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The Real Numbers . . . For Realsies!

The Numbers of the Real Number System

Vocabulary

Choose the word from the box that best completes each sentence.

counterexample	real numbers	whole numbers
natural numbers	closed	integers
rational numbers	Venn diagram	irrational numbers

1. The set of real numbers consists of the set of rational numbers and the set of irrational numbers.

2. The set of natural numbers consists of the numbers used to count objects.

3. A(n) Venn diagram uses circles to show how elements among sets of numbers or objects are related.

4. A set is said to be closed under an operation when you can perform the operation on any of the numbers in the set and the result is a number that is also in the same set.

5. The set of irrational numbers consists of all numbers that cannot be written as $\frac{a}{b}$, where a and b are integers.

6. The set of integers consists of the set of whole numbers and their opposites.

7. The set of rational numbers consists of all numbers that can be written as $\frac{a}{b}$, where a and b are integers, but b is not equal to 0.

8. The set of whole numbers consists of the set of natural numbers and the number 0.

9. To show that a set is not closed under an operation, one example that shows the result is not part of that set is needed. This example is called a counterexample.

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Problem Set

For each list of numbers, determine which are included in the given set.

1. The set of natural numbers:

   $10, -12, 0, 31, \frac{4}{5}, \sqrt{5}, -\frac{5}{3}, 1970$

   The numbers 10, 31, and 1970 are in the set of natural numbers.

2. The set of whole numbers:

   $-9, 18, 1, \frac{3}{4}, 0, 92, \sqrt{7}, 2096.5$

   The numbers 18, 1, 0, and 92 are in the set of whole numbers.

3. The set of integers:

   $54, \pi, \frac{2}{3}, -16, \sqrt{2}, 3.5, -\frac{7}{10}, -594$

   The numbers 54, −16, and −594 are in the set of integers.

4. The set of rational numbers:

   $8, -15, \frac{3}{8}, 0, \sqrt{3}, 9.5, -\frac{4}{5}, 857$

   The numbers 8, −15, $\frac{3}{8}$, 0, 9.5, $-\frac{4}{5}$, and 857 are in the set of rational numbers.

5. The set of irrational numbers:

   $-21, \frac{7}{8}, 3, \sqrt{2}, 2.5, 0, 99, \pi$

   The numbers $\sqrt{2}$ and π are in the set of irrational numbers.

6. The set of real numbers:

   $-18, 18, 1, \frac{2}{3}, \sqrt{3}, 1080, 5.4, -42$

   All the numbers in the set are in the set of real numbers.

Identify whether each given number set is closed or not closed under the operations addition, subtraction, multiplication, and division. Explain your reasoning.

7. the set of natural numbers

   The set of natural numbers is closed under addition and multiplication because when you add or multiply any two natural numbers, the sum or product is always a natural number.

   The set of natural numbers is not closed under subtraction because when you subtract a natural number from a natural number, the difference can be 0 or a negative integer.

   The set of natural numbers is not closed under division because when you divide a natural number by a natural number, the quotient can be a fraction.

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8. the set of integers

The set of integers is closed under addition, subtraction, and multiplication because when you add, subtract, or multiply any two integers, the sum, difference, or product is always an integer.

The set of integers is not closed under division because when you divide an integer by an integer, the quotient can be a decimal or fraction which are not integers.

9. the set of rational numbers

The set of rational numbers is closed under each of the four operations because when you add, subtract, multiply, or divide two rational numbers, the result is always a rational number. Note that division by zero is not defined to allow division to be closed for the set of rational numbers.

10. the set of real numbers

The set of real numbers is closed under each of the four operations because when you add, subtract, multiply, or divide two real numbers, the result is always a real number.

11. the set of whole numbers

The set of whole numbers is closed under addition and multiplication because when you add or multiply any whole numbers, the result is always a whole number.

The set of whole numbers is not closed under subtraction because when you subtract a whole number from a whole number, the difference can be a negative number.

The set of whole numbers is not closed under division because when you divide a whole number by a whole number, the quotient can be a fraction.

12. the set of irrational numbers

The set of irrational numbers is closed under each of the four operations because when you add, subtract, multiply, or divide two irrational numbers, the result is always an irrational number.

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Use the given equations to answer each question.

Equation A: $x + 4 = 10$	Equation B: $4x = 24$	Equation C: $x^2 = 16$
Equation D: $8 + x = 8$	Equation E: $x + 7 = 2$	Equation F: $5x = 0$
Equation G: $12x = 3$	Equation H: $x^2 = 3$	Equation J: $5x = 5$

13. Which equations could you solve if the only numbers you knew were natural numbers?
I could solve equations A, B, C, and J.

14. Which equations could you solve if the only numbers you knew were whole numbers?
I could solve equations A, B, C, D, F, and J.

15. Which equations could you solve if the only numbers you knew were integers?
I could solve equations A, B, C, D, E, F, and J.

16. Which equations could you solve if the only numbers you knew were rational numbers?
I could solve equations A, B, C, D, E, F, G, and J.

17. Which equations could you solve if the only numbers you knew were irrational numbers?
I could solve equation H.

18. Which equations could you solve if the only numbers you knew were real numbers?
I could solve equations A, B, C, D, E, F, G, H, and J.

Represent each given decimal as a fraction.

19. 0.4444 . . .

Let $x = 0.4444\ldots$

$$\begin{array}{r} 10x = 4.4444\ldots \\ -x = 0.4444\ldots \\ \hline 9x = 4 \end{array}$$

$$\frac{9x}{9} = \frac{4}{9}$$

$$x = \frac{4}{9}$$

The decimal 0.4444 . . . is equal to $\frac{4}{9}$.

20. 0.2525 . . .

Let $x = 0.2525\ldots$

$$\begin{array}{r} 100x = 25.2525\ldots \\ -x = 0.2525\ldots \\ \hline 99x = 25 \end{array}$$

$$\frac{99x}{99} = \frac{25}{99}$$

$$x = \frac{25}{99}$$

The decimal 0.2525 . . . is equal to $\frac{25}{99}$.

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21. 0.8181 . . .

