2 \approx 0.99$

Because the r^2 value is close to 1, the quadratic regression equation is a good fit for the data.

2.

x	y
0	2.1
0.5	3.4
1	4.1
1.5	4.3
2	3.9
2.5	2.3

$y = -1.35x^2 + 3.53x + 2.03$

$r^2 \approx 0.98$

Because the r^2 value is close to 1, the quadratic regression equation is a good fit for the data.

3.

x	y
−5	−0.5
−4	5
−1	7
0	3
0.5	−1
1	3

$y = -0.73x^2 - 2.93x + 3.84$

$r^2 \approx 0.55$

Because the r^2 value is not close to 1, the quadratic regression equation is not a good fit for the data.

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4.

x	y
−4	3.05
−3	−1.50
−2	−4.80
−1	−5.18
0	−3.75
1	−1.79

$y = 0.92x^2 + 1.86x - 4.28$

$r^2 \approx 0.99$

Because the r^2 value is close to 1, the quadratic regression equation is a good fit for the data.

5.

x	y
0	6.2
1	4.5
2	1.5
3	−0.5
4	2.4
5	3.9

$y = 0.71x^2 - 4.10x + 6.77$

$r^2 \approx 0.86$

Because the r^2 value is close to 1, the quadratic regression equation is a good fit for the data.

6.

x	y
−5	7.21
−4	1.80
−3	−2.40
−2	−5.92
−1	−1.40
0	2.73

$y = 1.47x^2 + 6.36x + 2.72$

$r^2 \approx 0.95$

Because the r^2 value is close to 1, the quadratic regression equation is a good fit for the data.

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They're A Lot More than Just Sparklers!

Solving Quadratic Inequalities

Problem Set

Determine the roots of each quadratic inequality. Use the interval method to determine the solution set of the inequality. Round your answer to the nearest thousandth if necessary.

1. $x^2 - 7x + 16 \geq 10$

$x^2 - 7x + 16 \geq 10$

$x^2 - 7x + 6 \geq 0$

$x^2 - 7x + 6 = 0$

$(x - 6)(x - 1) = 0$

$x - 6 = 0$ or $x - 1 = 0$

$x = 6$ or $x = 1$

Test 0, 2, and 7.

$x^2 - 7x + 16 \geq 10$

$(0)^2 - 7(0) + 16 \geq 10$

$16 \geq 10$ ✔

$x^2 - 7x + 16 \geq 10$

$(2)^2 - 7(2) + 16 \geq 10$

$4 - 14 + 16 \geq 10$

$6 \geq 10$ ✘

$x^2 - 7x + 16 \geq 10$

$(7)^2 - 7(7) + 16 \geq 10$

$49 - 49 + 16 \geq 10$

$16 \geq 10$ ✔

Solution: $x \in (-\infty, 1]$ or $x \in [6, \infty)$

2. $x^2 + 7x - 2 < -12$

$x^2 + 7x - 2 < -12$

$x^2 + 7x + 10 < 0$

$x^2 + 7x + 10 = 0$

$(x + 2)(x + 5) = 0$

$x + 2 = 0$ or $x + 5 = 0$

$x = -2$ $\quad$ $x = -5$

Test −6, −4, and 0.

$x^2 + 7x - 2 < -12$

$(-6)^2 + 7(-6) - 2 < -12$

$36 - 42 - 2 < -12$

$-8 < -12$ ✘

$x^2 + 7x - 2 < -12$

$(-4)^2 + 7(-4) - 2 < -12$

$16 - 28 - 2 < -12$

$-14 < -12$ ✔

$x^2 + 7x - 2 < -12$

$(0)^2 + 7(0) - 2 < -12$

$0 + 0 - 2 < -12$

$-2 < -12$ ✘

Solution: $x \in (-5, -2)$

3. $x^2 + x - 15 < 4$

$x^2 + x - 15 < 4$

$x^2 + x - 19 < 0$

$x^2 + x - 19 = 0$

$a = 1, b = 1, c = -19$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-19)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 76}}{2}$$

$$x = \frac{-1 \pm \sqrt{77}}{2}$$

$$x = \frac{-1 + \sqrt{77}}{2} \quad \text{or} \quad x = \frac{-1 - \sqrt{77}}{2}$$

$x \approx 3.887$ or $x \approx -4.887$

Test −6, 0, and 5.

