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The Coordinate Plane

Circles and Polygons on the Coordinate Plane

Problem Set

Use the given information to show that each statement is true. Justify your answers by using theorems and by using algebra.

- The center of circle O is at the origin. The coordinates of the given points are $A(-4, 0)$, $B(4, 0)$, and $C(0, 4)$. Show that $\triangle ABC$ is a right triangle.

$\triangle ABC$ is an inscribed triangle in circle O with the hypotenuse as the diameter of the circle, therefore the triangle is a right triangle by the Right Triangle Diameter Theorem.

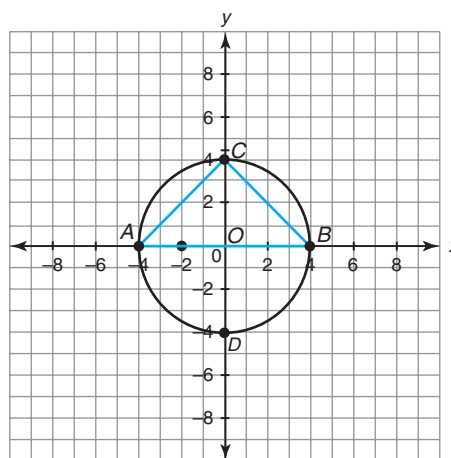
$$\begin{aligned} AB &= \sqrt{(-4 - 4)^2 + (0 - 0)^2} \\ &= \sqrt{(-8)^2} = \sqrt{64} = 8 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-4 - 0)^2 + (0 - 4)^2} \\ &= \sqrt{(-4)^2 + (-4)^2} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4 - 0)^2 + (0 - 4)^2} \\ &= \sqrt{4^2 + (-4)^2} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} (4\sqrt{2})^2 + (4\sqrt{2})^2 &\stackrel{?}{=} 8^2 \\ 32 + 32 &= 64 \end{aligned}$$

Therefore, by the Converse of the Pythagorean Theorem, $\triangle ABC$ is a right triangle.



2. The center of circle O is at the origin. \overleftrightarrow{AT} and \overleftrightarrow{AZ} are tangent to circle O . The coordinates of the given points are $A(-10, 30)$, $T(8, 6)$, and $Z(-10, 0)$. Show that the lengths of \overleftrightarrow{AT} and \overleftrightarrow{AZ} are equal.

$$AT = \sqrt{(-10 - 8)^2 + (30 - 6)^2}$$

$$= \sqrt{(-18)^2 + (24)^2}$$

$$= \sqrt{324 + 576}$$

$$= \sqrt{900} = 30$$

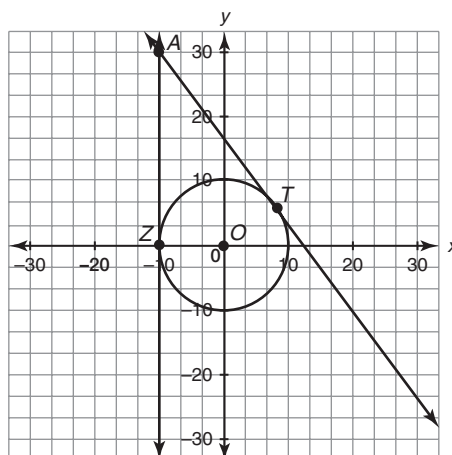
$$AZ = \sqrt{(-10 - (-10))^2 + (30 - 0)^2}$$

$$= \sqrt{0^2 + 30^2}$$

$$= \sqrt{900} = 30$$

$$AT = 30 = AZ.$$

Therefore, $AT = AZ$.

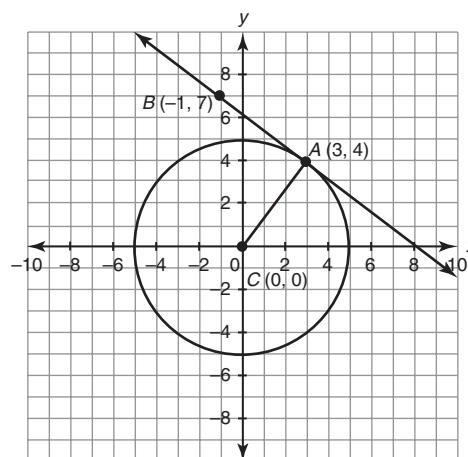


3. The center of circle C is at the origin. \overleftrightarrow{AB} is tangent to circle C at $A(3, 4)$. Show that the tangent line is perpendicular to \overleftrightarrow{CA} .