Let $x = 0.8181\ldots$

$$\begin{aligned} 100x &= 81.8181\ldots \\ -x &= 0.8181\ldots \\ \hline 99x &= 81 \\ \frac{99x}{99} &= \frac{81}{99} \\ x &= \frac{9}{11} \end{aligned}$$

The decimal 0.8181 . . . is equal to $\frac{9}{11}$.

22. 0.581581 . . .

Let $x = 0.581581\ldots$

$$\begin{aligned} 1000x &= 581.581581\ldots \\ -x &= 0.581581\ldots \\ \hline 999x &= 581 \\ \frac{999x}{999} &= \frac{581}{999} \\ x &= \frac{581}{999} \end{aligned}$$

The decimal 0.581581 . . . is equal to $\frac{581}{999}$.

23. 0.3939 . . .

Let $x = 0.3939\ldots$

$$\begin{aligned} 100x &= 39.3939\ldots \\ -x &= 0.3939\ldots \\ \hline 99x &= 39 \\ \frac{99x}{99} &= \frac{39}{99} \\ x &= \frac{39}{99} \\ x &= \frac{13}{33} \end{aligned}$$

The decimal 0.3939 . . . is equal to $\frac{13}{33}$.

24. 0.0909 . . .

Let $x = 0.0909\ldots$

$$\begin{aligned} 100x &= 9.0909\ldots \\ -x &= 0.0909\ldots \\ \hline 99x &= 9 \\ \frac{99x}{99} &= \frac{9}{99} \\ x &= \frac{9}{99} \\ x &= \frac{1}{11} \end{aligned}$$

The decimal 0.0909 . . . is equal to $\frac{1}{11}$.

25. 0.1212 . . .

Let $x = 0.1212\ldots$

$$\begin{aligned} 100x &= 12.1212\ldots \\ -x &= 0.1212\ldots \\ \hline 99x &= 12 \\ \frac{99x}{99} &= \frac{12}{99} \\ x &= \frac{12}{99} \\ x &= \frac{4}{33} \end{aligned}$$

The decimal 0.1212 . . . is equal to $\frac{4}{33}$.

26. 0.7373 . . .

Let $x = 0.7373\ldots$

$$\begin{aligned} 100x &= 73.7373\ldots \\ -x &= 0.7373\ldots \\ \hline 99x &= 73 \\ \frac{99x}{99} &= \frac{73}{99} \\ x &= \frac{73}{99} \end{aligned}$$

The decimal 0.7373 . . . is equal to $\frac{73}{99}$.

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27. 0.4848 . . .

Let $x = 0.4848\ldots$

$$\begin{array}{r} 10x = 48.4848\ldots \\ -x = 0.4848\ldots \\ \hline 99x = 48 \end{array}$$

$$\frac{99x}{99} = \frac{48}{99}$$

$$x = \frac{48}{99}$$

$$x = \frac{16}{33}$$

The decimal 0.4848 . . . is equal to $\frac{16}{33}$.

28. 1.400400 . . .

Let $x = 1.400400\ldots$

$$\begin{array}{r} 1000x = 1400.400400\ldots \\ -x = 1.400400\ldots \\ \hline 999x = 1399 \end{array}$$

$$\frac{999x}{999} = \frac{1399}{999}$$

$$x = \frac{1399}{999}$$

The decimal 1.400400 . . . is equal to $\frac{1399}{999}$.

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Getting Real, and Knowing How . . .

Real Number Properties

Problem Set

Identify the property demonstrated by each given equation.

1. $4 \cdot (3 \cdot 8) = (4 \cdot 3) \cdot 8$
   Associative Property of Multiplication

2. $9(8 - 5) = 9(8) - 9(5)$
   Distributive Property of Multiplication over Subtraction

3. $20 + 0 = 20$
   Additive Identity

4. $10 + 7 + 15 = 10 + 15 + 7$
   Commutative Property of Addition

5. $5 \cdot \frac{1}{5} = 1$
   Multiplicative Inverse

6. $4 \cdot 9 = 9 \cdot 4$
   Commutative Property of Multiplication

7. $99(1) = 99$
   Multiplicative Identity

8. $12 + (3 + 8) = (12 + 3) + 8$
   Associative Property of Addition

9. $5(x + 2) = 5x + 5(2)$
   Distributive Property of Multiplication over Addition

10. $8 + (-8) = 0$
    Additive Inverse

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Each expression has been simplified one step at a time. Next to each step, identify the property, transformation, or simplification used in the step.

11. $8x + 4(3x + 7)$

$8x + (12x + 28)$	Distributive Property of Multiplication over Addition
$(8x + 12x) + 28$	Associative Property of Addition
$20x + 28$	Combine like terms

12. $14(2x + 2 + x)$

$14(2x + x + 2)$	Commutative Property of Addition
$14(3x + 2)$	Combine like terms
$42x + 28$	Distributive Property of Multiplication over Addition

13. $11(13 - 13 + x - 9)$

$11(0 + x - 9)$	Subtract
$11(x - 9)$	Additive Identity
$11x - 99$	Distributive Property of Multiplication over Subtraction

14. $7(x - 4) + 28$

$7x - 28 + 28$	Distributive Property of Multiplication over Subtraction
$7x - 0$	Combine like terms
$7x$	Additive Identity

15. $3(5 + 7x - 5)$

$3(7x + 5 - 5)$	Commutative Property of Addition
$3(7x + 0)$	Combine like terms
$3(7x)$	Additive Identity
$21x$	Multiply

16. $4(10x + 2) - 40x$

$40x + 8 - 40x$	Distributive Property of Multiplication over Addition
$8 + 40x - 40x$	Commutative Property of Addition
$8 + 0$	Combine like terms
8	Additive Identity

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Each equation has been solved one step at a time. Next to each step, identify the property, transformation, or simplification used in the step.