$x^2 + x - 15 < 4$

$(-6)^2 + (-6) - 15 < 4$

$36 - 6 - 15 < 4$

$15 < 4$ ✘

$x^2 + x - 15 < 4$

$(0)^2 + (0) - 15 < 4$

$0 + 0 - 15 < 4$

$-15 < 4$ ✔

$x^2 + x - 15 < 4$

$(5)^2 + (5) - 15 < 4$

$25 + 5 - 15 < 4$

$15 < 4$ ✘

Solution: $x \in (-4.887, 3.887)$

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4. $-x^2 + 11x - 21 \leq 2$

$-x^2 + 11x - 21 \leq 2$

$-x^2 + 11x - 23 \leq 0$

$-x^2 + 11x - 23 = 0$

$a = -1, b = 11, c = -23$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(11) \pm \sqrt{(11)^2 - 4(-1)(-23)}}{2(-1)}$

$x = \dfrac{-11 \pm \sqrt{121 - 92}}{-2}$

$x = \dfrac{-11 \pm \sqrt{29}}{-2}$

$x = \dfrac{-11 + \sqrt{29}}{-2}$ or $x = \dfrac{-11 - \sqrt{29}}{-2}$

$x \approx 2.807$ or $x \approx 8.193$

Test 0, 4, and 9.

$-x^2 + 11x - 21 \leq 2$
$-(0)^2 + 11(0) - 21 \leq 2$
$0 + 0 - 21 \leq 2$
$-21 \leq 2$ ✔

$-x^2 + 11x - 21 \leq 2$
$-(4)^2 + 11(4) - 21 \leq 2$
$-16 + 44 - 21 \leq 2$
$7 \leq 2$ ✘

$-x^2 + 11x - 21 \leq 2$
$-(9)^2 + 11(9) - 21 \leq 2$
$-81 + 99 - 21 \leq 2$
$-3 \leq 2$ ✔

Solution: $x \in (-\infty, 2.807]$ or $x \in [8.193, \infty)$

5. $-x^2 + 4x - 5 \leq -2$

$-x^2 + 4x - 5 \leq -2$

$-x^2 + 4x - 3 \leq 0$

$-x^2 + 4x - 3 = 0$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x - 1 = 0$ or $x - 3 = 0$

$x = 1$ or $x = 3$

Test 0, 2, and 4.

$-x^2 + 4x - 5 \leq -2$
$-(0)^2 + 4(0) - 5 \leq -2$
$0 + 0 - 5 \leq -2$
$-5 \leq -2$ ✔

$-x^2 + 4x - 5 \leq -2$
$-(2)^2 + 4(2) - 5 \leq -2$
$-4 + 8 - 5 \leq -2$
$-1 \leq -2$ ✘

$-x^2 + 4x - 5 \leq -2$
$-(4)^2 + 4(4) - 5 \leq -2$
$-16 + 16 - 5 \leq -2$
$-5 \leq -2$ ✔

Solution: $x \in (-\infty, 1]$ or $x \in [3, \infty)$

6. $-x^2 - 3x + 14 > -3$

$-x^2 - 3x + 14 > -3$

$-x^2 - 3x + 17 > 0$

$-x^2 - 3x + 17 = 0$

$a = -1, b = -3, c = 17$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-1)(17)}}{2(-1)}$

$x = \frac{3 \pm \sqrt{9 + 68}}{-2}$

$x = \frac{3 \pm \sqrt{77}}{-2}$

$x = \frac{3 + \sqrt{77}}{-2}$ or $x = \frac{3 - \sqrt{77}}{-2}$

$x = -5.887$ or $x = 2.887$

Test −7, 0, and 4.