$$\text{slope of } \overleftrightarrow{AB} = \frac{7 - 4}{-1 - 3} = -\frac{3}{4}$$

$$\text{slope of } \overleftrightarrow{CA} = \frac{4 - 0}{3 - 0} = \frac{4}{3}$$

Because the slopes of \overleftrightarrow{AB} and \overleftrightarrow{CA} are opposite reciprocals, \overleftrightarrow{AB} is perpendicular to \overleftrightarrow{CA} .



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4. The center of circle O is at the origin. The coordinates of the given points are $A(3, 4)$, $B(-4, -3)$, $D(0, -5)$, $E(-3, 4)$, and $F(-1.5, -0.5)$. Show that $EF \cdot FD = AF \cdot FB$.

$$\begin{aligned} AF &= \sqrt{(3 + 1.5)^2 + (4 + 0.5)^2} \\ &= \sqrt{4.5^2 + 4.5^2} \\ &= \sqrt{40.5} \end{aligned}$$

$$\begin{aligned} FB &= \sqrt{(-1.5 + 4)^2 + (-0.5 + 3)^2} \\ &= \sqrt{2.5^2 + 2.5^2} = \sqrt{12.5} \end{aligned}$$

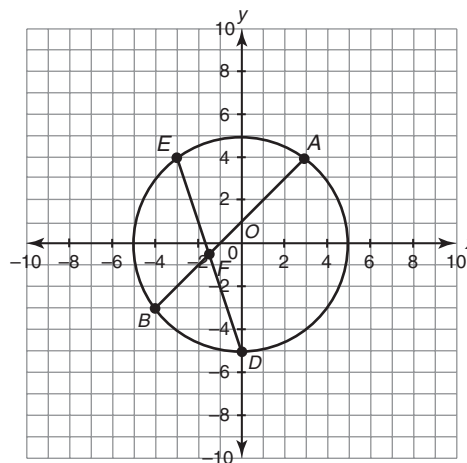
$$\begin{aligned} EF &= \sqrt{(-3 + 1.5)^2 + (4 + 0.5)^2} \\ &= \sqrt{(-1.5)^2 + 4.5^2} \\ &= \sqrt{22.5} \end{aligned}$$

$$\begin{aligned} FD &= \sqrt{(-1.5 - 0)^2 + (-0.5 + 5)^2} \\ &= \sqrt{(-1.5)^2 + (4.5)^2} \\ &= \sqrt{22.5} \end{aligned}$$

$$EF \cdot FD \stackrel{?}{=} AF \cdot FB$$

$$\sqrt{22.5} \cdot \sqrt{22.5} \stackrel{?}{=} \sqrt{40.5} \cdot \sqrt{12.5}$$

$$22.5 = 22.5 \quad \checkmark$$



5. The center of circle O is at the origin. The coordinates of the given points are $A(-5, 5)$, $B(-4, 3)$, $C(0, -5)$, and $D(0, 5)$. Show that $AD^2 = AB \cdot AC$.