17. $x + 19 = 23$

$x + 19 + (-19) = 23 + (-19)$	Addition Property of Equality
$x + 0 = 23 + (-19)$	Combine like terms
$x = 23 + (-19)$	Additive Identity
$x = 4$	Combine like terms

18. $x - 7 = 34$

$x - 7 + 7 = 34 + 7$	Addition Property of Equality
$x + 0 = 34 + 7$	Combine like terms
$x = 34 + 7$	Additive Identity
$x = 41$	Combine like terms

19. $13x = 52$

$13x \cdot \frac{1}{13} = 52 \cdot \frac{1}{13}$	Multiplication Property of Equality
$x(13) \cdot \frac{1}{13} = 52 \cdot \frac{1}{13}$	Commutative Property of Multiplication
$x(1) = 52 \cdot \frac{1}{13}$	Multiply
$x = 52 \cdot \frac{1}{13}$	Multiplicative Identity
$x = 4$	Multiply

20. $\frac{1}{7}x = 9$

$x\left(\frac{1}{7}\right) = 9$	Commutative Property of Multiplication
$x\left(\frac{1}{7}\right) \cdot 7 = 9 \cdot 7$	Multiplication Property of Equality
$x \cdot 1 = 9 \cdot 7$	Multiply
$x = 9 \cdot 7$	Multiplicative Identity
$x = 63$	Multiply

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21. $3(3x - 8) + 2 = 32$

$9x - 24 + 2 = 32$	Distributive Property of Multiplication over Subtraction
$9x - 22 = 32$	Combine like terms
$9x - 22 + 22 = 32 + 22$	Addition Property of Equality
$9x + 0 = 32 + 22$	Combine like terms
$9x = 32 + 22$	Additive Identity
$9x = 54$	Combine like terms
$9x \cdot \frac{1}{9} = 54 \cdot \frac{1}{9}$	Multiplication Property of Equality
$x(9) \cdot \frac{1}{9} = 54 \cdot \frac{1}{9}$	Commutative Property of Multiplication
$x(1) = 54 \cdot \frac{1}{9}$	Multiply
$x = 54 \cdot \frac{1}{9}$	Multiplicative Identity
$x = 6$	Multiply

22. $5(3 + 6x) - 25 = 20$

$15 + 30x - 25 = 20$	Distributive Property of Multiplication over Addition
$30x + 15 - 25 = 20$	Commutative Property of Addition
$30x - 10 = 20$	Combine like terms
$30x - 10 + 10 = 20 + 10$	Addition Property of Equality
$30x + 0 = 20 + 10$	Combine like terms
$30x = 20 + 10$	Additive Identity
$30x = 30$	Combine like terms
$30x \cdot \frac{1}{30} = 30 \cdot \frac{1}{30}$	Multiplication Property of Equality
$x(30) \cdot \frac{1}{30} = 30 \cdot \frac{1}{30}$	Commutative Property of Multiplication
$x(1) = 1$	Multiply
$x = 1$	Multiplicative Identity

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23.

$7x + 1 = \frac{12x + 6}{2}$	
$7x + 1 = \frac{12x}{2} + \frac{6}{2}$	Distributive Property of Division over Addition
$7x + 1 = 6x + 3$	Simplify
$7x + 1 - 1 = 6x + 3 - 1$	Addition Property of Equality
$7x = 6x + 2$	Combine like terms
$7x - 6x = 6x + 2 - 6x$	Addition Property of Equality
$7x - 6x = 2 + 6x - 6x$	Commutative Property of Addition
$x = 2$	Combine like terms

24.

$\frac{4x - 8}{2} = 11 - 3x$	
$\frac{4x}{2} - \frac{8}{2} = 11 - 3x$	Distributive Property of Division over Subtraction
$2x - 4 = 11 - 3x$	Simplify
$2x - 4 + 4 = 11 - 3x + 4$	Addition Property of Equality
$2x - 4 + 4 = 11 + 4 - 3x$	Commutative Property of Addition
$2x = 15 - 3x$	Combine like terms
$2x + 3x = 15 - 3x + 3x$	Addition Property of Equality
$5x = 15$	Combine like terms
$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 15$	Multiplication Property of Equality
$x = 3$	Multiply

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25. $\frac{2x - 5}{3} = -2x + 17$

$3\left(\frac{2x - 5}{3}\right) = 3(-2x + 17)$	Multiplication Property of Equality
$2x - 5 = 3(-2x + 17)$	Simplify
$2x - 5 = -6x + 51$	Distributive Property of Multiplication over Addition
$2x - 5 + 5 = -6x + 51 + 5$	Addition Property of Equality
$2x = -6x + 56$	Combine like terms
$2x + 6x = -6x + 56 + 6x$	Addition Property of Equality
$2x + 6x = -6x + 6x + 56$	Commutative Property of Addition
$8x = 56$	Combine like terms
$\frac{1}{8} \cdot 8x = \frac{1}{8} \cdot 56$	Multiplication Property of Equality
$x = 7$	Multiply

26. $2x - 3 = \frac{(4x + 9)}{5}$

$5(2x - 3) = 5\left(\frac{4x + 9}{5}\right)$	Multiplication Property of Equality
$5(2x - 3) = 4x + 9$	Simplify
$10x - 15 = 4x + 9$	Distributive Property of Multiplication over Subtraction
$10x - 15 + 15 = 4x + 9 + 15$	Addition Property of Equality
$10x = 4x + 24$	Combine like terms
$10x - 4x = 4x + 24 - 4x$	Addition Property of Equality
$10x - 4x = 24 + 4x - 4x$	Commutative Property of Addition
$6x = 24$	Combine like terms
$\frac{1}{6} \cdot 6x = \frac{1}{6} \cdot 24$	Multiplication Property of Equality
$x = 4$	Multiply

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Imagine the Possibilities
Imaginary and Complex Numbers

Vocabulary

Match each definition to the corresponding term.

1. the set of all numbers written in the form $a + bi$, where a and b are real numbers
   e. complex numbers

2. the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0
   c. imaginary numbers

3. the term bi in a complex number written as $a + bi$
   g. imaginary part of a complex number

4. a number equal to $\sqrt{-1}$
   b. the number i

5. to raise a quantity to a power
   a. exponentiation

6. a number of the form bi where b is a real number and is not equal to 0
   d. pure imaginary number

7. the term a in a complex number written as $a + bi$
   f. real part of a complex number

a. exponentiation

b. the number i

c. imaginary numbers

d. pure imaginary number

e. complex numbers

f. real part of a complex number

g. imaginary part of a complex number

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Problem Set

Calculate each power of i.