$-x^2 - 3x + 14 > -3$

$-(-7)^2 - 3(-7) + 14 > -3$

$-49 + 21 + 14 > -3$

$-14 > -3$ ✘

$-x^2 - 3x + 14 > -3$

$-(0)^2 - 3(0) + 14 > -3$

$0 - 0 + 14 > -3$

$14 > -3$ ✔

$-x^2 - 3x + 14 > -3$

$-(4)^2 - 3(4) + 14 > -3$

$-16 - 12 + 14 > -3$

$-14 > -3$ ✘

Solution: $x \in (-5.887, 2.887)$

A water balloon is thrown upward from a height of 5 feet with an initial velocity of 35 feet per second. The quadratic function $h(t) = -16t^2 + 35t + 5$ represents the height of the balloon, h, in feet t seconds after it is thrown. Use this information to answer each question.

7. How long does it take for the balloon to reach the ground? Round your answer to the nearest thousandth.

$0 = -16t^2 + 35t + 5$

$a = -16, b = 35, c = 5$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(5)}}{2(-16)}$

$t = \frac{-35 \pm \sqrt{1225 + 320}}{-32}$

$t = \frac{-35 \pm \sqrt{1545}}{-32}$

$t = \frac{-35 + 39.306}{-32}$ or $t = \frac{-35 - 39.306}{-32}$

$t \approx -0.1346$ or $t \approx 2.322$

It will take just over 2.3 seconds for the balloon to reach the ground.

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8. Determine when the balloon is less than 10 feet above the ground. Round your answer to the nearest thousandth.

$-16t^2 + 35t + 5 < 10$

$-16t^2 + 35t - 5 < 0$

$a = -16, b = 35, c = -5$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(-5)}}{2(-16)}$

$t = \frac{-35 \pm \sqrt{1225 - 320}}{-32}$

$t = \frac{-35 \pm \sqrt{905}}{-32}$

$t = \frac{-35 + \sqrt{905}}{-32}$ or $t = \frac{-35 - \sqrt{905}}{-32}$

$t \approx 0.1536$ or $t \approx 2.034$

Test 0, 1, and 3.

$-16t^2 + 35t + 5 < 10$

$-16(0)^2 + 35(0) + 5 < 10$

$0 + 0 + 5 < 10$

$5 < 10$ ✔

$-16t^2 + 35t + 5 < 10$

$-16(1)^2 + 35(1) + 5 < 10$

$-16 + 35 + 5 < 10$

$24 < 10$ ✘

$-16t^2 + 35t + 5 < 10$

$-16(3)^2 + 35(3) + 5 < 10$

$-144 + 105 + 5 < 10$

$-34 < 10$ ✔

The balloon is less than 10 feet above the ground before 0.1536 seconds and after 2.034 seconds.

9. Determine when the balloon is more than 10 feet above the ground. Round your answer to the nearest thousandth.

$-16t^2 + 35t + 5 > 10$

$-16t^2 + 35t - 5 > 0$

$a = -16, b = 35, c = -5$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(-5)}}{2(-16)}$

$t = \frac{-35 \pm \sqrt{1225 - 320}}{-32}$

$t = \frac{-35 \pm \sqrt{905}}{-32}$

$t = \frac{-35 + \sqrt{905}}{-32}$ or $t = \frac{-35 - \sqrt{905}}{-32}$

$t \approx 0.1536$ or $t \approx 2.034$

Test 0, 1, and 3.

$-16t^2 + 35t + 5 > 10$

$-16(0)^2 + 35(0) + 5 > 10$

$0 + 0 + 5 > 10$

$5 > 10$ ✘

$-16t^2 + 35t + 5 > 10$

$-16(1)^2 + 35(1) + 5 > 10$

$-16 + 35 + 5 > 10$

$24 > 10$ ✔

$-16t^2 + 35t + 5 > 10$

$-16(3)^2 + 35(3) + 5 > 10$

$-144 + 105 + 5 > 10$

$-34 > 10$ ✘

The balloon is more than 10 feet above the ground between 0.1536 seconds and 2.034 seconds.

10. Determine when the balloon is less than 20 feet above the ground. Round your answer to the nearest thousandth.

$-16t^2 + 35t + 5 < 20$

$-16t^2 + 35t - 15 < 0$

$a = -16, b = 35, c = -15$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(-15)}}{2(-16)}$

$t = \frac{-35 \pm \sqrt{1225 - 960}}{-32}$

$t = \frac{-35 \pm \sqrt{265}}{-32}$

$t = \frac{-35 + \sqrt{265}}{-32}$ or $t = \frac{-35 - \sqrt{265}}{-32}$

$t \approx 0.5850$ or $t \approx 1.602$

Test 0, 1, and 2.