$$AD = \sqrt{(-5 - 0)^2 + (5 - 5)^2}$$

$$= \sqrt{(-5)^2 + 0^2}$$

$$= \sqrt{25} = 5$$

$$AB = \sqrt{(-5 + 4)^2 + (5 - 3)^2}$$

$$= \sqrt{(-1)^2 + 2^2}$$

$$= \sqrt{5}$$

$$AC = \sqrt{(-5 - 0)^2 + (5 + 5)^2}$$

$$= \sqrt{(-5)^2 + 10^2}$$

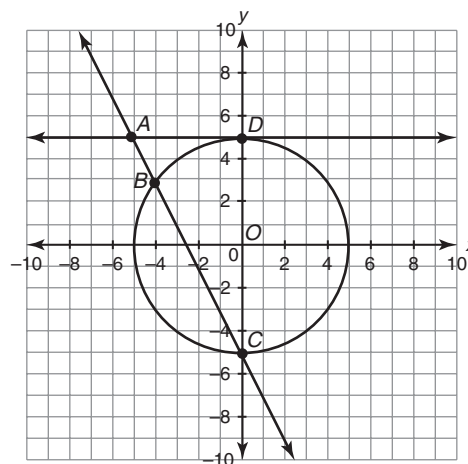
$$= \sqrt{125}$$

$$AD^2 \stackrel{?}{=} AB \cdot AC$$

$$5^2 \stackrel{?}{=} (\sqrt{5})(\sqrt{125})$$

$$5^2 \stackrel{?}{=} \sqrt{625}$$

$$25 = 25 \quad \checkmark$$



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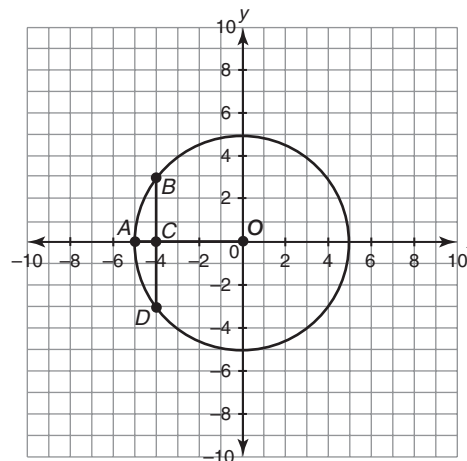
6. The center of circle O is at the origin. \overline{BD} is perpendicular to \overline{OA} at point C . The coordinates of the given points are $A(-5, 0)$, $B(-4, 3)$, $C(-4, 0)$, and $D(-4, -3)$. Show that \overline{OA} bisects \overline{BD} .

$$\begin{aligned} BD &= \sqrt{(-4 + 4)^2 + (-3 - 3)^2} \\ &= \sqrt{0^2 + (-6)^2} \\ &= 6 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-4 + 4)^2 + (3 - 0)^2} \\ &= \sqrt{0^2 + 3^2} \\ &= 3 \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-4 + 4)^2 + (0 + 3)^2} \\ &= \sqrt{0^2 + 3^2} \\ &= 3 \end{aligned}$$

\overline{BD} and \overline{OA} intersect at C . $BC = CD = 3 = \frac{1}{2}BD$. Therefore, \overline{OA} bisects \overline{BD} .



Classify the polygon formed by connecting the midpoints of the sides of each quadrilateral. Show all your work.

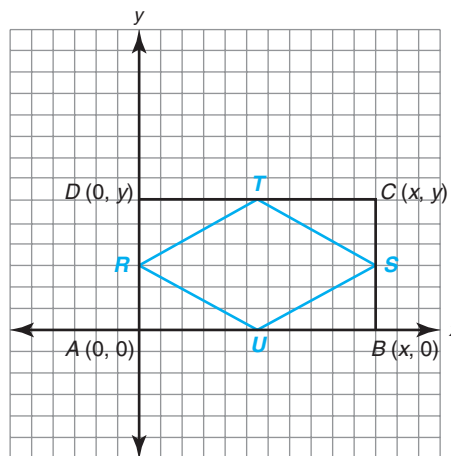
7. The rectangle shown has vertices $A(0, 0)$, $B(x, 0)$, $C(x, y)$, and $D(0, y)$.

Midpoint \overline{AB} : $U\left(\frac{x}{2}, 0\right)$

Midpoint \overline{BC} : $S\left(x, \frac{y}{2}\right)$

Midpoint \overline{CD} : $T\left(\frac{x}{2}, y\right)$

Midpoint \overline{DA} : $R\left(0, \frac{y}{2}\right)$



$$\text{Slope } \overline{RT} = \frac{y - \frac{y}{2}}{\frac{x}{2} - 0} = \frac{\frac{y}{2}}{\frac{x}{2}} = \frac{y}{x}$$

$$\text{Slope } \overline{US} = \frac{0 - \frac{y}{2}}{\frac{x}{2} - x} = \frac{-\frac{y}{2}}{-\frac{x}{2}} = \frac{y}{x}$$

$$\text{Slope } \overline{TS} = \frac{\frac{y}{2} - y}{x - \frac{x}{2}} = \frac{-\frac{y}{2}}{\frac{x}{2}} = -\frac{y}{x}$$

$$\text{Slope } \overline{RU} = \frac{\frac{y}{2} - 0}{0 - \frac{x}{2}} = \frac{\frac{y}{2}}{-\frac{x}{2}} = -\frac{y}{x}$$

\overline{RT} and \overline{US} are parallel since they have the same slope of $\frac{y}{x}$.