1. i^{12}

$$\begin{aligned} i^{12} &= (i^4)^3 \\ &= (1)^3 \\ &= 1 \end{aligned}$$

2. i^{13}

$$\begin{aligned} i^{13} &= i^{12} \cdot i \\ &= (i^4)^3 \cdot i \\ &= (1)^3 \cdot i \\ &= 1 \cdot i \\ &= i \end{aligned}$$

3. i^{15}

$$\begin{aligned} i^{15} &= i^{12} \cdot i^3 \\ &= (i^4)^3 \cdot (-\sqrt{-1}) \\ &= (1)^3 \cdot (-i) \\ &= 1 \cdot (-i) \\ &= -i \end{aligned}$$

4. i^{20}

$$\begin{aligned} i^{20} &= (i^4)^5 \\ &= (1)^5 \\ &= 1 \end{aligned}$$

5. i^{22}

$$\begin{aligned} i^{22} &= i^{20} \cdot i^2 \\ &= (i^4)^5 \cdot (-1) \\ &= (1)^5 \cdot (-1) \\ &= 1 \cdot (-1) \\ &= -1 \end{aligned}$$

6. i^{25}

$$\begin{aligned} i^{25} &= i^{24} \cdot i \\ &= (i^4)^6 \cdot i \\ &= (1)^6 \cdot i \\ &= 1 \cdot i \\ &= i \end{aligned}$$

7. i^{44}

$$\begin{aligned} i^{44} &= (i^4)^{11} \\ &= (1)^{11} \\ &= 1 \end{aligned}$$

8. i^{46}

$$\begin{aligned} i^{46} &= i^{44} \cdot i^2 \\ &= (i^4)^{11} \cdot (-1) \\ &= (1)^{11} \cdot (-1) \\ &= 1 \cdot (-1) \\ &= -1 \end{aligned}$$

9. i^{84}

$$\begin{aligned} i^{84} &= (i^4)^{21} \\ &= (1)^{21} \\ &= 1 \end{aligned}$$

10. i^{99}

$$\begin{aligned} i^{99} &= i^{96} \cdot i^3 \\ &= (i^4)^{24} \cdot (-\sqrt{-1}) \\ &= (1)^{24} \cdot (-i) \\ &= 1 \cdot (-i) \\ &= -i \end{aligned}$$

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Simplify each expression using i.

11. $\sqrt{-9}$

$\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1}$

$= 3i$

12. $\sqrt{-36}$

$\sqrt{-36} = \sqrt{36} \cdot \sqrt{-1}$

$= 6i$

13. $\sqrt{-20}$

$\sqrt{-20} = \sqrt{20} \cdot \sqrt{-1}$

$= \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{-1}$

$= 2\sqrt{5}i$

14. $3 + \sqrt{-18}$

$3 + \sqrt{-18} = 3 + \sqrt{18} \cdot \sqrt{-1}$

$= 3 + \sqrt{9} \cdot \sqrt{2} \cdot \sqrt{-1}$

$= 3 + 3\sqrt{2}i$

15. $9 - \sqrt{-64}$

$9 - \sqrt{-64} = 9 - \sqrt{64} \cdot \sqrt{-1}$

$= 9 - 8i$

16. $\frac{10 + \sqrt{-12}}{2}$

$\frac{10 + \sqrt{-12}}{2} = \frac{10 + \sqrt{12} \cdot \sqrt{-1}}{2}$

$= \frac{10 + \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{-1}}{2}$

$= \frac{10 + 2\sqrt{3}i}{2}$

$= \frac{10}{2} + \frac{2\sqrt{3}i}{2}$

$= 5 + \sqrt{3}i$

17. $\frac{8 - \sqrt{-32}}{4}$

$\frac{8 - \sqrt{-32}}{4} = \frac{8 - \sqrt{32} \cdot \sqrt{-1}}{4}$

$= \frac{8 - \sqrt{16} \cdot \sqrt{2} \cdot \sqrt{-1}}{4}$

$= \frac{8 - 4\sqrt{2}i}{4}$

$= \frac{8}{4} - \frac{4\sqrt{2}i}{4}$

$= 2 - \sqrt{2}i$

18. $\frac{16 + \sqrt{-48}}{2}$

$\frac{16 + \sqrt{-48}}{2} = \frac{16 + \sqrt{48} \cdot \sqrt{-1}}{2}$

$= \frac{16 + \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1}}{2}$

$= \frac{16 + 4\sqrt{3}i}{2}$

$= \frac{16}{2} + \frac{4\sqrt{3}i}{2}$

$= 8 + 2\sqrt{3}i$

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Simplify each algebraic expression.

19. $5xi - 2xi$

$5xi - 2xi = 3xi$

20. $10xi + 8i - 6xi - i$

$$\begin{aligned} 10xi + 8i - 6xi - i &= (10xi - 6xi) + (8i - i) \\ &= 4xi + 7i \end{aligned}$$

21. $5x + 10i - 2 + 3x - 2i - 7$

$$\begin{aligned} 5x + 10i - 2 + 3x - 2i - 7 &= (5x + 3x) + (10i - 2i) + (-2 - 7) \\ &= 8x + 8i - 9 \end{aligned}$$

22. $(x - i)^2$

$$\begin{aligned} (x - i)^2 &= (x - i)(x - i) \\ &= x^2 - xi - xi + i^2 \\ &= x^2 - 2xi + (-1) \\ &= x^2 - 2xi - 1 \end{aligned}$$

23. $(x - i)(x + 3i)$

$$\begin{aligned} (x - i)(x + 3i) &= x^2 + 3xi - xi - 3i^2 \\ &= x^2 + 2xi - 3(-1) \\ &= x^2 + 2xi + 3 \end{aligned}$$

24. $(4x + i)(2x - 2i)$

$$\begin{aligned} (4x + i)(2x - 2i) &= 8x^2 - 8xi + 2xi - 2i^2 \\ &= 8x^2 - 6xi - 2(-1) \\ &= 8x^2 - 6xi + 2 \end{aligned}$$

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Determine the real part and the imaginary part of each complex number.

25. 24

The real part is 24. The imaginary part is 0*i*.

26. $8i$

The real part is 0. The imaginary part is 8*i*.

27. $7 + 3i$

The real part is 7. The imaginary part is 3*i*.

