Name ____________________ Date ________

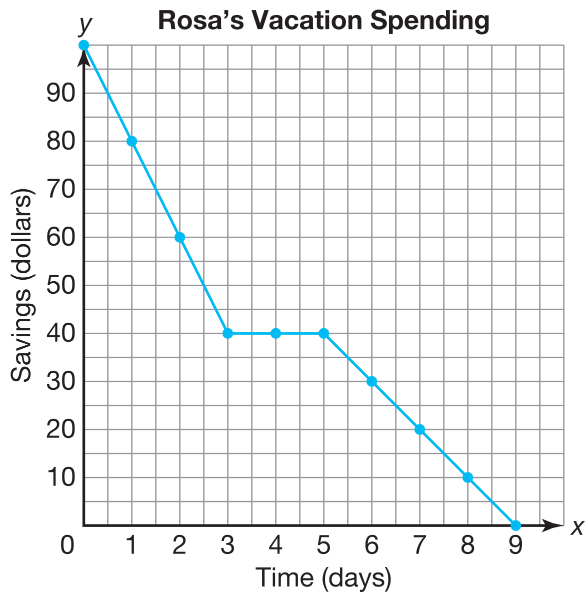
I Graph in Pieces
Linear Piecewise Functions

Problem Set

Complete each table. Then, sketch a graph that represents the problem situation.

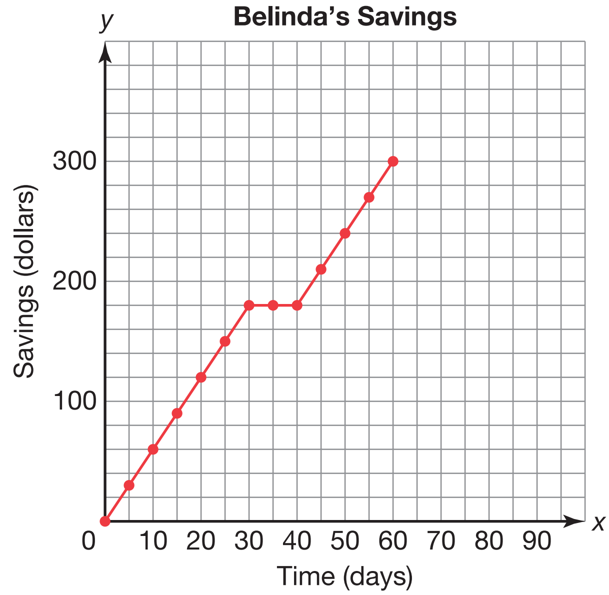
1. Rosa saved $100 to spend on vacation. For the first 3 days of her vacation she spent $20 each day. Then for the next 2 days, she spent nothing. After those 5 days, she spent $10 each day until her savings were depleted.

Time (days)	Savings (dollars)
0	100
1	80
2	60
3	40
4	40
5	40
6	30
7	20
8	10
9	0



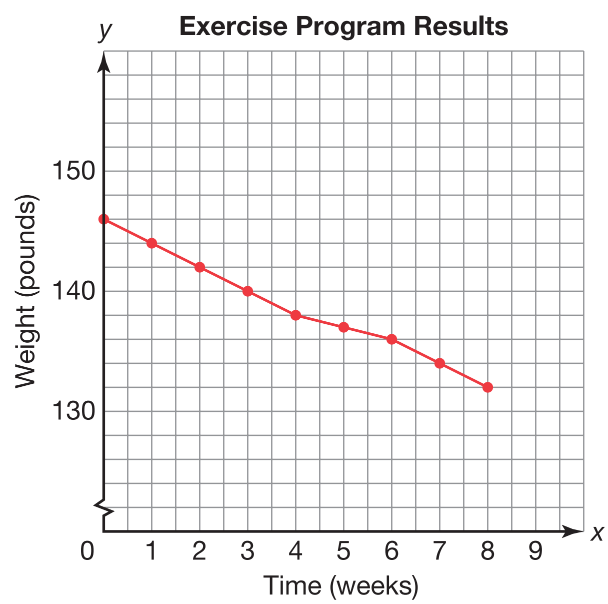
2. Belinda is saving money for a new snowboard. She earns \$30 every 5 days she tutors. After 30 days, she takes a break from tutoring and does not earn any money for 10 days. After those 10 days she begins tutoring again and earns \$30 every 5 days until she reaches her goal of \$300.

Time (days)	Savings (dollars)
0	0
5	30
10	60
15	90
20	120
25	150
30	180
35	180
40	180
45	210
50	240
55	270
60	300



3. Shanise starts a new exercise program to lose weight. Before starting the program her weight is 146 pounds. She loses 2 pounds each of the first 4 weeks of her new program. Then, for the next 2 weeks she loses 1 pound per week. After those 2 weeks she adds swimming to her program and again loses 2 pounds per week for the next 2 weeks until she reaches her goal.

Time (weeks)	Weight (pounds)
0	146
1	144
2	142
3	140
4	138
5	137
6	136
7	134
8	132



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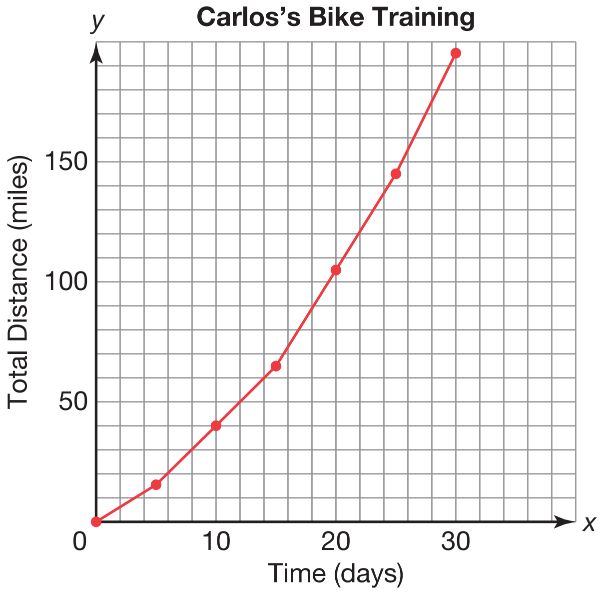
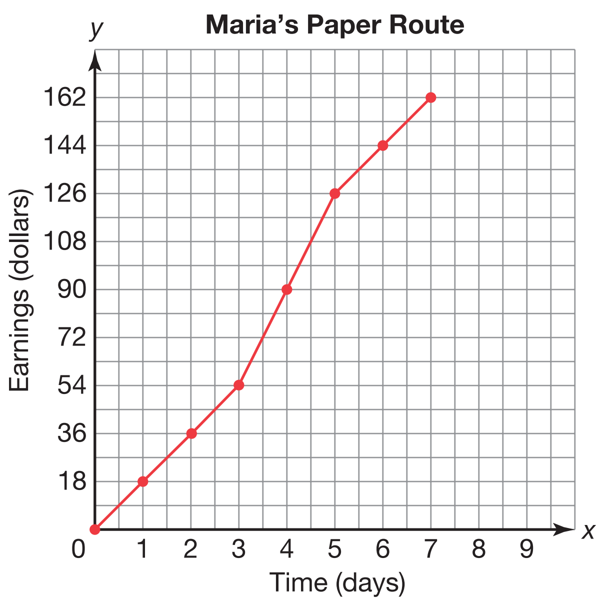
4. Carlos is training for a bike race in 30 days. For the first 5 days of his training he bikes 3 miles each day. For the next 10 days he bikes 5 miles each day. For the next 10 days of his training he bikes 8 miles each day. For the last 5 days of his training he bikes 10 miles a day.

Time (days)	Total Distance (miles)
0	0
5	15
10	40
15	65
20	105
25	145
30	195



5. Maria earns money delivering newspapers each morning. For the first 3 days she earns $18 each day. For the next 2 days, she takes on an additional route to cover a coworker who is out sick and earns $36 each day. For the next 2 days she returns to her original route and earns $18 each day.

Time (days)	Earnings (dollars)
0	0
1	18
2	36
3	54
4	90
5	126
6	144
7	162



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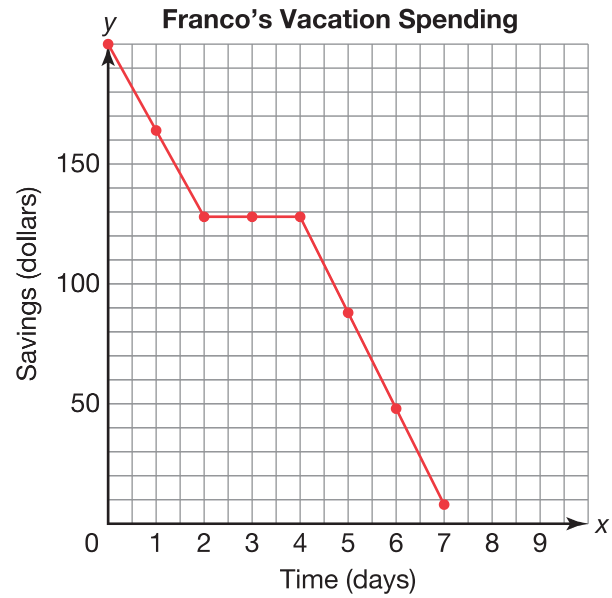
6. Franco saved \$200 to spend at an amusement park while on vacation. For the first 2 days of his vacation he spent \$36 each day. Then for the next 2 days, he spent nothing. After those 4 days, he stayed 3 more days and spent \$40 each day.

Time (days)	Savings (dollars)
0	200
1	164
2	128
3	128
4	128
5	88
6	48
7	8



Write a piecewise function to represent the data shown in each table.