\overline{TS} and \overline{RU} are parallel since they have the same slope of $-\frac{y}{x}$.

There are no perpendicular sides. The slopes are not opposite reciprocals.

$$\begin{aligned} RT &= \sqrt{\left(0 - \frac{x}{2}\right)^2 + \left(\frac{y}{2} - y\right)^2} \\ &= \sqrt{\left(-\frac{x}{2}\right)^2 + \left(-\frac{y}{2}\right)^2} \\ &= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}} \end{aligned}$$

$$\begin{aligned} TS &= \sqrt{\left(\frac{x}{2} - x\right)^2 + \left(y - \frac{y}{2}\right)^2} \\ &= \sqrt{\left(-\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2} \\ &= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}} \end{aligned}$$

$$\begin{aligned} US &= \sqrt{\left(x - \frac{x}{2}\right)^2 + \left(\frac{y}{2} - 0\right)^2} \\ &= \sqrt{\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2} \\ &= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}} \end{aligned}$$

$$\begin{aligned} RU &= \sqrt{\left(0 - \frac{x}{2}\right)^2 + \left(\frac{y}{2} - 0\right)^2} \\ &= \sqrt{\left(-\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2} \\ &= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}} \end{aligned}$$

All four sides of $RTSU$ are congruent.

Opposite sides are parallel and all sides are congruent, so the quadrilateral formed by connecting the midpoints of the rectangle is a rhombus.

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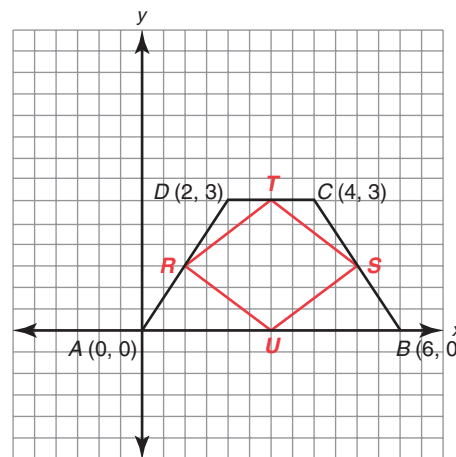
8. The isosceles trapezoid shown has vertices $A(0, 0)$, $B(6, 0)$, $C(4, 3)$, and $D(2, 3)$.

Midpoint \overline{AB} : $U(3, 0)$

Midpoint \overline{BC} : $S(5, \frac{3}{2})$

Midpoint \overline{CD} : $T(3, 3)$

Midpoint \overline{DA} : $R(1, \frac{3}{2})$



$$\text{Slope } \overline{RT} = \frac{3 - \frac{3}{2}}{3 - 1} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$$

$$\text{Slope } \overline{TS} = \frac{\frac{3}{2} - 3}{5 - 3} = \frac{-\frac{3}{2}}{2} = -\frac{3}{4}$$

$$\text{Slope } \overline{SU} = \frac{\frac{3}{2} - 0}{5 - 3} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$$

$$\text{Slope } \overline{RU} = \frac{0 - \frac{3}{2}}{3 - 1} = \frac{-\frac{3}{2}}{2} = -\frac{3}{4}$$

\overline{RT} and \overline{SU} are parallel since they have the same slope of $\frac{3}{4}$.

\overline{TS} and \overline{RU} are parallel since they have the same slope of $-\frac{3}{4}$.

There are no perpendicular sides. The slopes are not opposite reciprocals.

$$\begin{aligned} RT &= \sqrt{(1 - 3)^2 + \left(\frac{3}{2} - 3\right)^2} \\ &= \sqrt{(-2)^2 + \left(-\frac{3}{2}\right)^2} \\ &= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} TS &= \sqrt{(3 - 5)^2 + \left(3 - \frac{3}{2}\right)^2} \\ &= \sqrt{(-2)^2 + \left(\frac{3}{2}\right)^2} \\ &= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} SU &= \sqrt{(5 - 3)^2 + \left(\frac{3}{2} - 0\right)^2} \\ &= \sqrt{2^2 + \left(\frac{3}{2}\right)^2} \\ &= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} RU &= \sqrt{(1 - 3)^2 + \left(\frac{3}{2} - 0\right)^2} \\ &= \sqrt{(-2)^2 + \left(\frac{3}{2}\right)^2} \\ &= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} \end{aligned}$$

All four sides of $RTSU$ are congruent.