28. $\sqrt{8}$

The real part is $\sqrt{8}$. The imaginary part is 0*i*.

29. $-35i$

The real part is 0. The imaginary part is $-35i$.

30. $14 - \sqrt{5}i$

The real part is 14. The imaginary part is $-\sqrt{5}i$.

31. 52

The real part is 52. The imaginary part is 0*i*.

32. $2.5 + 3\sqrt{2}i$

The real part is 2.5. The imaginary part is $3\sqrt{2}i$.

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Identify each given number using words from the box.

natural number	whole number	integer
rational number	irrational number	real number
imaginary number	complex number	

33. -25

integer, rational number, real number, complex number

34. $\sqrt{3}$

irrational number, real number, complex number

35. 9

natural number, whole number, integer, rational number, real number, complex number

36. $6 + 7i$

imaginary number, complex number

37. $\frac{2}{5}$

rational number, real number, complex number

38. $14i$

imaginary number, complex number

39. $0.\overline{18}$

rational number, real number, complex number

40. $\sqrt{-4}$

imaginary number, complex number

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15

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Now It's Getting Complex . . . But It's Really Not Difficult!
Complex Number Operations

Vocabulary

Match each term to its corresponding definition.

1. the number *i*
   E.
2. imaginary number
   A.
3. pure imaginary number
   H.
4. complex number
   F.
5. real part of a complex number
   B.
6. imaginary part of a complex number
   I.
7. complex conjugates
   D.
8. monomial
   J.
9. binomial
   C.
10. trinomial
    G.

A. a number in the form $a + bi$ where a and b are real numbers and b is not equal to 0

B. term a of a number written in the form $a + bi$

C. a polynomial with two terms

D. pairs of numbers of the form $a + bi$ and $a - bi$

E. a number such that its square equals -1

F. a number in the form $a + bi$ where a and b are real numbers

G. a polynomial with three terms

H. a number of the form bi where b is not equal to 0

I. term bi of a number written in the form $a + bi$

J. a polynomial with one term

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Problem Set

Calculate each power of i.

1. i^{48}

$$i^{48} = (i^4)^{12} = 1^{12} = 1$$

2. i^{361}

$$i^{361} = (i^4)^{90}(i^1) = 1^{90}(\sqrt{-1}) = \sqrt{-1} = i$$

3. i^{55}

$$i^{55} = (i^4)^{13}(i^2)(i^1) = 1^{13}(-1)(\sqrt{-1}) = -\sqrt{-1} = -i$$

4. i^{1000}

$$i^{1000} = (i^4)^{250} = 1^{250} = 1$$

5. i^{-22}

$$i^{-22} = \frac{1}{(i^4)^5 \cdot (i^2)} = \frac{1}{1^5(-1)} = -1$$

6. i^{-7}

$$i^{-7} = \frac{1}{(i^2)^3 \cdot i} = \frac{1}{(-1)^3 \cdot i} = \frac{1}{-i} = \frac{1}{i^{-1}} = i = \sqrt{-1}$$

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Rewrite each expression using i.

7. $\sqrt{-72}$

$\sqrt{-72} = \sqrt{36(2)(-1)}$
$= 6\sqrt{2}i$

8. $\sqrt{-49} + \sqrt{-23}$

$\sqrt{-49} + \sqrt{-23} = \sqrt{49(-1)} + \sqrt{23(-1)}$
$= 7i + \sqrt{23}i$

9. $38 - \sqrt{-200} + \sqrt{121}$

$38 - \sqrt{-200} + \sqrt{121} = 38 - \sqrt{100(2)(-1)} + 11$
$= 49 - 10\sqrt{2}i$

10. $\sqrt{-45} + 21$

$\sqrt{-45} + 21 = \sqrt{9(5)(-1)} + 21$
$= 3\sqrt{5}i + 21$

11. $\dfrac{\sqrt{-48} - 12}{4}$

$\dfrac{\sqrt{-48} - 12}{4} = \dfrac{\sqrt{16(3)(-1)}}{4} - \dfrac{12}{4}$

$= \dfrac{4\sqrt{3}i}{4} - 3$

$= \sqrt{3}i - 3$

12. $\dfrac{1 + \sqrt{4} - \sqrt{-15}}{3}$

$\dfrac{1 + \sqrt{4} - \sqrt{-15}}{3} = \dfrac{1}{3} + \dfrac{2}{3} - \dfrac{\sqrt{15(-1)}}{3}$

$= 1 - \dfrac{\sqrt{15}i}{3}$

13. $-\sqrt{-28} + \dfrac{\sqrt{21}}{3} - \dfrac{\sqrt{12}}{6}$

$-\sqrt{-28} + \dfrac{\sqrt{21}}{3} - \dfrac{\sqrt{12}}{6} = -\sqrt{4(7)(-1)} + \dfrac{2\sqrt{21}}{6} - \dfrac{\sqrt{4(3)}}{6}$

$= -2\sqrt{7}i + \dfrac{2\sqrt{21} - 2\sqrt{3}}{6}$

$= -2\sqrt{7}i + \dfrac{2(\sqrt{21} - \sqrt{3})}{6}$

$= -2\sqrt{7}i + \dfrac{(\sqrt{21} - \sqrt{3})}{3}$

14. $\dfrac{\sqrt{-75} + \sqrt{80}}{10}$

$\dfrac{\sqrt{-75} + \sqrt{80}}{10} = \dfrac{\sqrt{25(3)(-1)}}{10} + \dfrac{\sqrt{16(5)}}{10}$

$= \dfrac{5\sqrt{3}i}{10} + \dfrac{4\sqrt{5}}{10}$

$= \dfrac{1}{2}\sqrt{3}i + \dfrac{2}{5}\sqrt{5}$

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Simplify each expression.