7.

x	$f(x)$
0	60
1	55
2	50
3	45
4	45
5	45
6	45
7	43
8	41
9	39

From 0 to 3:

The y-intercept is 60.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{55 - 60}{1 - 0} = -\frac{5}{1} = -5$$

$$y = mx + b$$
$$y = -5x + 60$$

From 3 to 6:

The slope is 0.

$$y = 45$$

From 6 to 9:

A point is (6, 45).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{41 - 43}{8 - 7} = \frac{-2}{1} = -2$$

$$y - y_1 = m(x - x_1)$$
$$y - 45 = -2(x - 6)$$
$$y - 45 = -2x + 12$$
$$y = -2x + 57$$

$$f(x) = \begin{cases} -5x + 60, & 0 \le x \le 3 \\ 45, & 3 < x \le 6 \\ -2x + 57, & 6 < x \le 9 \end{cases}$$

Name ______________________________ Date __________

8.

x	$f(x)$
0	0
2	3
4	6
6	9
8	12
10	12
12	12
14	18
16	24
18	30

From 0 to 8:

The y-intercept is 0

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{2 - 0} = \frac{3}{2}$$

$$y = mx + b$$
$$y = \frac{3}{2}x$$

From 8 to 12:

The slope is 0. $y = 12$

From 12 to 18:

A point is (12, 12).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 12}{14 - 12} = \frac{6}{2} = 3$$

$$y - y_1 = m(x - x_1)$$
$$y - 12 = 3(x - 12)$$
$$y - 12 = 3x - 36$$
$$y = 3x - 24$$

$$f(x) = \begin{cases} \frac{3}{2}x, & 0 \le x \le 8 \\ 12, & 8 < x \le 12 \\ 3x - 24, & 12 < x \le 18 \end{cases}$$

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9.

x	$f(x)$
0	80
1	75
2	70
3	65
4	64
5	63
6	62
7	61
8	60
9	58

$$f(x) = \begin{cases} -5x + 80, & 0 \le x \le 3 \\ -x + 68, & 3 < x \le 8 \\ -2x + 76, & 8 < x \le 9 \end{cases}$$

From 0 to 3:

The y-intercept is 80.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{75 - 80}{1 - 0} = \frac{-5}{1} = -5$$

$$y = mx + b$$
$$y = -5x + 80$$

From 3 to 8:

A point is (3, 65).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{64 - 65}{4 - 3} = \frac{-1}{1} = -1$$

$$y - y_1 = m(x - x_1)$$
$$y - 65 = -1(x - 3)$$
$$y - 65 = -x + 3$$
$$y = -x + 68$$

From 8 to 9:

A point is (8, 60).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{58 - 60}{9 - 8} = \frac{-2}{1} = -2$$

$$y - y_1 = m(x - x_1)$$
$$y - 60 = -2(x - 8)$$
$$y - 60 = -2x + 16$$
$$y = -2x + 76$$

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10.

x	$f(x)$
0	4
3	6
6	8
9	12
12	16
15	20
18	22
21	24
24	26
27	28

$$f(x) = \begin{cases} \frac{2}{3}x + 4, & 0 \le x \le 6 \\ \frac{4}{3}x, & 6 < x \le 15 \\ \frac{2}{3}x + 10, & 15 < x \le 27 \end{cases}$$

From 0 to 6:

The y-intercept is 4.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4}{3 - 0} = \frac{2}{3}$$

$$y = mx + b$$
$$y = \frac{2}{3}x + 4$$

From 6 to 15:

A point is (6, 8).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 8}{9 - 6} = \frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - 8 = \frac{4}{3}(x - 6)$$
$$3(y - 8) = 4(x - 6)$$
$$3y - 24 = 4x - 24$$
$$3y = 4x$$
$$y = \frac{4}{3}x$$

From 15 to 27:

A point is (15, 20).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{22 - 20}{18 - 15} = \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - 20 = \frac{2}{3}(x - 15)$$
$$3(y - 20) = 2(x - 15)$$
$$3y - 60 = 2x - 30$$
$$3y = 2x + 30$$
$$y = \frac{2}{3}x + 10$$

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11.

x	$f(x)$
0	100
2	80
4	60
6	60
8	60
10	60
12	54
14	48
16	42
18	36

From 0 to 4:

The y-intercept is 100.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{80 - 100}{2 - 0} = \frac{-20}{2} = -10$$

$$y = mx + b$$
$$y = -10x + 100$$

From 4 to 10:

The slope is 0. $y = 60$

From 10 to 18:

A point is (10, 60).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{54 - 60}{12 - 10} = \frac{-6}{2} = -3$$

$$y - y_1 = m(x - x_1)$$
$$y - 60 = -3(x - 10)$$
$$y - 60 = -3x + 30$$
$$y = -3x + 90$$

$$f(x) = \begin{cases} -10x + 100, & 0 \le x \le 4 \\ 60, & 4 < x \le 10 \\ -3x + 90, & 10 < x \le 18 \end{cases}$$

12.

x	$f(x)$
0	74
1	70
2	66
3	62
4	64
5	66
6	68
7	60
8	52
9	44

From 0 to 3:

The y-intercept is 74.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 74}{1 - 0} = \frac{-4}{1} = -4$$

$$y = mx + b$$
$$y = -4x + 74$$

From 3 to 6:

A point is (3, 62).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{64 - 62}{4 - 3} = \frac{2}{1} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 62 = 2(x - 3)$$
$$y - 62 = 2x - 6$$
$$y = 2x + 56$$

From 6 to 9:

A point is (6, 68).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 68}{7 - 6} = \frac{-8}{1} = -8$$

$$y - y_1 = m(x - x_1)$$
$$y - 68 = -8(x - 6)$$
$$y - 68 = -8x + 48$$
$$y = -8x + 116$$

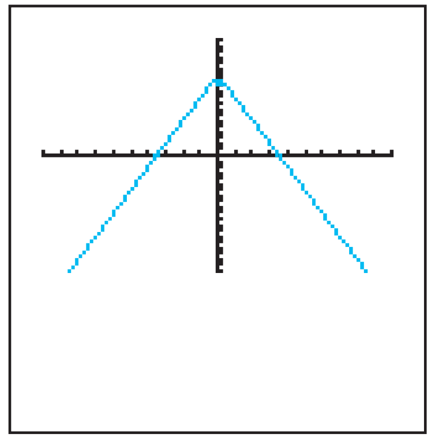
$$f(x) = \begin{cases} -4x + 74, & 0 \le x \le 3 \\ 2x + 56, & 3 < x \le 6 \\ -8x + 116, & 6 < x \le 9 \end{cases}$$

Name ____ Date ____________

Sketch a graph that represents the data shown in each table. Write a function to represent the graph.

13.

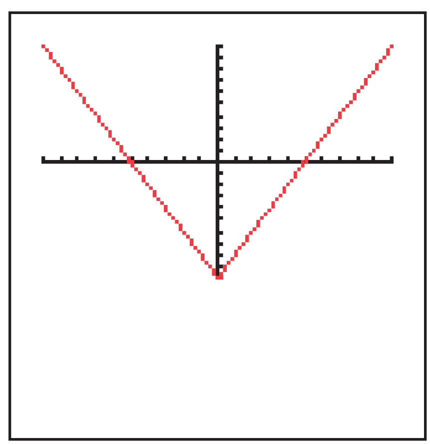
x	$f(x)$
−3	2
−2	6
−1	10
0	14
1	10
2	6
3	2



$f(x) = -|4x| + 14$

14.

x	$f(x)$
−3	−4
−2	−6
−1	−8
0	−10
1	−8
2	−6
3	−4



$f(x) = |2x| - 10$

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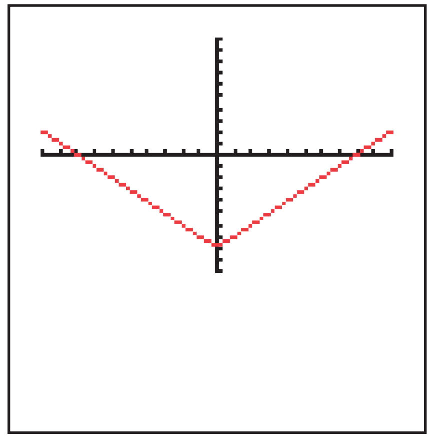
15.

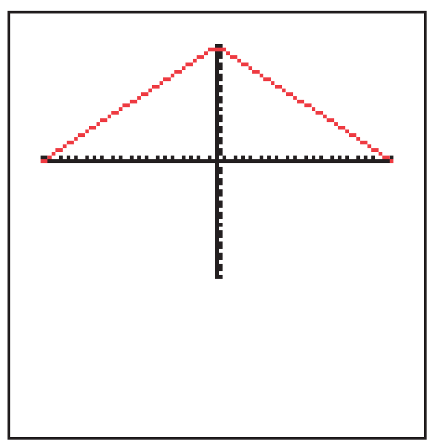
x	$f(x)$
-6	-2
-4	-4
-2	-6
0	-8
2	-6
4	-4
6	-2



$f(x) = |x| - 8$

16.

x	$f(x)$
-15	5
-10	10
-5	15
0	20
5	15
10	10
15	5



$f(x) = -|x| + 20$

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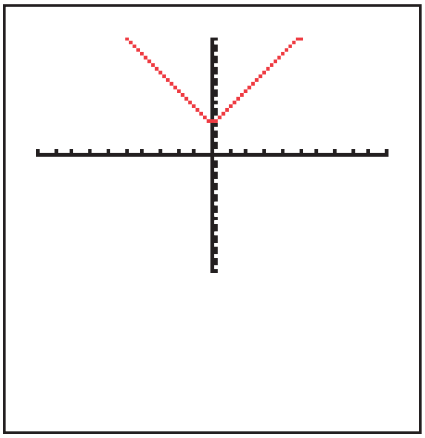
17.

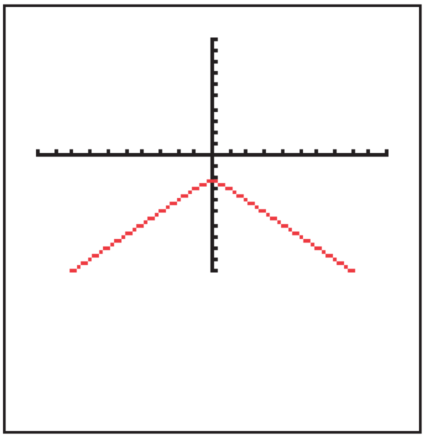
x	$f(x)$
−3	14
−2	11
−1	8
0	5
1	8
2	11
3	14



$f(x) = |3x| + 5$

18.

x	$f(x)$
−3	−5
−2	−4
−1	−3
0	−2
1	−3
2	−4
3	−5



$f(x) = -|x| - 2$

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Step By Step
Step Functions

16

Vocabulary

For each function, write a definition and give an example.