Opposite sides are parallel and all sides are congruent, so the quadrilateral formed by the midpoints of the isosceles trapezoid is a rhombus.

9. The parallelogram shown has vertices $A(4, 5)$, $B(7, 5)$, $C(3, 0)$, and $D(0, 0)$.

$$\text{Midpoint } \overline{AB} = E(5.5, 5)$$

$$\text{Midpoint } \overline{BC} = F(5, 2.5)$$

$$\text{Midpoint } \overline{CD} = G(1.5, 0)$$

$$\text{Midpoint } \overline{DA} = H(2, 2.5)$$

$$\text{Slope } \overline{EF} = \frac{5 - 2.5}{5.5 - 5} = \frac{2.5}{0.5} = 5$$

$$\text{Slope } \overline{FG} = \frac{2.5 - 0}{5 - 1.5} = \frac{2.5}{3.5} = \frac{5}{7}$$

$$\text{Slope } \overline{GH} = \frac{2.5 - 0}{2 - 1.5} = \frac{2.5}{0.5} = 5$$

$$\text{Slope } \overline{EH} = \frac{5 - 2.5}{5.5 - 2} = \frac{2.5}{3.5} = \frac{5}{7}$$

\overline{FG} and \overline{EH} are parallel since they have the same slope of $\frac{5}{7}$.

\overline{EF} and \overline{GH} are parallel since they have the same slope of 5.

There are no perpendicular sides. The slopes are not opposite reciprocals.

$$\begin{aligned} EF &= \sqrt{(5.5 - 5)^2 + (5 - 2.5)^2} \\ &= \sqrt{0.5^2 + 2.5^2} \\ &= \sqrt{6.5} \end{aligned}$$

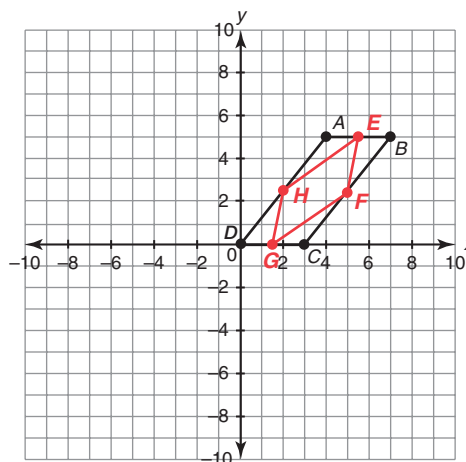
$$\begin{aligned} FG &= \sqrt{(5 - 1.5)^2 + (2.5 - 0)^2} \\ &= \sqrt{3.5^2 + 2.5^2} \\ &= \sqrt{18.5} \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(2 - 1.5)^2 + (2.5 - 0)^2} \\ &= \sqrt{0.5^2 + 2.5^2} \\ &= \sqrt{6.5} \end{aligned}$$

$$\begin{aligned} EH &= \sqrt{(5.5 - 2)^2 + (5 - 2.5)^2} \\ &= \sqrt{3.5^2 + 2.5^2} \\ &= \sqrt{18.5} \end{aligned}$$

Opposite sides of $EFGH$ are congruent.

Opposite sides are parallel and opposite sides are congruent, so the quadrilateral formed by the midpoints of the parallelogram is a parallelogram.