$-16t^2 + 35t + 5 < 20$
$-16(0)^2 + 35(0) + 5 < 20$
$0 + 0 + 5 < 20$
$5 < 20$ ✔

$-16t^2 + 35t + 5 < 20$
$-16(1)^2 + 35(1) + 5 < 20$
$-16 + 35 + 5 < 20$
$24 < 20$ ✘

$-16t^2 + 35t + 5 < 20$
$-16(2)^2 + 35(2) + 5 < 20$
$-64 + 70 + 5 < 20$
$11 < 20$ ✔

The balloon is less than 20 feet above the ground before 0.5850 seconds and after 1.602 seconds.

11. Determine when the balloon is more than 20 feet above the ground. Round your answer to the nearest thousandth.

$-16t^2 + 35t + 5 > 20$

$-16t^2 + 35t - 15 > 0$

$a = -16, b = 35, c = -15$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(-15)}}{2(-16)}$

$t = \frac{-35 \pm \sqrt{1225 - 960}}{-32}$

$t = \frac{-35 \pm \sqrt{265}}{-32}$

$t = \frac{-35 + \sqrt{265}}{-32}$ or $t = \frac{-35 - \sqrt{265}}{-32}$

$t \approx 0.5850$ or $t \approx 1.602$

Test 0, 1, and 2.

$-16t^2 + 35t + 5 > 20$
$-16(0)^2 + 35(0) + 5 > 20$
$0 + 0 + 5 > 20$
$5 > 20$ ✘

$-16t^2 + 35t + 5 > 20$
$-16(1)^2 + 35(1) + 5 > 20$
$-16 + 35 + 5 > 20$
$24 > 20$ ✔

$-16t^2 + 35t + 5 > 20$
$-16(2)^2 + 35(2) + 5 > 20$
$-64 + 70 + 5 > 20$
$11 > 20$ ✘

The balloon is more than 20 feet above the ground between 0.5850 seconds and 1.602 seconds.

Name _____ Date _____

12. Determine when the balloon is less than 30 feet above the ground. Round your answer to the nearest thousandth.

$-16t^2 + 35t + 5 < 30$

$-16t^2 + 35t - 25 < 0$

$a = -16, b = 35, c = -25$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(-25)}}{2(-16)}$$

$$t = \frac{-35 \pm \sqrt{1225 - 1600}}{-32}$$

$$t = \frac{-35 \pm \sqrt{-375}}{-32}$$

The equation that corresponds to the inequality has no real roots. The balloon is always less than 30 feet above the ground.

Name _____ Date _____

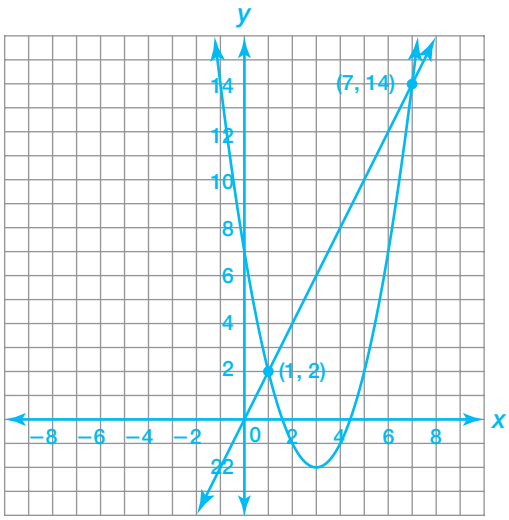
You Must Have a System

Systems of Quadratic Equations

Problem Set

Solve each system of equations algebraically. Then verify each solution graphically.