1. step function

A step function is a piecewise function whose pieces are disjoint constant functions.

$$f(x) = \begin{cases} 1, & 0 < x \le 1 \\ 2, & 1 < x \le 2 \\ 3, & 2 < x \le 3 \end{cases}$$

2. greatest integer (floor) function

The greatest integer function, also known as the floor function, is defined as the greatest integer less than or equal to x.

$f(x) = \lfloor x \rfloor$

3. least integer (ceiling) function

The least integer function, also known as the ceiling function, is defined as the least integer greater than or equal to x.

$f(x) = \lceil x \rceil$

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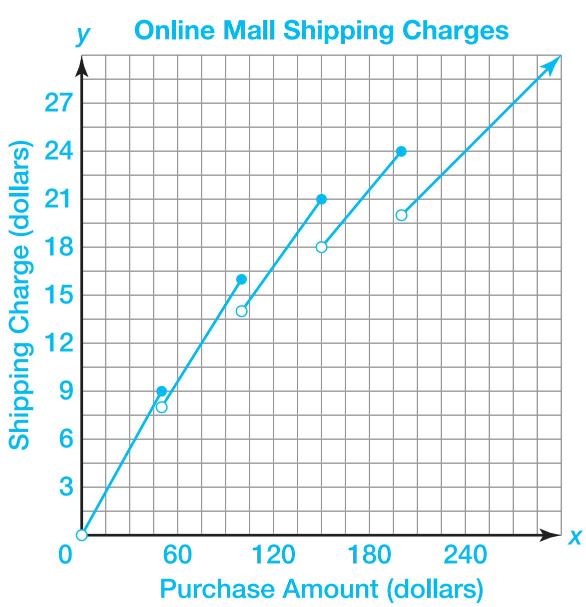
Problem Set

Write and graph a function to represent each problem situation.

1. An online mall assigns shipping charges based on the total value of merchandise purchased. The shipping charges are as follows:

- 18% for purchases more than \$0 and up to and including \$50,
- 16% for purchases more than \$50 and up to and including \$100,
- 14% for purchases more than \$100 and up to and including \$150,
- 12% for purchases more than \$150 and up to and including \$200, and
- 10% for purchases more than \$200.

$$f(x) = \begin{cases} 0.18x, & 0 < x \leq 50 \\ 0.16x, & 50 < x \leq 100 \\ 0.14x, & 100 < x \leq 150 \\ 0.12x, & 150 < x \leq 200 \\ 0.10x, & 200 < x \end{cases}$$



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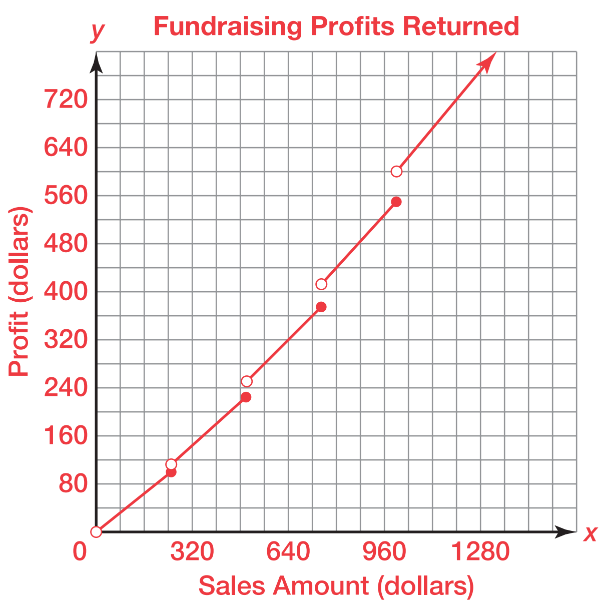
2. A fundraising company bases the profit returned to organizations on the total value of products sold. The profit returned is calculated as follows:

- 40% for sales more than \$0 and up to and including \$250,
- 45% for sales more than \$250 and up to and including \$500,
- 50% for sales more than \$500 and up to and including \$750,
- 55% for sales more than \$750 and up to and including \$1000, and
- 60% for sales more than \$1000.

$$f(x) = \begin{cases} 0.40x, & 0 < x \le 250 \\ 0.45x, & 250 < x \le 500 \\ 0.50x, & 500 < x \le 750 \\ 0.55x, & 750 < x \le 1000 \\ 0.60x, & 1000 < x \end{cases}$$



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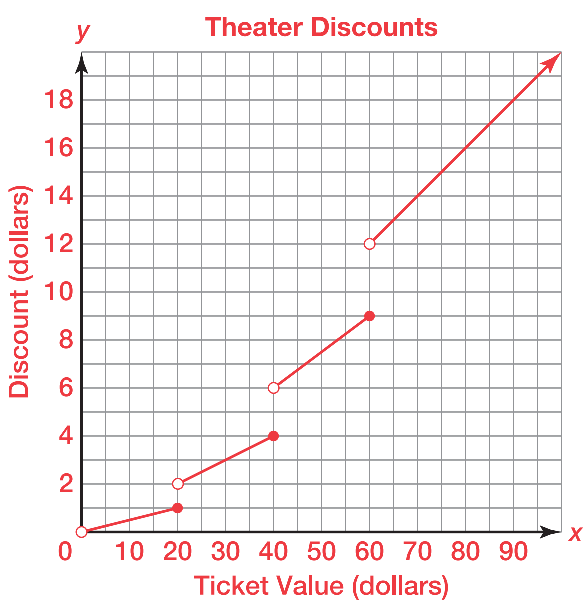
3. A theater company offers discounts based on the value of tickets purchased. The discounts are as follows:

- 5% for purchases more than \$0 and up to and including \$20,
- 10% for purchases more than \$20 and up to and including \$40,
- 15% for purchases more than \$40 and up to and including \$60, and
- 20% for purchases more than \$60.

$$f(x) = \begin{cases} 0.05x, & 0 < x \le 20 \\ 0.10x, & 20 < x \le 40 \\ 0.15x, & 40 < x \le 60 \\ 0.20x, & 60 < x \end{cases}$$



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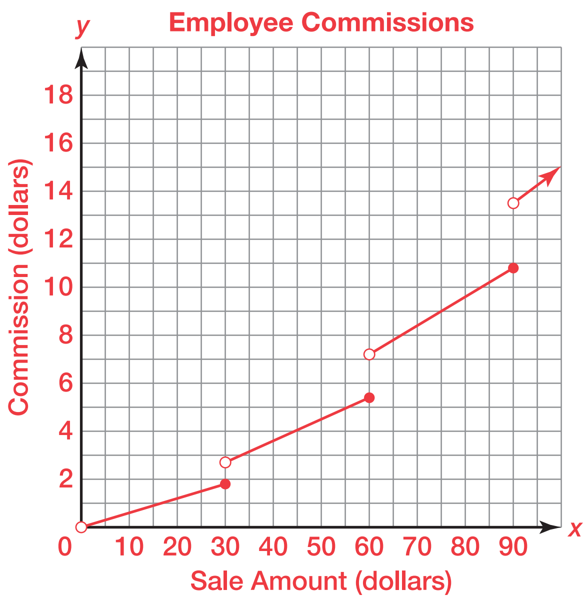
4. A small clothing company pays its employees a commission based on the total value of clothing sold. The commission for each sale is calculated as follows:

- 6% for sales more than \$0 and up to and including \$30,
- 9% for sales more than \$30 and up to and including \$60,
- 12% for sales more than \$60 and up to and including \$90, and
- 15% for sales more than \$90.

$$f(x) = \begin{cases} 0.06x, & 0 < x \le 30 \\ 0.09x, & 30 < x \le 60 \\ 0.12x, & 60 < x \le 90 \\ 0.15x, & 90 < x \end{cases}$$



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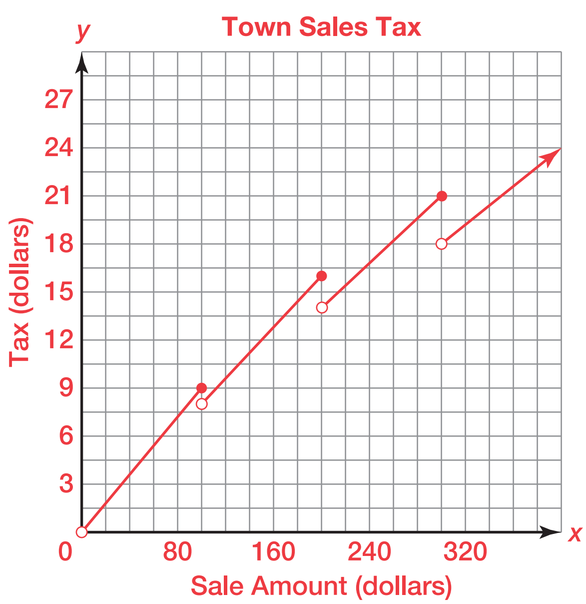
5. A small town calculates its local sales tax rate based on the total value of the goods sold. The local sales tax is calculated as follows:

- 9% for sales more than \$0 and up to and including \$100,
- 8% for sales more than \$100 and up to and including \$200,
- 7% for sales more than \$200 and up to and including \$300, and
- 6% for sales more than \$300.

$$f(x) = \begin{cases} 0.09x, & 0 < x \le 100 \\ 0.08x, & 100 < x \le 200 \\ 0.07x, & 200 < x \le 300 \\ 0.06x, & 300 < x \end{cases}$$