15. $(2 + 5i) - (7 - 9i)$

$(2 + 5i) - (7 - 9i) = 2 + 5i - 7 + 9i$

$= (2 - 7) + (5i + 9i)$

$= -5 + 14i$

16. $-6 + 8i - 1 - 11i + 13$

$-6 + 8i - 1 - 11i - 13$

$= (-6 - 1 + 13) + (8i - 11i)$

$= 6 - 3i$

17. $-(4i - 1 + 3i) + (6i - 10 + 17)$

$-(4i - 1 + 3i) + (6i - 10 + 17)$

$= (-4i - 3i + 6i) + (1 - 10 + 17)$

$= -i + 8$

18. $22i + 13 - (7i + 3 + 12i) + 16i - 25$

$22i + 13 - (7i + 3 + 12i) + 16i - 25$

$= (22i - 7i - 12i + 16i) + (13 - 3 - 25)$

$= 19i - 15$

19. $9 + 3i(7 - 2i)$

$9 + 3i(7 - 2i) = 9 + 21i - 6i^2$

$= 9 + 21i - 6(-1)$

$= (9 + 6) + 21i$

$= 15 + 21i$

20. $(4 - 5i)(8 + i)$

$(4 - 5i)(8 + i) = 32 + 4i - 40i - 5i^2$

$= 32 + 4i - 40i - 5(-1)$

$= (32 + 5) + (4i - 40i)$

$= 37 - 36i$

21. $-0.5(14i - 6) - 4i(0.75 - 3i)$

$-0.5(14i - 6) - 4i(0.75 - 3i)$

$= -7i + 3 - 3i + 12i^2$

$= (-7i - 3i) + 3 + 12(-1)$

$= (-7i - 3i) + (3 - 12)$

$= -10i - 9$

22. $\left(\frac{1}{2}i - \frac{3}{4}\right) \times \left(\frac{1}{8} - \frac{3}{4}i\right)$

$\left(\frac{1}{2}i - \frac{3}{4}\right) \times \left(\frac{1}{8} - \frac{3}{4}i\right) = \frac{1}{16}i - \frac{3}{8}i^2 - \frac{3}{32} + \frac{9}{16}i$

$= \left(\frac{1}{16}i + \frac{9}{16}i\right) - \frac{3}{8}(-1) - \frac{3}{32}$

$= \frac{10}{16}i + \frac{3}{8} - \frac{3}{32}$

$= \frac{5}{8}i + \frac{9}{32}$

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Determine each product.

23. $(3 + i)(3 - i)$

$$\begin{aligned}(3 + i)(3 - i) &= 9 - 3i + 3i - i^2\\ &= 9 - (-1)\\ &= 10\end{aligned}$$

24. $(4i - 5)(4i + 5)$

$$\begin{aligned}(4i - 5)(4i + 5) &= 16i^2 + 20i - 20i - 25\\ &= 16(-1) - 25\\ &= -41\end{aligned}$$

25. $(7 - 2i)(7 + 2i)$

$$\begin{aligned}(7 - 2i)(7 + 2i) &= 49 + 14i - 14i - 4i^2\\ &= 49 - 4(-1)\\ &= 53\end{aligned}$$

26. $\left(\frac{1}{3} + 3i\right)\left(\frac{1}{3} - 3i\right)$

$$\begin{aligned}\left(\frac{1}{3} + 3i\right)\left(\frac{1}{3} - 3i\right) &= \frac{1}{9} - i + i - 9i^2\\ &= \frac{1}{9} - 9(-1)\\ &= 9\frac{1}{9}\end{aligned}$$

27. $(0.1 + 0.6i)(0.1 - 0.6i)$

$$\begin{aligned}&(0.1 + 0.6i)(0.1 - 0.6i)\\ &= 0.01 - 0.06i + 0.06i - 0.36i^2\\ &= 0.01 - 0.36(-1)\\ &= 0.01 + 0.36\\ &= 0.37\end{aligned}$$

28. $-2[(-i - 8)(-i + 8)]$

$$\begin{aligned}-2[(-i - 8)(-i + 8)] &= -2(i^2 - 8i + 8i - 64)\\ &= -2(-1 - 64)\\ &= 130\end{aligned}$$

Identify each expression as a monomial, binomial, or trinomial. Explain your reasoning.

29. $4xi + 7x$

The expression is a monomial because it can be rewritten as $(4i + 7)x$, which shows one x term.

30. $-3x + 5 - 8xi + 1$

The expression is a binomial because it can be rewritten as $(-3 - 8i)x + 6$, which shows one x term and one constant term.

31. $6x^2i + 3x^2$

The expression is a monomial because it can be rewritten as $(6i + 3)x^2$, which shows one x^2 term.

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32. $8i - x^3 + 7x^2i$

The expression is a trinomial because it shows one x^3 term, one x^2 term, and one constant term.

33. $xi - x + i + 2 - 4i$

The expression is binomial because it can be rewritten as $(i - 1)x + (2 - 3i)$, which shows one x term and one constant term.

34. $-3x^3i - x^2 + 6x^3 + 9i - 1$

The expression is a trinomial because it can be rewritten as $(-3i + 6)x^3 + (-1)x^2 + (9i - 1)$, which shows one x^3 term, one x^2 term, and one constant term.

Simplify each expression, if possible.

35. $(x - 6i)^2$

$$\begin{aligned}(x - 6i)^2 &= x^2 - 6xi - 6xi + 36i^2 \\ &= x^2 - 12xi + 36(-1) \\ &= x^2 - 12xi - 36\end{aligned}$$

36. $(2 + 5xi)(7 - xi)$

$$\begin{aligned}(2 + 5xi)(7 - xi) &= 14 - 2xi + 35xi - 5x^2i^2 \\ &= 14 + 33xi - 5x^2(-1) \\ &= 14 + 33xi + 5x^2\end{aligned}$$

37. $3xi - 4yi$

This expression cannot be simplified.