1. $\begin{cases} y = x^2 - 6x + 7 \\ y = 2x \end{cases}$



$2x = x^2 - 6x + 7$

$0 = x^2 - 8x + 7$

$0 = (x - 7)(x - 1)$

$x - 7 = 0$ or $x - 1 = 0$

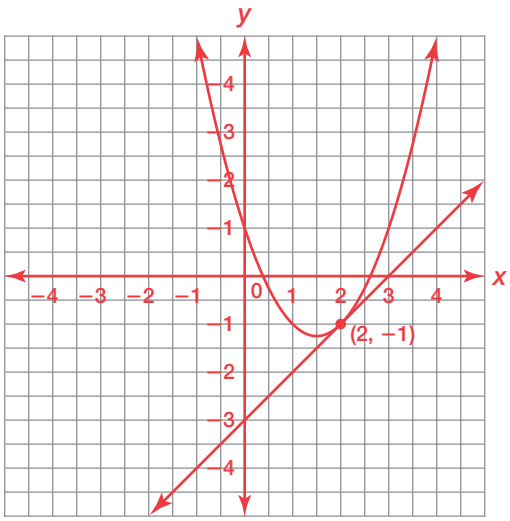
$x = 7$ $\quad$ $x = 1$

$y = 2(7)$ $\quad$ $y = 2(1)$

$y = 14$ $\quad$ $y = 2$

The system has two solutions: (7, 14) and (1, 2).

2. $\begin{cases} y = x^2 - 3x + 1 \\ y = x - 3 \end{cases}$



$x - 3 = x^2 - 3x + 1$

$0 = x^2 - 4x + 4$

$0 = (x - 2)(x - 2)$

$x - 2 = 0$

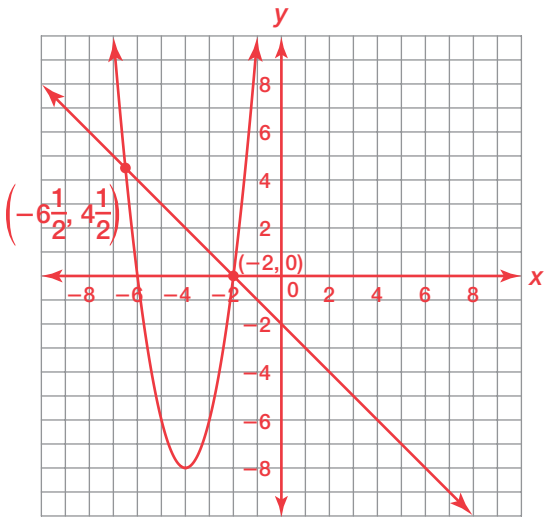
$x = 2$

$y = (2) - 3$

$y = -1$

The system has one solution: (2, −1).

3. $\begin{cases} y = 2x^2 + 16x + 24 \\ y = -x - 2 \end{cases}$



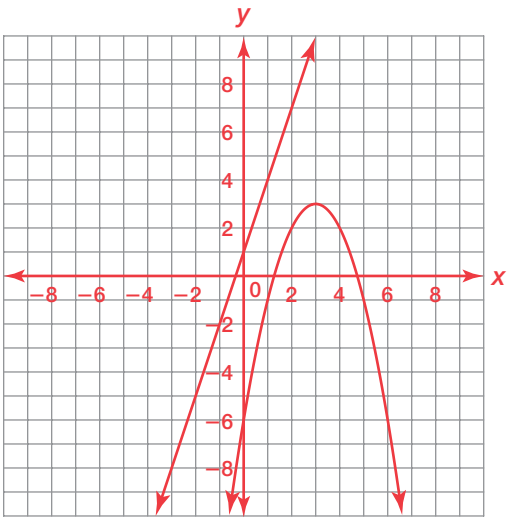
$$-x - 2 = 2x^2 + 16x + 24$$
$$0 = 2x^2 + 17x + 26$$
$$0 = (2x + 13)(x + 2)$$

$2x + 13 = 0$ or $x + 2 = 0$

$$2x = -13 \qquad x = -2$$
$$x = -\frac{13}{2} \qquad y = -(-2) - 2$$
$$y = 2 - 2$$
$$y = 0$$
$$y = -\left(\frac{-13}{2}\right) - 2$$
$$y = \frac{13}{2} - \frac{4}{2}$$
$$y = \frac{9}{2}$$

The system has two solutions:
$\left(-6\frac{1}{2}, 4\frac{1}{2}\right)$ and $(-2, 0)$.