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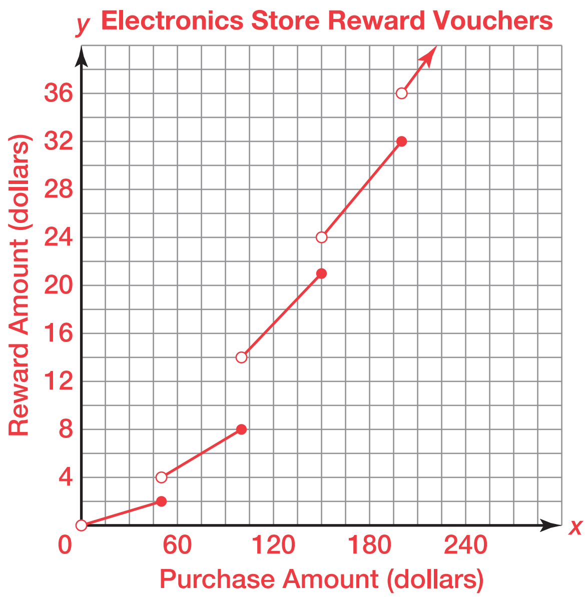
6. An electronics store rewards customers with in-store reward vouchers. The value of the reward vouchers are based on the total value of merchandise purchased. The rewards are calculated as follows:

- 4% for purchases more than $0 and up to and including $50,
- 8% for purchases more than $50 and up to and including $100,
- 14% for purchases more than $100 and up to and including $150,
- 16% for purchases more than $150 and up to and including $200, and
- 18% for purchases more than $200.

$$f(x) = \begin{cases} 0.04x, & 0 < x \le 50 \\ 0.08x, & 50 < x \le 100 \\ 0.14x, & 100 < x \le 150 \\ 0.16x, & 150 < x \le 200 \\ 0.18x, & 200 < x \end{cases}$$



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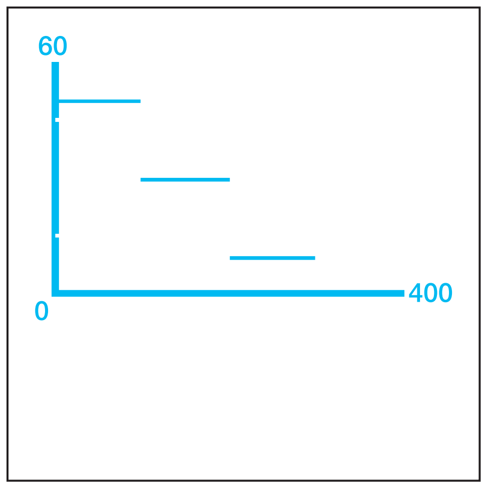
Write a function to represent each problem situation. Then use your graphing calculator to graph the function.

7. To encourage quality and minimize defects, a manufacturer pays his employees a bonus based on the value of defective merchandise produced. The fewer defective merchandise produced, the greater the employee's bonus. The bonuses are calculated as follows:

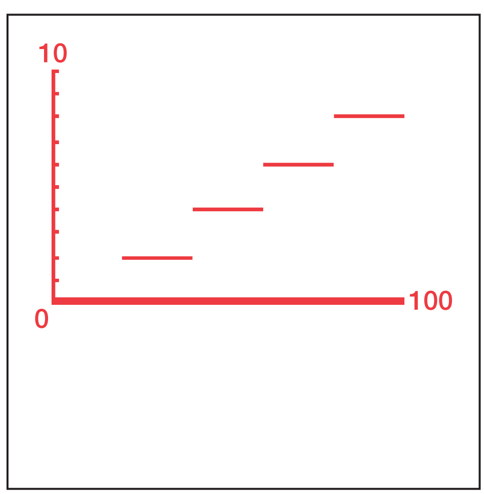
- \$50 for more than \$0 and up to and including \$100 of defective merchandise,
- \$30 for more than \$100 and up to and including \$200 of defective merchandise,
- \$10 for more than \$200 and up to and including \$300 of defective merchandise, and
- \$0 for more than \$300 of defective merchandise.

$$f(x) = \begin{cases} 50, & 0 < x \le 100 \\ 30, & 100 < x \le 200 \\ 10, & 200 < x \le 300 \\ 0, & 300 < x \end{cases}$$



8. A jewelry store offers reward coupons to its customers. A \$2 reward coupon is awarded for each \$20 spent. Write a function that represents the value of reward coupons awarded for up to \$100 spent.

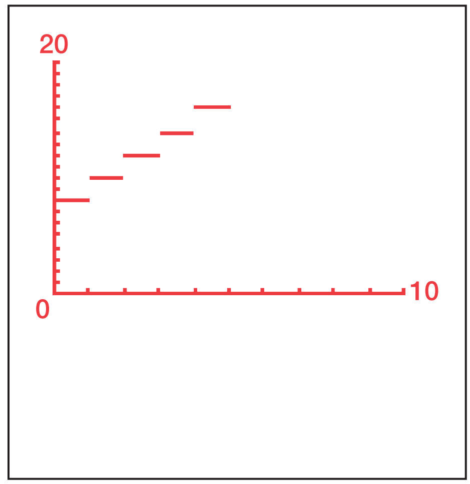
$$f(x) = \begin{cases} 0, & 0 < x < 20 \\ 2, & 20 \le x < 40 \\ 4, & 40 \le x < 60 \\ 6, & 60 \le x < 80 \\ 8, & 80 \le x < 100 \\ 10, & x = 100 \end{cases}$$



Name ____ Date ________________

9. A kids bounce house charges \$8 for the first hour and \$2 for each additional hour of playtime. Write a function that represents the charges for up to 5 hours of playtime.

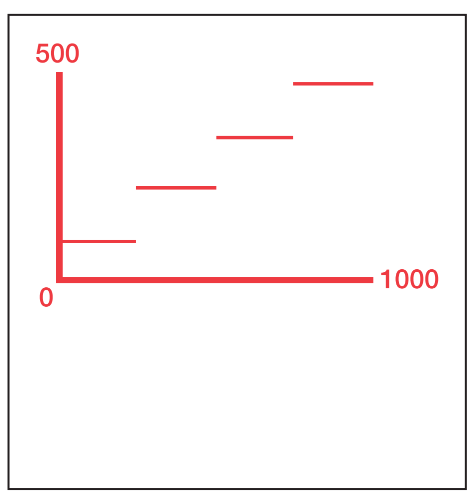
$$f(x) = \begin{cases} 8, & 0 < x \leq 1 \\ 10, & 1 < x \leq 2 \\ 12, & 2 < x \leq 3 \\ 14, & 3 < x \leq 4 \\ 16, & 4 < x \leq 5 \end{cases}$$



10. A fundraising company bases the profit returned to organizations on the total value of products sold. The profit returned is calculated as follows:

- \$100 for sales more than \$0 and up to and including \$250,
- \$225 for sales more than \$250 and up to and including \$500,
- \$350 for sales more than \$500 and up to and including \$750, and
- \$475 for sales more than \$750 and up to and including \$1000.

$$f(x) = \begin{cases} 100, & 0 < x \leq 250 \\ 225, & 250 < x \leq 500 \\ 350, & 500 < x \leq 750 \\ 475, & 750 < x \leq 1000 \end{cases}$$



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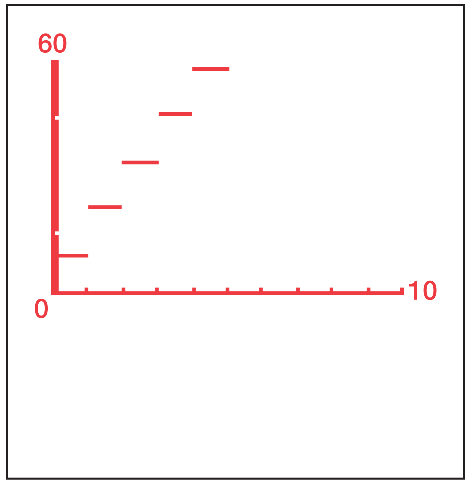
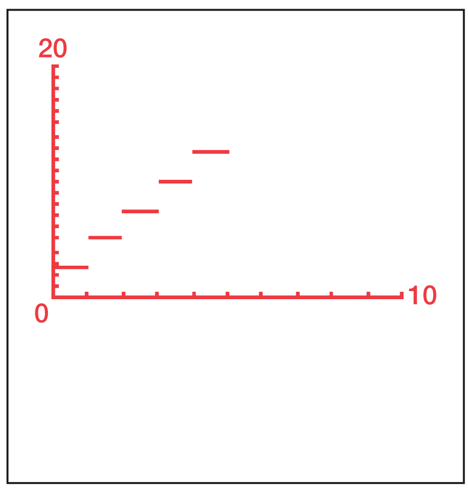
11. An ice rink charges hockey teams for ice time to practice. The ice rink charges $10 for the first hour and $12 for each additional hour. Write a function that represents the charges for up to 5 hours.

$$f(x) = \begin{cases} 10, & 0 < x \leq 1 \\ 22, & 1 < x \leq 2 \\ 34, & 2 < x \leq 3 \\ 46, & 3 < x \leq 4 \\ 58, & 4 < x \leq 5 \end{cases}$$



12. Ava is participating in a walk for charity. Her sponsors agree to donate $2.50 plus $2.50 for each whole mile that she walks. Write a function that represents the donation amount for up to 5 miles.

$$f(x) = \begin{cases} 2.50, & 0 \leq x < 1 \\ 5.00, & 1 \leq x < 2 \\ 7.50, & 2 \leq x < 3 \\ 10.00, & 3 \leq x < 4 \\ 12.50, & 4 \leq x < 5 \\ 15, & x = 5 \end{cases}$$



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Evaluate.

13. $\lfloor 4.5 \rfloor$

$\lfloor 4.5 \rfloor = 4$

14. $\lceil 5.1 \rceil$

$\lceil 5.1 \rceil = 6$

15. $\lceil -8.3 \rceil$

$\lceil -8.3 \rceil = -8$

16. $\lfloor -3.2 \rfloor$

$\lfloor -3.2 \rfloor = -4$

17. $\lfloor 7.3 \rfloor$

$\lfloor 7.3 \rfloor = 7$

18. $\lfloor 0.6 \rfloor$

$\lfloor 0.6 \rfloor = 0$

19. $\lceil 7.9 \rceil$

$\lceil 7.9 \rceil = 8$

20. $\lceil 0.03 \rceil$

$\lceil 0.03 \rceil = 1$

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The Inverse Undoes What a Function Does
Inverses of Linear Functions

Vocabulary

Match each definition with the corresponding term.