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10. The rhombus shown has vertices $A(4, 10)$, $B(8, 5)$, $C(4, 0)$, and $D(0, 5)$.

$$\text{Midpoint } \overline{AB} = E(6, 7.5)$$

$$\text{Midpoint } \overline{BC} = F(6, 2.5)$$

$$\text{Midpoint } \overline{CD} = G(2, 2.5)$$

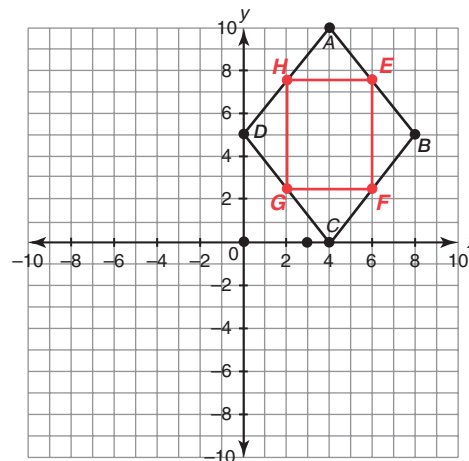
$$\text{Midpoint } \overline{DA} = H(2, 7.5)$$

$$\text{Slope } \overline{EF} = \frac{7.5 - 2.5}{6 - 6} = \frac{5}{0} = \text{undefined}$$

$$\text{Slope } \overline{FG} = \frac{2.5 - 2.5}{6 - 2} = \frac{0}{4} = 0$$

$$\text{Slope } \overline{GH} = \frac{2.5 - 7.5}{2 - 2} = \frac{-5}{0} = \text{undefined}$$

$$\text{Slope } \overline{EH} = \frac{7.5 - 7.5}{6 - 2} = \frac{0}{4} = 0$$



\overline{EF} and \overline{GH} are parallel since they have an undefined slope. The lines are vertical.

\overline{FG} and \overline{EH} are parallel since they have the same slope of 0. The lines are horizontal.

Consecutive sides are perpendicular since the segments are horizontal and vertical.

$$\begin{aligned} EF &= \sqrt{(6 - 6)^2 + (7.5 - 2.5)^2} \\ &= \sqrt{0^2 + 5^2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} FG &= \sqrt{(6 - 2)^2 + (2.5 - 2.5)^2} \\ &= \sqrt{4^2 + 0^2} \\ &= 4 \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(2 - 2)^2 + (2.5 - 7.5)^2} \\ &= \sqrt{0^2 + (-5)^2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} EH &= \sqrt{(6 - 2)^2 + (7.5 - 7.5)^2} \\ &= \sqrt{4^2 + 0^2} \\ &= 4 \end{aligned}$$

Opposite sides of $EFGH$ are congruent.

Opposite sides are parallel, consecutive sides are perpendicular, and opposite sides are congruent, so the quadrilateral formed by the midpoints of the rhombus is a rectangle.

11. The square shown has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 4)$, and $D(0, 4)$.

$$\text{Midpoint } \overline{AB} = E(2, 0)$$

$$\text{Midpoint } \overline{BC} = F(4, 2)$$

$$\text{Midpoint } \overline{CD} = G(2, 4)$$

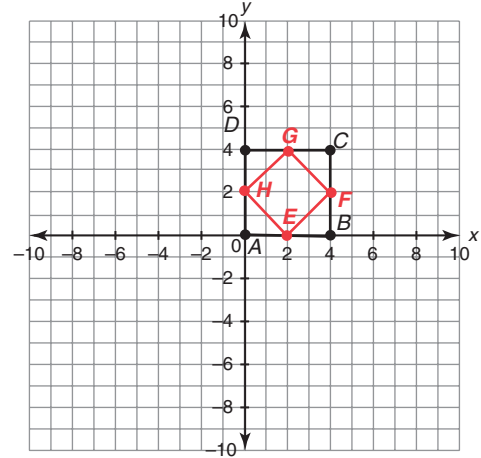
$$\text{Midpoint } \overline{DA} = H(0, 2)$$

$$\text{Slope } \overline{EF} = \frac{2 - 0}{4 - 2} = \frac{2}{2} = 1$$

$$\text{Slope } \overline{FG} = \frac{4 - 2}{2 - 4} = \frac{2}{-2} = -1$$

$$\text{Slope } \overline{GH} = \frac{2 - 4}{0 - 2} = \frac{-2}{-2} = 1$$

$$\text{Slope } \overline{EH} = \frac{2 - 0}{0 - 2} = \frac{2}{-2} = -1$$



\overline{FG} and \overline{EH} are parallel since they have the same slope of -1 .

\overline{EF} and \overline{GH} are parallel since they have the same slope of 1 .