4. $\begin{cases} y = -x^2 + 6x - 6 \\ y = 3x + 1 \end{cases}$



$$3x + 1 = -x^2 + 6x - 6$$
$$0 = -x^2 + 3x - 7$$
$$x = \frac{-(3) \pm \sqrt{3^2 - 4(-1)(-7)}}{2(-1)}$$
$$x = \frac{-(3) \pm \sqrt{9 - 28}}{-2}$$
$$x = \frac{-(3) \pm \sqrt{-19}}{-2}$$

The system has no real solution.

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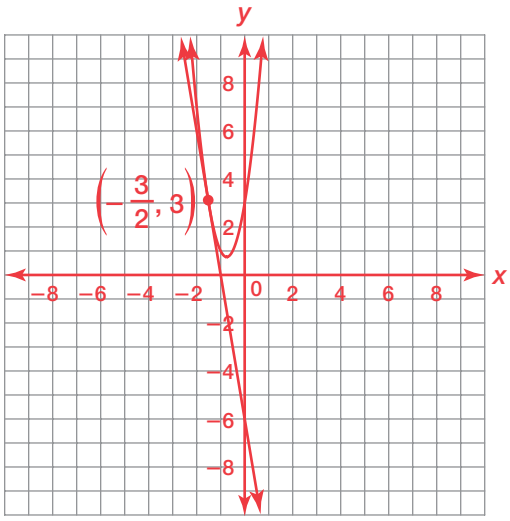
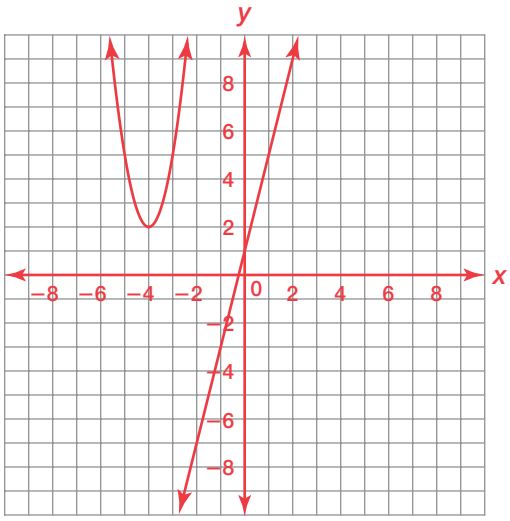
5. $\begin{cases} y = 4x^2 + 6x + 3 \\ y = -6x - 6 \end{cases}$



$$-6x - 6 = 4x^2 + 6x + 3$$
$$0 = 4x^2 + 12x + 9$$
$$0 = (2x + 3)(2x + 3)$$
$$2x + 3 = 0$$
$$2x = -3$$
$$x = -\frac{3}{2}$$
$$y = -6\left(\frac{-3}{2}\right) - 6$$
$$y = \frac{18}{2} - 6$$
$$y = 9 - 6$$
$$y = 3$$

The system has one solution: $\left(-\frac{3}{2}, 3\right)$.

6. $\begin{cases} y = 3x^2 + 24x + 50 \\ y = 4x + 1 \end{cases}$

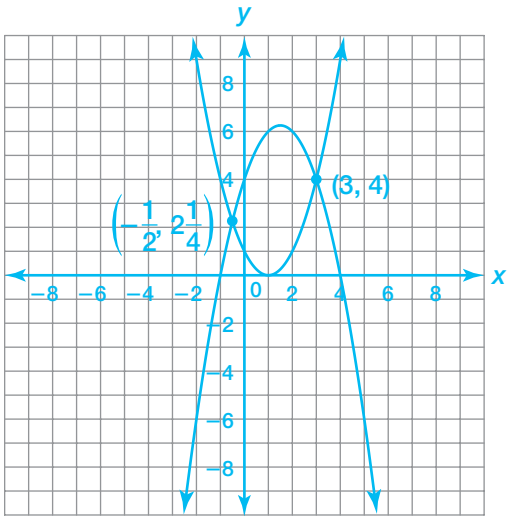


$$4x + 1 = 3x^2 + 24x + 50$$
$$0 = 3x^2 + 20x + 49$$
$$x = \frac{-20 \pm \sqrt{20^2 - 4(3)(49)}}{2(3)}$$
$$x = \frac{-20 \pm \sqrt{400 - 588}}{6}$$
$$x = \frac{-20 \pm \sqrt{-188}}{6}$$

The system has no real solutions.