1. inverse operation
   b.
2. inverse function
   c.
3. composition of functions
   a.

a. the combination of functions such that the output from one function becomes the input for the next function

b. working backwards or retracing steps to return to an original value or position

c. a function which takes an output value, performs some operation(s) on the value, and arrives back at the original function's input value

Problem Set

Identify the domain and range of each relationship and the reverse relationship. Determine if the relationship and the reverse relationship are functions.

1. Each student in your school chooses his or her favorite sport.
   Relationship domain: students in your school
   Relationship range: all of the sports chosen
   The relationship is a function because for each student there is exactly one favorite sport.
   Reverse relationship domain: all of the sports chosen
   Reverse range: students in your school
   The reverse relationship is not a function because for each sport there may be more than one student who chose it as their favorite.

2. Each student in your school is assigned a unique student ID number.
   Relationship domain: students in your school
   Relationship range: assigned student ID numbers
   The relationship is a function because for each student there is exactly one student ID number.
   Reverse relationship domain: assigned student ID numbers
   Reverse range: students in your school
   The reverse relationship is a function because for each student ID number there is exactly one student.

3. Each of the 24 students in your class chooses a red, blue, orange, green, or yellow marble from a bag of assorted marbles.
   Relationship domain: students in your class
   Relationship range: red, blue, orange, green, and yellow marbles
   The relationship is a function because for each student there is exactly one marble chosen.
   Reverse relationship domain: red, blue, orange, green, and yellow marbles
   Reverse range: students in your class
   The relationship is not a function because for each color of marble there may be more than one student who chose it.

4. Every member of the basketball team is assigned a jersey number.
   Relationship domain: members of the basketball team
   Relationship range: assigned jersey numbers
   The relationship is a function because for each member of the basketball team there is exactly one assigned jersey number.
   Reverse relationship domain: assigned jersey numbers
   Reverse range: members of the basketball team
   The reverse relationship is a function because for each jersey number there is exactly one member of the basketball team.

5. Each member of your family chooses their favorite game for game night.
   Relationship domain: members of your family
   Relationship range: all of the games chosen
   The relationship is a function because for each member of your family there is exactly one favorite game.
   Reverse relationship domain: all of the games chosen
   Reverse range: members of your family
   The reverse relationship is not a function because for each game there may be more than one family member who chose it as their favorite.

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6. Each student in your class is assigned a letter grade for their final exam.
Relationship domain: students in your class
Relationship range: all of the possible letter grades
The relationship is a function because for each student there is exactly one letter grade.
Reverse relationship domain: all of the possible letter grades
Reverse range: students in your class
The reverse relationship is not a function because for each letter grade there may be more than one student to whom it was assigned.

Write a phrase, expression, or sentence to describe the inverse of each situation.

7. Close a dresser drawer.
Open the dresser drawer.

8. Light a candle.
Extinguish the candle.

9. Jog 3 blocks north and 5 blocks east.
Jog 5 blocks west and 3 blocks south.

10. Open the garage door and drive out of the garage.
Back into the garage and close the garage door.

11. Divide a number by 2 then add 7.
Subtract 7 then multiply by 2.

12. Multiply a number by 3 then add 1.
Subtract 1 then divide by 3.

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Complete each table. Write an equation to represent the relationship. Write an equation for the inverse of the problem situation.

13. One foot is equivalent to 12 inches.

Feet	Inches
1	12
2	24
3	36
4	48
5	60

Let i = the number of inches.

Let f = the number of feet.

$i = 12f$

Inverse: $f = \frac{i}{12}$

14. One meter is equivalent to 100 centimeters.

Meters	Centimeters
1	100
2	200
3	300
4	400
5	500

Let m = the number of meters.

Let c = the number of centimeters.

$c = 100m$

Inverse: $m = \frac{c}{100}$

15. One pint is equivalent to 2 cups.

Pints	Cups
2	4
4	8
6	12
8	16
10	20

Let p = the number of pints.

Let c = the number of cups.

$c = 2p$

Inverse: $p = \frac{c}{2}$

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16. Four quarters is equivalent to 1 dollar.

Quarters	Dollars
4	1
16	4
32	8
64	16
128	32

Let q = the number of quarters.

Let d = the number of dollars.

$d = \frac{q}{4}$

Inverse: $q = 4d$

17. Three feet is equivalent to 1 yard.

Feet	Yards
3	1
9	3
12	4
18	6
24	8

Let f = the number of feet.

Let y = the number of yards.

$y = \frac{f}{3}$

Inverse: $f = 3y$

18. One US dollar is equivalent to 13 Mexican pesos.

Dollars	Pesos
1	13
2	26
3	39
4	52
5	65

Let d = the number of dollars.

Let p = the number of pesos.

$p = 13d$

Inverse: $d = \frac{p}{13}$

Determine the inverse of each function. Graph the original function and its inverse.

19. $f(x) = 4x$

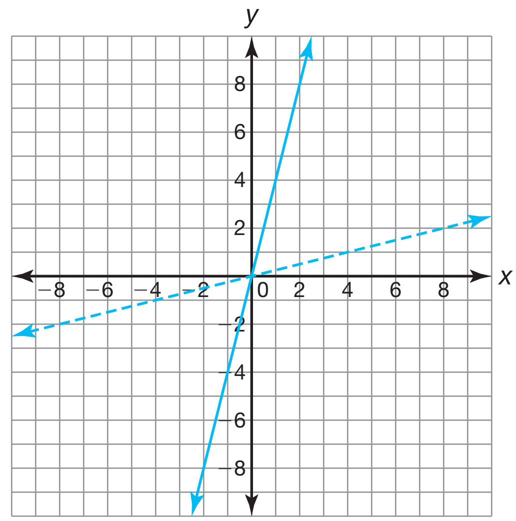
$f(x) = 4x$

$y = 4x$

$x = 4y$

$\frac{x}{4} = y$

$f^{-1}(x) = \frac{x}{4}$



20. $f(x) = \frac{1}{3}x$

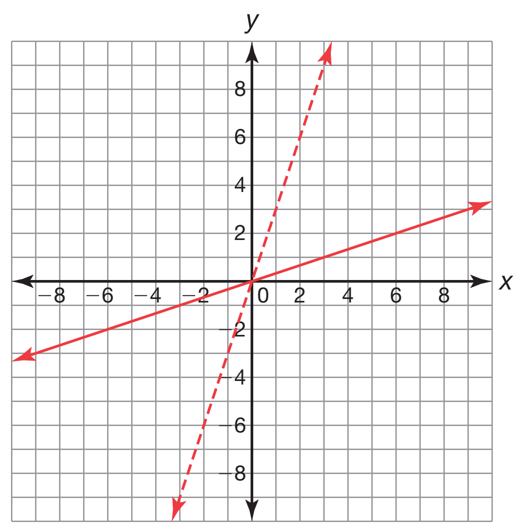
$f(x) = \frac{1}{3}x$

$y = \frac{1}{3}x$

$x = \frac{1}{3}y$

$3x = y$

$f^{-1}(x) = 3x$



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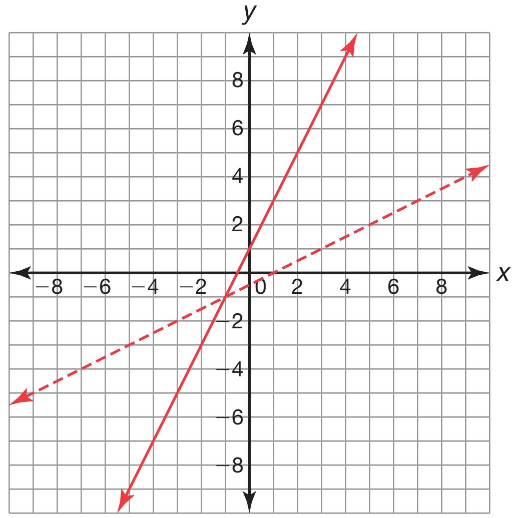
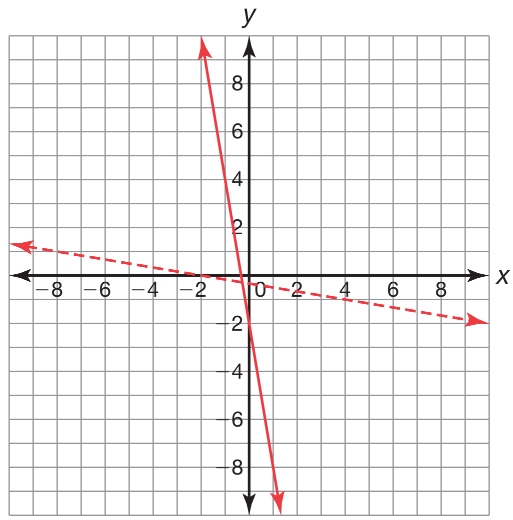
21. $f(x) = 2x + 1$

$$f(x) = 2x + 1$$
$$y = 2x + 1$$
$$x = 2y + 1$$
$$x - 1 = 2y$$
$$\frac{x-1}{2} = \frac{2y}{2}$$
$$\frac{1}{2}x - \frac{1}{2} = y$$
$$f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$$



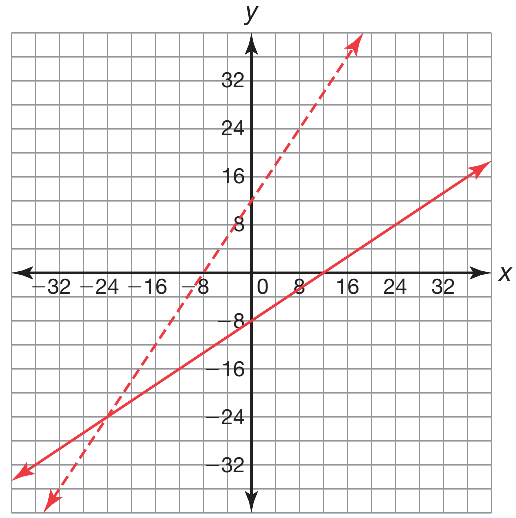
22. $f(x) = -6x - 2$

$$f(x) = -6x - 2$$
$$y = -6x - 2$$
$$x = -6y - 2$$
$$x + 2 = -6y$$
$$\frac{x+2}{-6} = \frac{-6y}{-6}$$
$$-\frac{1}{6}x - \frac{1}{3} = y$$
$$f^{-1}(x) = -\frac{1}{6}x - \frac{1}{3}$$