38. $(2xi - 9)(3x + 5i)$

$$\begin{aligned}(2xi - 9)(3x + 5i) &= 6x^2i + 10xi^2 - 27x - 45i \\ &= 6x^2i + (-1)10x - 27x - 45i \\ &= 6x^2i - 37x - 45i\end{aligned}$$

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39. $(x + 4i)(x - 4i)(x + 4i)$

$$\begin{aligned}(x + 4i)(x - 4i)(x + 4i) &= (x^2 - 16i^2)(x + 4i)\\ &= (x^2 - 16(-1))(x + 4i)\\ &= (x^2 + 16)(x + 4i)\\ &= (x^3 + 4x^2i + 16x + 64i\end{aligned}$$

40. $(3i - 2xi)(3i - 2xi) + (2i - 3xi)(2 - 3xi)$

$$\begin{aligned}(3i - 2xi)(3i - 2xi) + (2i - 3xi)(2 - 3xi) &= (9i^2 - 6xi^2 - 6xi^2 + 4x^2i^2) + (4i - 6xi^2 - 6xi + 9x^2i^2)\\ &= (9(-1) - 12x(-1) + 4x^2(-1)) + (4i - 6x(-1) - 6xi + 9x^2(-1))\\ &= (-9 + 12x - 4x^2) + (4i + 6x - 6xi - 9x^2)\\ &= (-4x^2 - 9x^2) + (12x + 6x - 6xi) + (-9 + 4i)\\ &= -13x^2 + 18x - 6xi - 9 + 4i\end{aligned}$$

For each complex number, write its conjugate.

41. $7 + 2i$

$7 - 2i$

42. $3 + 5i$

$3 - 5i$

43. $8i$

$-8i$

44. $-7i$

$7i$

45. $2 - 11i$

$2 + 11i$

46. $9 - 4i$

$9 + 4i$

47. $-13 - 6i$

$-13 + 6i$

48. $-21 + 4i$

$-21 - 4i$

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Calculate each quotient.

49. $\frac{3 + 4i}{5 + 6i}$

$$\frac{3 + 4i}{5 + 6i} = \frac{3 + 4i}{5 + 6i} \cdot \frac{5 - 6i}{5 - 6i} = \frac{15 - 18i + 20i - 24i^2}{25 - 30i + 30i - 36i^2}$$

$$= \frac{15 + 2i + 24}{25 + 36} = \frac{39 + 2i}{61} = \frac{39}{61} + \frac{2}{61}i$$

50. $\frac{8 + 7i}{2 + i}$

$$\frac{8 + 7i}{2 + i} = \frac{8 + 7i}{2 + i} \cdot \frac{2 - i}{2 - i} = \frac{16 - 8i + 14i - 7i^2}{4 + 2i - 2i - i^2}$$

$$= \frac{16 + 6i + 7}{4 + 1} = \frac{23 + 6i}{5} = \frac{23}{5} + \frac{6}{5}i$$

51. $\frac{-6 + 2i}{2 - 3i}$

$$\frac{-6 + 2i}{2 - 3i} = \frac{-6 + 2i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{-12 - 18i + 4i + 6i^2}{4 + 6i - 6i - 9i^2}$$

$$= \frac{-12 - 14i - 6}{4 + 9} = \frac{-18 - 14i}{13} = -\frac{18}{13} - \frac{14}{13}i$$

52. $\frac{-1 + 5i}{1 - 4i}$

$$\frac{-1 + 5i}{1 - 4i} = \frac{-1 + 5i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i} = \frac{-1 - 4i + 5i + 20i^2}{1 + 4i - 4i - 16i^2}$$

$$= \frac{-1 + i - 20}{1 + 16} = \frac{-21 + i}{17} = -\frac{21}{17} + \frac{1}{17}i$$

53. $\frac{6 - 3i}{2 - i}$

$$\frac{6 - 3i}{2 - i} = \frac{6 - 3i}{2 - i} \cdot \frac{2 + i}{2 + i} = \frac{12 - 6i - 6i - 3i^2}{4 + 2i - 2i - i^2}$$

$$= \frac{12 + 3}{4 + 1} = \frac{15}{5} = 3$$

54. $\frac{4 - 2i}{-1 + 2i}$

$$\frac{4 - 2i}{-1 + 2i} = \frac{4 - 2i}{-1 + 2i} \cdot \frac{-1 - 2i}{-1 - 2i} = \frac{-4 - 8i + 2i + 4i^2}{1 + 2i - 2i - 4i^2}$$

$$= \frac{-4 - 6i - 4}{1 + 4} = \frac{-8 - 6i}{5} = -\frac{8}{5} - \frac{6}{5}i$$

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It's Not Complex—Just Its Solutions Are Complex!

Solving Quadratics with Complex Solutions

Vocabulary

Define the term in your own words.

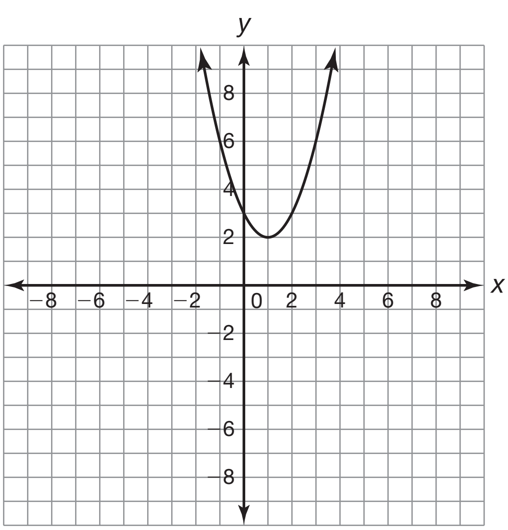
1. imaginary roots (imaginary zeros)

   Imaginary roots, or imaginary zeros, are the solutions of functions and equations which have imaginary solutions.

Problem Set

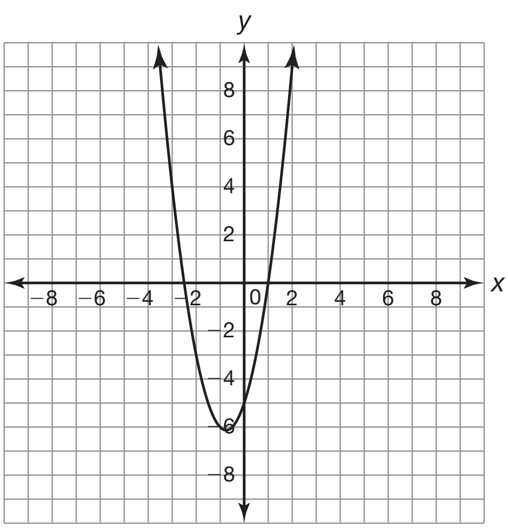
For each given graph, determine the number of roots for the quadratic equation then determine whether the roots are real or imaginary.

1.



The equation has two imaginary roots.

2.



The equation has two real roots.