Solve each system of equations algebraically. Then verify each solution graphically.

7. $\begin{cases} y = x^2 - 2x + 1 \\ y = -x^2 + 3x + 4 \end{cases}$



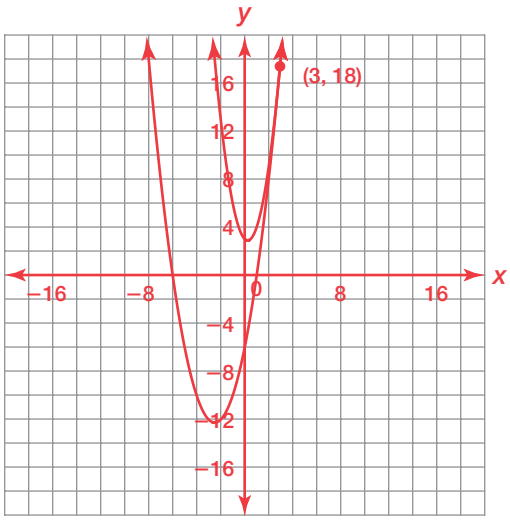
$$-x^2 + 3x + 4 = x^2 - 2x + 1$$
$$0 = 2x^2 - 5x - 3$$
$$0 = (2x + 1)(x - 3)$$
$$2x + 1 = 0 \quad \text{or}$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

$$y = \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$$
$$y = \frac{1}{4} + 1 + 1$$
$$y = \frac{9}{4} = 2\frac{1}{4}$$

$$x - 3 = 0$$
$$x = 3$$
$$y = (3)^2 - 2(3) + 1$$
$$y = 9 - 6 + 1$$
$$y = 4$$

The system has two solutions: $\left(-\frac{1}{2}, 2\frac{1}{4}\right)$ and (3, 4).

8. $\begin{cases} y = 2x^2 - x + 3 \\ y = x^2 + 5x - 6 \end{cases}$

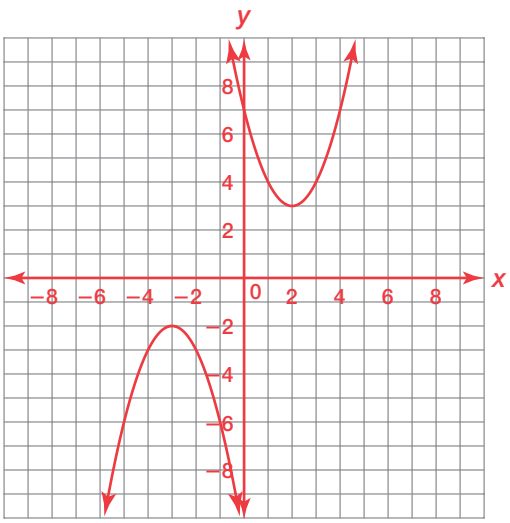


$$x^2 + 5x - 6 = 2x^2 - x + 3$$
$$0 = x^2 - 6x + 9$$
$$0 = (x - 3)(x - 3)$$
$$x - 3 = 0$$
$$x = 3$$
$$y = (3)^2 + 5(3) - 6$$
$$y = 9 + 15 - 6$$
$$y = 18$$

The system has one solution: (3, 18).

Name _____ Date _____

9. $\begin{cases} y = x^2 - 4x + 7 \\ y = -x^2 - 6x - 11 \end{cases}$



$$-x^2 - 6x - 11 = x^2 - 4x + 7$$

$$0 = 2x^2 + 2x + 18$$

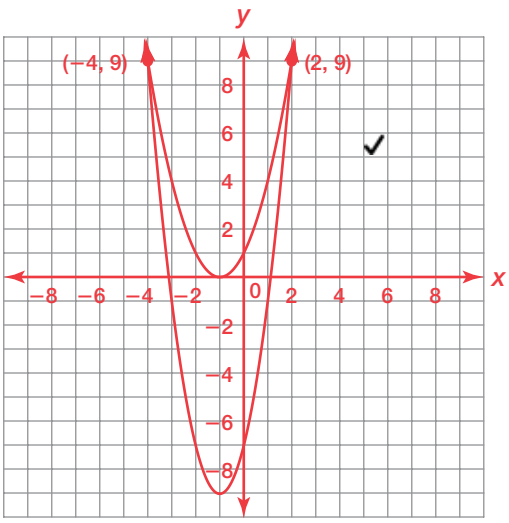
$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(18)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{4 - 144}}{4}$$