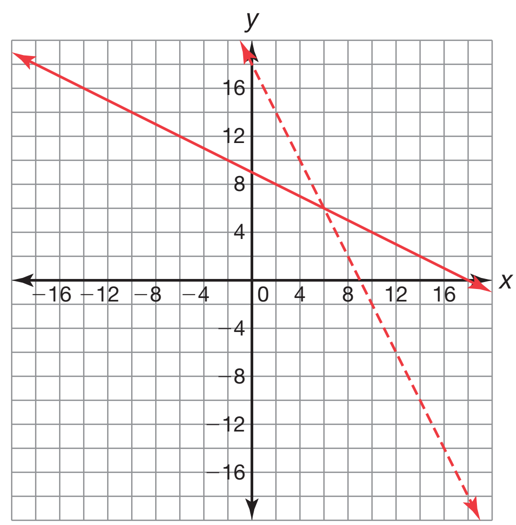
23. $f(x) = \frac{2}{3}x - 8$

$$f(x) = \frac{2}{3}x - 8$$
$$y = \frac{2}{3}x - 8$$
$$x = \frac{2}{3}y - 8$$
$$x + 8 = \frac{2}{3}y$$
$$\left(\frac{3}{2}\right) \cdot (x + 8) = \left(\frac{3}{2}\right) \cdot \left(\frac{2}{3}y\right)$$
$$\frac{3}{2}x + 12 = y$$
$$f^{-1}(x) = \frac{3}{2}x + 12$$



24. $f(x) = -0.5x + 9$

$$f(x) = -0.5x + 9$$
$$y = -0.5x + 9$$
$$x = -0.5y + 9$$
$$x - 9 = -0.5y$$
$$\frac{x - 9}{-0.5} = \frac{-0.5y}{-0.5}$$
$$-2x + 18 = y$$
$$f^{-1}(x) = -2x + 18$$



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Determine the corresponding point on the graph of each inverse function.

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25. Given that (2, 5) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (5, 2).

26. Given that (−3, 1) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (1, −3).

27. Given that (−4, −1) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (−1, −4).

28. Given that (0, 8) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (8, 0).

29. Given that (1, −7) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (−7, 1).

30. Given that (−6, 0) is a point on the graph of $f(x)$, what is the corresponding point on the graph of $f^{-1}(x)$?

The corresponding point on the graph of $f^{-1}(x)$ is (0, −6).

Determine if the functions in each pair are inverses.

31. $f(x) = 5x + 1$ and $g(x) = \frac{1}{5}x - \frac{1}{5}$

$f(x) = 5x + 1$

$f(g(x)) = f\left(\frac{1}{5}x - \frac{1}{5}\right)$

$f(g(x)) = 5\left(\frac{1}{5}x - \frac{1}{5}\right) + 1$

$= (x - 1) + 1$

$= x$

$g(x) = \frac{1}{5}x - \frac{1}{5}$

$g(f(x)) = g(5x + 1)$

$g(f(x)) = \frac{1}{5}(5x + 1) - \frac{1}{5}$

$= \left(x + \frac{1}{5}\right) - \frac{1}{5}$

$= x$

The functions are inverses because $f(g(x)) = g(f(x)) = x$.

32. $f(x) = 8x - 2$ and $g(x) = \frac{1}{8}x - \frac{1}{4}$

$f(x) = 8x - 2$

$f(g(x)) = f\left(\frac{1}{8}x - \frac{1}{4}\right)$

$f(g(x)) = 8\left(\frac{1}{8}x - \frac{1}{4}\right) - 2$

$= (x - 2) - 2$

$= x - 4$

$g(x) = \frac{1}{8}x - \frac{1}{4}$

$g(f(x)) = g(8x - 2)$

$g(f(x)) = \frac{1}{8}(8x - 2) - \frac{1}{4}$

$= \left(x - \frac{1}{4}\right) - \frac{1}{4}$

$= x - \frac{1}{2}$

The functions are not inverses because $f(g(x)) \neq g(f(x)) \neq x$.

33. $f(x) = -\frac{1}{2}x + 5$ and $g(x) = -2x + 10$

$f(x) = -\frac{1}{2}x + 5$

$f(g(x)) = f(-2x + 10)$

$f(g(x)) = -\frac{1}{2}(-2x + 10) + 5$

$= (x - 5) + 5$

$= x$

$g(x) = -2x + 10$

$g(f(x)) = g\left(-\frac{1}{2}x + 5\right)$

$g(f(x)) = -2\left(-\frac{1}{2}x + 5\right) + 10$

$= (x - 10) + 10$

$= x$

The functions are inverses because $f(g(x)) = g(f(x)) = x$.

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34. $f(x) = -\frac{2}{3}x - 2$ and $g(x) = -\frac{3}{2}x - 3$

$f(x) = -\frac{2}{3}x - 2$

$f(g(x)) = f\left(-\frac{3}{2}x - 3\right)$

$f(g(x)) = -\frac{2}{3}\left(-\frac{3}{2}x - 3\right) - 2$

$= (x + 2) - 2$

$= x$

$g(x) = -\frac{3}{2}x - 3$

$g(f(x)) = g\left(-\frac{2}{3}x - 2\right)$

$g(f(x)) = -\frac{3}{2}\left(-\frac{2}{3}x - 2\right) - 3$

$= (x + 3) - 3$

$= x$

The functions are inverses because $f(g(x)) = g(f(x)) = x$.

35. $f(x) = 0.4x - 8$ and $g(x) = 2.5x + 20$

$f(x) = 0.4x - 8$

$f(g(x)) = f(2.5x + 20)$

$f(g(x)) = 0.4(2.5x + 20) - 8$

$= (x + 8) - 8$

$= x$

$g(x) = 2.5x + 20$

$g(f(x)) = g(0.4x - 8)$

$g(f(x)) = 2.5(0.4x - 8) + 20$

$= (x - 20) + 20$

$= x$

The functions are inverses because $f(g(x)) = g(f(x)) = x$.

36. $f(x) = -0.2x + 6$ and $g(x) = 5x - 30$

$f(x) = -0.2x + 6$

$f(g(x)) = f(5x - 30)$

$f(g(x)) = -0.2(5x - 30) + 6$

$= (-x + 6) + 6$

$= -x + 12$

$g(x) = 5x - 30$

$g(f(x)) = g(-0.2x + 6)$

$g(f(x)) = 5(-0.2x + 6) - 30$

$= (-x + 30) - 30$

$= -x$

The functions are not inverses because $f(g(x)) \neq g(f(x)) \neq x$.

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16

Taking the Egg Plunge!
Inverses of Non-Linear Functions

Vocabulary

Write a definition for each term in your own words.

1. one-to-one function
A function is a one-to-one function if both the function and its inverse are functions.

2. restrict the domain
To restrict the domain of a function means to define a new domain for the function that is a subset of the original domain.

Problem Set

Complete each table of values for the function and its inverse. Determine whether the function is a one-to-one function.

1. $f(x) = 2x + 5$

x	$f(x)$
-2	1
-1	3
0	5
1	7
2	9

x	$f^{-1}(x)$
1	-2
3	-1
5	0
7	1
9	2

The function is one-to-one because both the original function and its inverse are functions.

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2. $f(x) = -6x + 1$

x	$f(x)$
−2	13
−1	7
0	1
1	−5
2	−11

x	$f^{-1}(x)$
13	−2
7	−1
1	0
−5	1
−11	2

The function is one-to-one because both the original function and its inverse are functions.

3. $f(x) = 5x^2 - 8$

x	$f(x)$
−2	12
−1	−3
0	−8
1	−3
2	12

x	$f^{-1}(x)$
12	−2
−3	−1
−8	0
−3	1
12	2

The function is not one-to-one because its inverse is not a function.

4. $f(x) = 4^x$

x	$f(x)$
−2	0.0625
−1	0.25
0	1
1	4
2	16

x	$f^{-1}(x)$
0.0625	−2
0.25	−1
1	0
4	1
16	2

The function is one-to-one because both the original function and its inverse are functions.

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5. $f(x) = -3$

x	$f(x)$
−2	−3
−1	−3
0	−3
1	−3
2	−3

x	$f^{-1}(x)$
−3	−2
−3	−1
−3	0
−3	1
−3	2

The function is not one-to-one because its inverse is not a function.

6. $f(x) = |4x|$

x	$f(x)$
−2	8
−1	4
0	0
1	4
2	8

x	$f^{-1}(x)$
8	−2
4	−1
0	0
4	1
8	2

The function is not one-to-one because its inverse is not a function.

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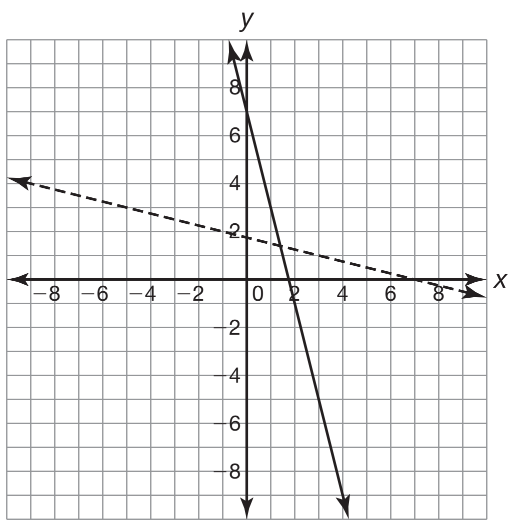
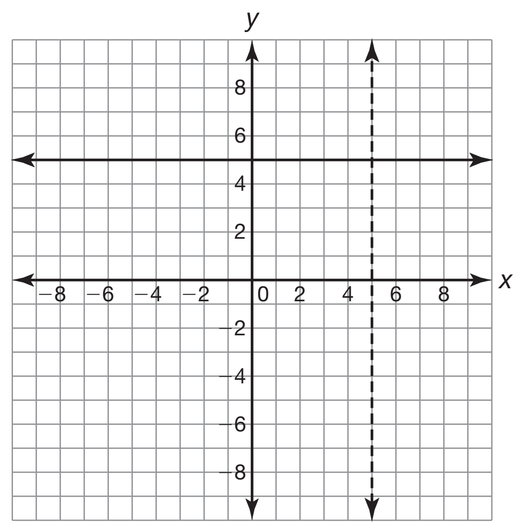
Determine whether each function is a one-to-one function by examining the graph of the function and its inverse.