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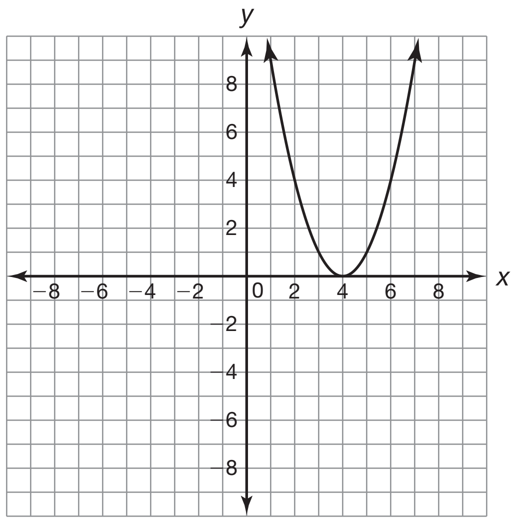
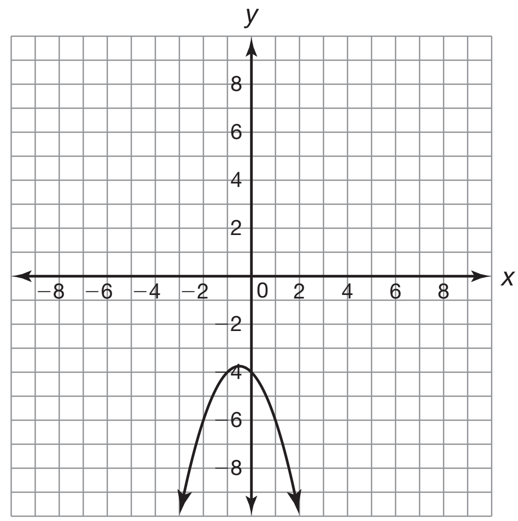
3.



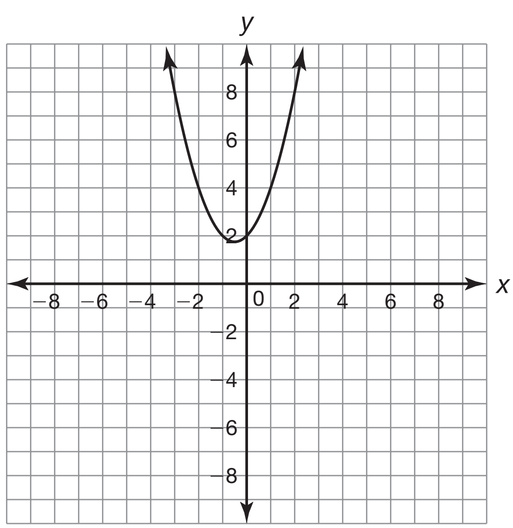
The equation has a one real root.

4.



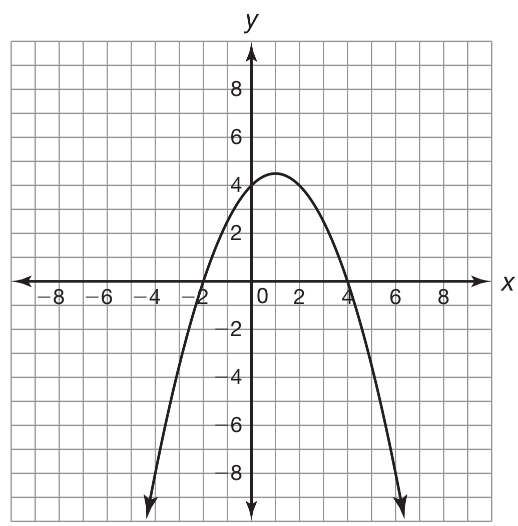
The equation has two imaginary roots.

5.



The equation has two imaginary roots.

6.



The equation has two real roots.

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Determine the zeros of each given function.

7. $f(x) = 4x^2 + 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{0 \pm \sqrt{-16}}{8}$$

$$x = \frac{0 \pm 4i}{8}$$

$$x = \pm \frac{1}{2}i$$

The zeros are $\frac{1}{2}i$ and $-\frac{1}{2}i$.

8. $f(x) = x^2 + 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{-36}}{2}$$

$$x = \frac{0 \pm 6i}{2}$$

$$x = \pm 3i$$

The zeros are $3i$ and $-3i$.

9. $f(x) = x^2 + 2x + 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$$x = -1 \pm 2i$$

The zeros are $-1 + 2i$ and $-1 - 2i$.

10. $f(x) = -x^2 + 4x - 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-6)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 24}}{-2}$$

$$x = \frac{-4 \pm \sqrt{-8}}{-2}$$

$$x = \frac{-4 \pm 2\sqrt{2}i}{-2}$$

$$x = 2 \pm \sqrt{2}i$$

The zeros are $2 + \sqrt{2}i$ and $2 - \sqrt{2}i$.

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11. $f(x) = x^2 + 2x + 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2}$$

$$x = \frac{-2 \pm \sqrt{-4}}{2}$$

$$x = \frac{-2 \pm 2i}{2}$$

$$x = -1 \pm i$$

The zeros are $-1 + i$ and $-1 - i$.

12. $f(x) = -x^2 + 6x - 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-25)}}{2(-1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 100}}{-2}$$

$$x = \frac{-6 \pm \sqrt{-64}}{-2}$$

$$x = \frac{-6 \pm 8i}{-2}$$

$$x = 3 \pm 4i$$

The zeros are $3 + 4i$ and $3 - 4i$.

13. $f(x) = x^2 - 4x + 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 36}}{2}$$

$$x = \frac{4 \pm \sqrt{-20}}{2}$$

$$x = \frac{4 \pm 2\sqrt{5}i}{2}$$

$$x = 2 \pm \sqrt{5}i$$

The zeros are $2 + \sqrt{5}i$ and $2 - \sqrt{5}i$.

14. $f(x) = 2x^2 + 8x + 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(10)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 - 80}}{4}$$

$$x = \frac{-8 \pm \sqrt{-16}}{4}$$

$$x = \frac{-8 \pm 4i}{4}$$

$$x = -2 \pm i$$

The zeros are $-2 + i$ and $-2 - i$.

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