$$x = \frac{-2 \pm \sqrt{-140}}{4}$$

The system has no real solutions.

10. $\begin{cases} y = 2x^2 + 4x - 7 \\ y = x^2 + 2x + 1 \end{cases}$



$$x^2 + 2x + 1 = 2x^2 + 4x - 7$$

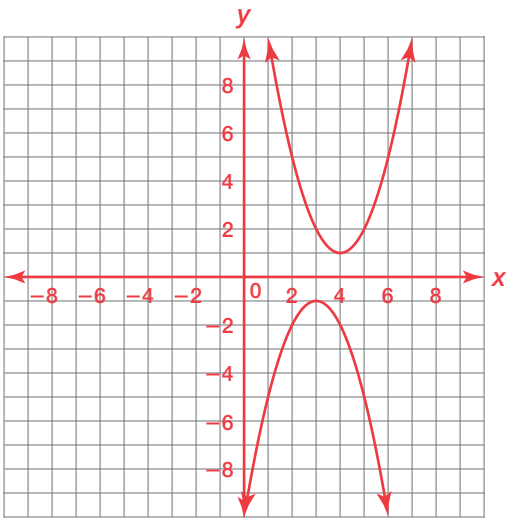
$$0 = x^2 + 2x - 8$$

$$0 = (x + 4)(x - 2)$$

$x + 4 = 0$	or	$x - 2 = 0$
$x = -4$		$x = 2$
$y = (-4)^2 + 2(-4) + 1$		$y = (2)^2 + 2(2) + 1$
$y = 16 - 8 + 1$		$y = 4 + 4 + 1$
$y = 9$		$y = 9$

The system has two solutions: (−4, 9) and (2, 9).

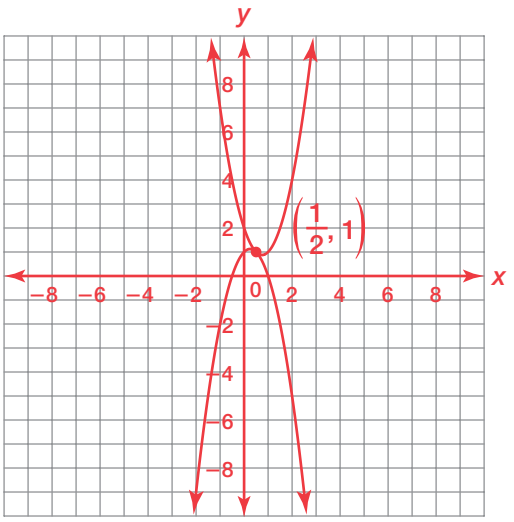
11. $\begin{cases} y = x^2 - 8x + 17 \\ y = -x^2 + 6x - 10 \end{cases}$



$$-x^2 + 6x - 10 = x^2 - 8x + 17$$
$$0 = 2x^2 - 14x + 27$$
$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(2)(27)}}{2(2)}$$
$$x = \frac{14 \pm \sqrt{196 - 216}}{4}$$
$$x = \frac{14 \pm \sqrt{-20}}{4}$$

The system has no real solutions.

12. $\begin{cases} y = 2x^2 - 3x + 2 \\ y = -2x^2 + x + 1 \end{cases}$



$$-2x^2 + x + 1 = 2x^2 - 3x + 2$$
$$0 = 4x^2 - 4x + 1$$
$$0 = (2x - 1)(2x - 1)$$
$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$
$$y = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 2$$
$$y = \frac{1}{2} - \frac{3}{2} + \frac{4}{2}$$
$$y = 1$$

The system has one solution: $\left(\frac{1}{2}, 1\right)$