7. $f(x) = -4x + 7$



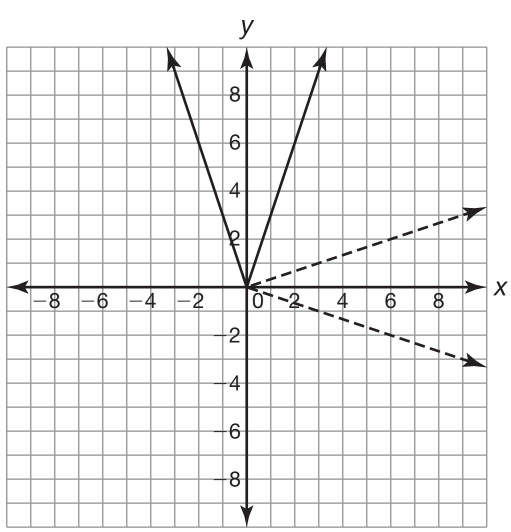
The function is one-to-one because both the original function and its inverse are functions.

8. $f(x) = 5$



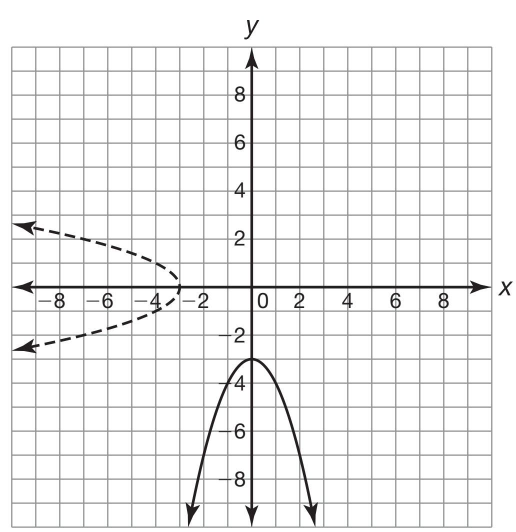
The inverse function does not pass the Vertical Line Test. So, the function is not one-to-one because its inverse is not a function.

9. $f(x) = |3x|$



The inverse function does not pass the Vertical Line Test. So, the function is not one-to-one because its inverse is not a function.

10. $f(x) = -x^2 - 3$



The inverse function does not pass the Vertical Line Test. So, the function is not one-to-one because its inverse is not a function.

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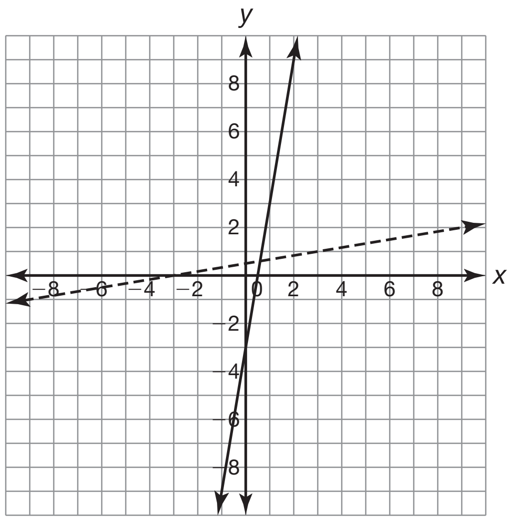
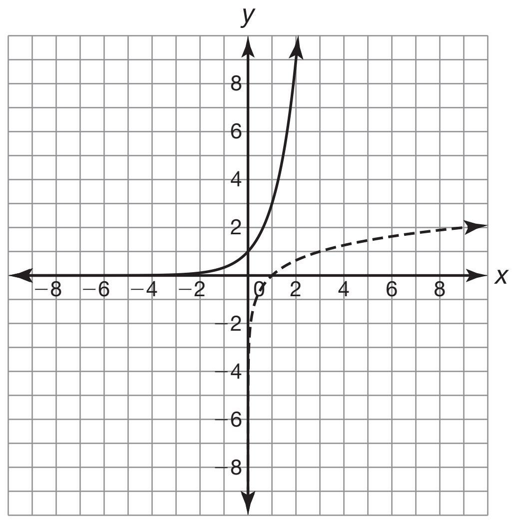
11. $f(x) = 6x - 3$



The function is one-to-one because both the original function and its inverse are functions.

12. $f(x) = 3^x$



The function is one-to-one because both the original function and its inverse are functions.

Identify each equation as linear, exponential, quadratic, or linear absolute value. Determine whether the function is a one-to-one function.

13. $f(x) = 2x - 9$

The function is a linear function. A linear function that is not a constant function is a one-to-one function. So, the function is one-to-one.

14. $f(x) = -6$

The function is a linear function. A linear function that is a constant function is not a one-to-one function. So, the function is not one-to-one.

15. $f(x) = -3x + 10$

The function is a linear function. A linear function that is not a constant function is a one-to-one function. So, the function is one-to-one.

16. $f(x) = 5^x$

The function is an exponential function. An exponential function is always a one-to-one function. So, the function is one-to-one.

17. $f(x) = -|6x|$

The function is a linear absolute value function. A linear absolute value function is never a one-to-one function. So, the function is not one-to-one.

18. $f(x) = 9x^2 + 3$

The function is a quadratic function. A quadratic function is never a one-to-one function. So, the function is not one-to-one.

Determine the equation of the inverse for each quadratic function.

19. $f(x) = 7x^2$

$$f(x) = 7x^2$$
$$y = 7x^2$$
$$x = 7y^2$$
$$\frac{x}{7} = y^2$$
$$\pm\sqrt{\frac{x}{7}} = y$$

20. $f(x) = -x^2$

$$f(x) = -x^2$$
$$y = -x^2$$
$$x = -y^2$$
$$-x = y^2$$
$$\pm\sqrt{-x} = y$$

21. $f(x) = 6x^2 + 11$

$$f(x) = 6x^2 + 11$$
$$y = 6x^2 + 11$$
$$x = 6y^2 + 11$$
$$x - 11 = 6y^2$$
$$\frac{x - 11}{6} = y^2$$
$$\pm\sqrt{\frac{x - 11}{6}} = y$$

22. $f(x) = 2x^2 - 12$

$$f(x) = 2x^2 - 12$$
$$y = 2x^2 - 12$$
$$x = 2y^2 - 12$$
$$x + 12 = 2y^2$$
$$\frac{x + 12}{2} = y^2$$
$$\pm\sqrt{\frac{x + 12}{2}} = y$$

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23. $f(x) = -4x^2 - 6$

$$f(x) = -4x^2 - 6$$
$$y = -4x^2 - 6$$
$$x = -4y^2 - 6$$
$$x + 6 = -4y^2$$
$$\frac{x+6}{-4} = y^2$$
$$\pm\sqrt{\frac{x+6}{-4}} = y$$

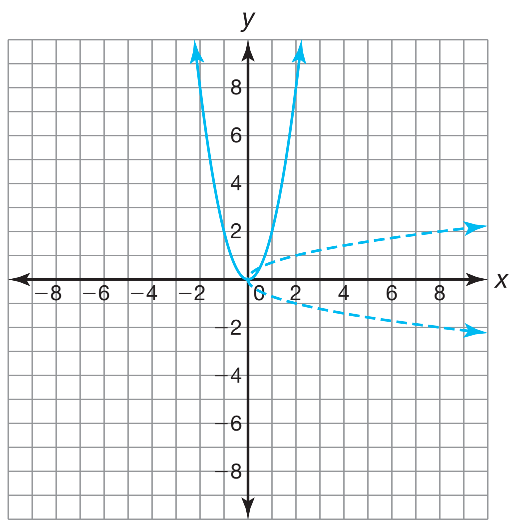
24. $f(x) = -3x^2 + 20$

$$f(x) = -3x^2 + 20$$
$$y = -3x^2 + 20$$
$$x = -3y^2 + 20$$
$$x - 20 = -3y^2$$
$$\frac{x-20}{-3} = y^2$$
$$\pm\sqrt{\frac{x-20}{-3}} = y$$

Determine the equation of the inverse for each given function. Graph the function and its inverse. Restrict the domain of the original function and the inverse so that the inverse is also a function.

25. $f(x) = 2x^2$

$$f(x) = 2x^2$$
$$y = 2x^2$$
$$x = 2y^2$$
$$\frac{x}{2} = y^2$$
$$\pm\sqrt{\frac{x}{2}} = y$$



$$f(x) = \begin{cases} 2x^2, & \text{domain: } x \geq 0, \text{ range: } y \geq 0, \\ 2x^2, & \text{domain: } x \leq 0, \text{ range: } y \geq 0, \end{cases}$$

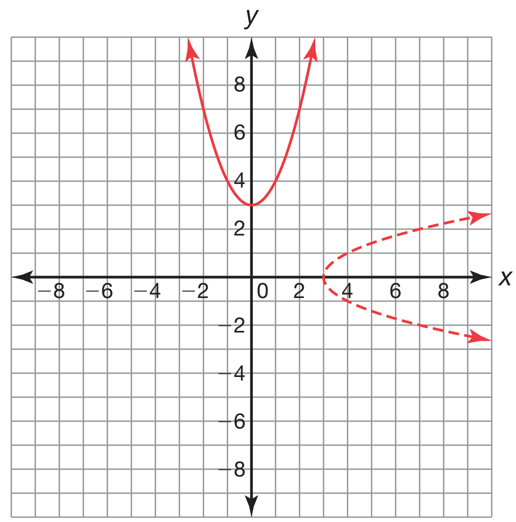
$$\text{Inverse of } f(x) = \begin{cases} \sqrt{\frac{x}{2}}, & \text{domain: } x \geq 0, \text{ range: } y \geq 0, \\ -\sqrt{\frac{x}{2}}, & \text{domain: } x \geq 0, \text{ range: } y \leq 0, \end{cases}$$

For the function $y = 2x^2$ with $x \geq 0$, the inverse is $y = \sqrt{\frac{x}{2}}$.