Consecutive sides are perpendicular since the product of the slopes of each pair of consecutive sides is -1 .

$$\begin{aligned} EF &= \sqrt{(2 - 4)^2 + (0 - 2)^2} \\ &= \sqrt{(-2)^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} FG &= \sqrt{(4 - 2)^2 + (2 - 4)^2} \\ &= \sqrt{2^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(2 - 0)^2 + (4 - 2)^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} EH &= \sqrt{(0 - 2)^2 + (2 - 0)^2} \\ &= \sqrt{(-2)^2 + 2^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

All four sides of $EFGH$ are congruent.

Opposite sides are parallel, consecutive sides are perpendicular, and all four sides are congruent, so the quadrilateral formed by the midpoints of the square is a square.

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12. The kite shown has vertices $A(0, 2)$, $B(2, 0)$, $C(0, -3)$, and $D(-2, 0)$.

$$\text{Midpoint } \overline{AB} = E(1, 1)$$

$$\text{Midpoint } \overline{BC} = F(1, -1.5)$$

$$\text{Midpoint } \overline{CD} = G(-1, -1.5)$$

$$\text{Midpoint } \overline{DA} = H(-1, 1)$$

$$\text{Slope } \overline{EF} = \frac{1 - (-1.5)}{1 - 1} = \frac{2.5}{0} = \text{undefined}$$

$$\text{Slope } \overline{FG} = \frac{-1.5 + 1.5}{1 + 1} = \frac{0}{2} = 0$$

$$\text{Slope } \overline{GH} = \frac{-1.5 - 1}{-1 + 1} = \frac{-2.5}{0} = \text{undefined}$$

$$\text{Slope } \overline{EH} = \frac{1 - 1}{-1 - 1} = \frac{0}{-2} = 0$$

\overline{FG} and \overline{EH} are parallel since they have the same slope of 0. The lines are horizontal.

\overline{EF} and \overline{GH} are parallel since they have the same undefined slope. The lines are vertical.

Consecutive sides are perpendicular since the segments are horizontal and vertical.

$$\begin{aligned} EF &= \sqrt{(1 - 1)^2 + (1 + 1.5)^2} \\ &= \sqrt{0^2 + 2.5^2} \\ &= 2.5 \end{aligned}$$

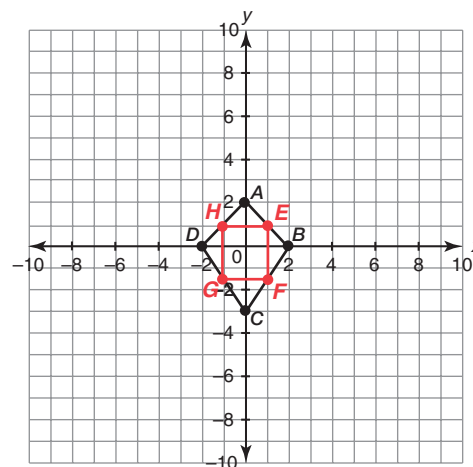
$$\begin{aligned} FG &= \sqrt{(1 + 1)^2 + (-1.5 + 1.5)^2} \\ &= \sqrt{2^2 + 0^2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(-1 + 1)^2 + (-1.5 - 1)^2} \\ &= \sqrt{0^2 + (-2.5)^2} \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} EH &= \sqrt{(1 + 1)^2 + (1 - 1)^2} \\ &= \sqrt{2^2 + 0^2} \\ &= 2 \end{aligned}$$

Opposite sides of $EFGH$ are congruent.

Opposite sides are parallel, consecutive sides are perpendicular, and opposite sides are congruent, so the quadrilateral formed by the midpoints of the kite is a rectangle.



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Bring On the Algebra

Deriving the Equation for a Circle

Problem Set

Write an equation in standard form of each circle.