For the function $y = 2x^2$ with $x \leq 0$, the inverse is $y = -\sqrt{\frac{x}{2}}$.

26. $f(x) = x^2 + 3$

$$f(x) = x^2 + 3$$
$$y = x^2 + 3$$
$$x = y^2 + 3$$
$$x - 3 = y^2$$
$$\pm\sqrt{x - 3} = y$$



$$f(x) = \begin{cases} x^2 + 3, & \text{domain: } x \geq 0\text{, range: } y \geq 3, \\ x^2 + 3, & \text{domain: } x \leq 0\text{, range: } y \geq 3, \end{cases}$$

$$\text{Inverse of } f(x) = \begin{cases} \sqrt{x - 3}, & \text{domain: } x \geq 3\text{, range: } y \geq 0, \\ -\sqrt{x - 3}, & \text{domain: } x \geq 3\text{, range: } y \leq 0, \end{cases}$$

For the function $y = x^2 + 3$ with $x \geq 0$, the inverse is $y = \sqrt{x - 3}$.

For the function $y = x^2 + 3$ with $x \leq 0$, the inverse is $y = -\sqrt{x - 3}$.

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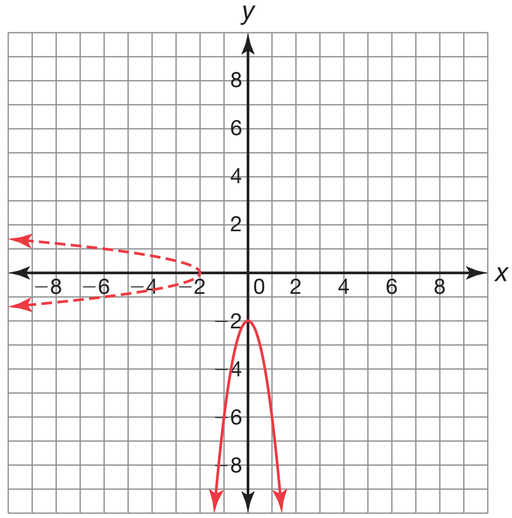
27. $f(x) = -4x^2 - 2$

$$f(x) = -4x^2 - 2$$
$$y = -4x^2 - 2$$
$$x = -4y^2 - 2$$
$$x + 2 = -4y^2$$
$$\frac{x+2}{-4} = y^2$$
$$\pm\sqrt{\frac{x+2}{-4}} = y$$



$$f(x) = \begin{cases} -4x^2 - 2, & \text{domain: } x \geq 0\text{, range: } y \leq -2 \\ -4x^2 - 2, & \text{domain: } x \leq 0\text{, range: } y \leq -2 \end{cases}$$

$$\text{Inverse of } f(x) = \begin{cases} \sqrt{\frac{x+2}{-4}}, & \text{domain: } x \leq -2\text{, range: } y \geq 0, \\ -\sqrt{\frac{x+2}{-4}}, & \text{domain: } x \leq -2\text{, range: } y \leq 0, \end{cases}$$

For the function $y = -4x^2 - 2$ with $x \geq 0$, the inverse is $y = \sqrt{\frac{x+2}{-4}}$.

For the function $y = -4x^2 - 2$ with $x \leq 0$, the inverse is $y = -\sqrt{\frac{x+2}{-4}}$.

28. $f(x) = |2x|$

$f(x) = |2x|$

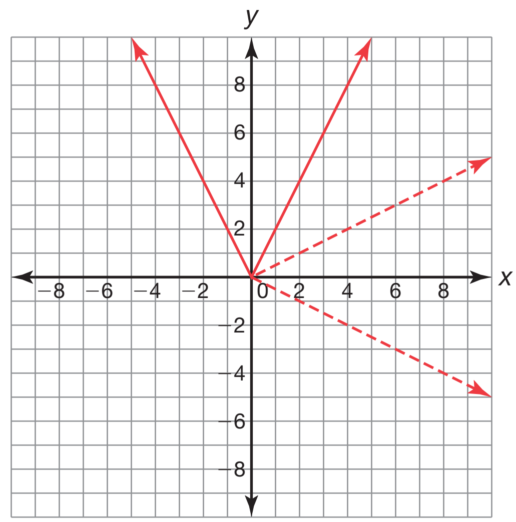
$$f(x) = \begin{cases} 2x, & x \ge 0 \\ -2x, & x \le 0 \end{cases}$$

$$y = \begin{cases} 2x, & x \ge 0 \\ -2x, & x \le 0 \end{cases}$$

$$x = \begin{cases} 2y, & y \ge 0 \\ -2y, & y \le 0 \end{cases}$$

$x = 2y$ $\qquad$ $x = -2y$

$\frac{x}{2} = y$ $\qquad$ $-\frac{x}{2} = y$



$$f(x) = \begin{cases} 2x, & \text{domain: } x \ge 0\text{, range: } y \ge 0 \\ -2x, & \text{domain: } x \le 0\text{, range: } y \ge 0 \end{cases}$$

$$\text{Inverse of } f(x) = \begin{cases} \frac{x}{2}, & \text{domain: } x \ge 0\text{, range: } y \ge 0\text{,} \\ -\frac{x}{2}, & \text{domain: } x \ge 0\text{, range: } y \le 0\text{,} \end{cases}$$

For the function $y = 2x$ with $x \ge 0$, the inverse is $y = \frac{x}{2}$.

For the function $y = -2x$ with $x \le 0$, the inverse is $y = -\frac{x}{2}$.

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29. $f(x) = -|x|$

$f(x) = -|x|$

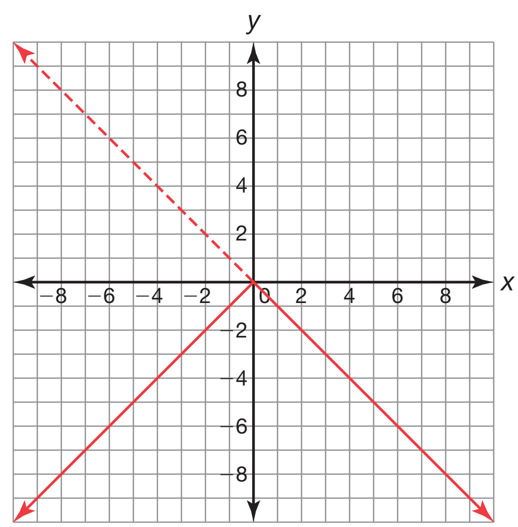
$f(x) = \begin{cases} x, & x \le 0 \\ -x, & x \ge 0 \end{cases}$

$y = \begin{cases} x, & x \le 0 \\ -x, & x \ge 0 \end{cases}$

$x = \begin{cases} y, & y \le 0 \\ -y, & y \le 0 \end{cases}$

$x = y \qquad x = -y$

$\qquad\qquad -x = y$



$f(x) = \begin{cases} x, & \text{domain: } x \le 0, \text{ range: } y \le 0 \\ -x, & \text{domain: } x \ge 0, \text{ range: } y \le 0 \end{cases}$

$\text{Inverse of } f(x) = \begin{cases} x, & \text{domain: } x \le 0, \text{ range: } y \le 0 \\ -x, & \text{domain: } x \le 0, \text{ range: } y \ge 0 \end{cases}$

For the function $y = x$ with $x \le 0$, the inverse is $y = x$.

For the function $y = -x$ with $x \ge 0$, the inverse is $y = -x$.

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30. $f(x) = -|5x|$

$f(x) = -|5x|$

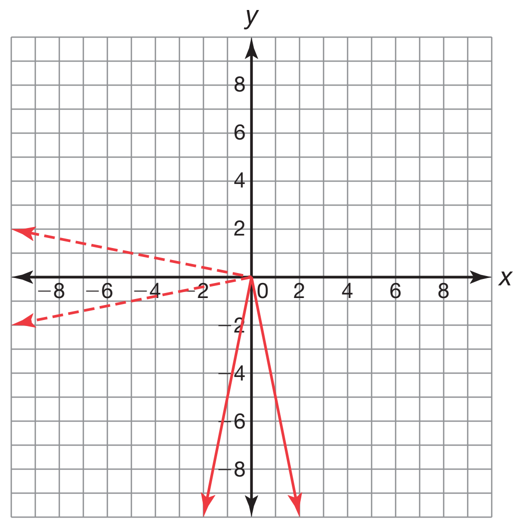
$f(x) = \begin{cases} 5x, & x \le 0 \\ -5x, & x \ge 0 \end{cases}$

$y = \begin{cases} 5x, & x \le 0 \\ -5x, & x \ge 0 \end{cases}$

$x = \begin{cases} 5y, & y \le 0 \\ -5y, & y \ge 0 \end{cases}$

$x = 5y \qquad x = -5y$

$\frac{x}{5} = y \qquad -\frac{x}{5} = y$



$f(x) = \begin{cases} 5x, & \text{domain: } x \le 0\text{, range: } y \le 0 \\ -5x, & \text{domain: } x \ge 0\text{, range: } y \le 0 \end{cases}$

Inverse of $f(x) = \begin{cases} \frac{x}{5}, & \text{domain: } x \le 0\text{, range: } y \le 0 \\ -\frac{x}{5}, & \text{domain: } x \le 0\text{, range: } y \ge 0 \end{cases}$

For the function $y = 5x$ with $x \le 0$, the inverse is $y = \frac{x}{5}$.

For the function $y = -5x$ with $x \ge 0$, the inverse is $y = -\frac{x}{5}$.