1. a circle with center point at the origin when $r = 4$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 4^2$$

$$x^2 + y^2 = 16$$

2. a circle with center point at the origin when $r = \frac{2}{3}$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = \left(\frac{2}{3}\right)^2$$

$$x^2 + y^2 = \frac{4}{9}$$

3. a circle with center point $(6, 5)$ when $r = 1$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 6)^2 + (y - 5)^2 = 1^2$$

$$(x - 6)^2 + (y - 5)^2 = 1$$

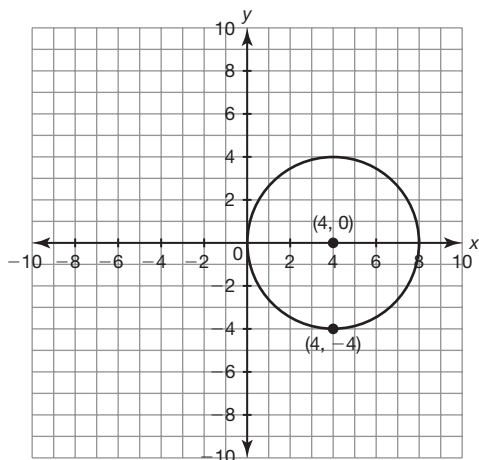
4. a circle with center point $(-8, -12)$ when $r = 7$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 8)^2 + (y + 12)^2 = 7^2$$

$$(x + 8)^2 + (y + 12)^2 = 49$$

5.

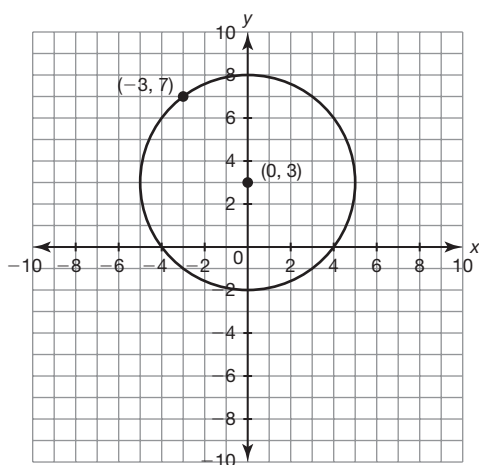


$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 4)^2 + (y - 0)^2 = 4^2$$

$$(x - 4)^2 + y^2 = 16$$

6.



$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + (y - 3)^2 = 5^2$$

$$x^2 + (y - 3)^2 = 25$$

Write the equation of each circle in standard form. Then identify the center point and radius of the circle.

7. $x^2 + y^2 + 6x - 2y + 1 = 0$

$$x^2 + y^2 + 6x - 2y + 1 = 0$$

$$x^2 + 6x + y^2 - 2y = -1$$

$$(x^2 + 6x + 9) + (y^2 - 2y + 1) = -1 + 9 + 1$$

$$(x + 3)^2 + (y - 1)^2 = 9$$

center: $(-3, 1)$, radius: 3

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8. $x^2 + y^2 - 14x + 4y + 49 = 0$

$$x^2 + y^2 - 14x + 4y + 49 = 0$$

$$x^2 - 14x + y^2 + 4y = -49$$

$$(x^2 - 14x + 49) + (y^2 + 4y + 4) = -49 + 49 + 4$$

$$(x - 7)^2 + (y + 2)^2 = 4$$

 center: $(7, -2)$, radius: 2

9. $x^2 + y^2 - 2x - 2y + 1 = 0$

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

$$x^2 - 2x + y^2 - 2y = -1$$

$$(x^2 - 2x + 1) + (y^2 - 2y + 1) = -1 + 1 + 1$$

$$(x - 1)^2 + (y - 1)^2 = 1$$

 center: $(1, 1)$, radius: 1

10. $81x^2 + 81y^2 + 36x - 324y + 327 = 0$

$$81x^2 + 81y^2 + 36x - 324y + 327 = 0$$

$$81x^2 + 36x + 81y^2 - 324y = -327$$

$$x^2 + \frac{4}{9}x + y^2 - 4y = -\frac{109}{27}$$

$$\left(x^2 + \frac{4}{9}x + \frac{4}{81}\right) + (y^2 - 4y + 4) = -\frac{109}{27} + \frac{4}{81} + 4$$

$$\left(x + \frac{2}{9}\right)^2 + (y - 2)^2 = \frac{1}{81}$$

 center: $\left(-\frac{2}{9}, 2\right)$, radius: $\frac{1}{9}$

11. $9x^2 + 9y^2 + 72x - 12y + 147 = 0$

$$9x^2 + 9y^2 + 72x - 12y + 147 = 0$$

$$9x^2 + 72x + 9y^2 - 12y = -147$$

$$x^2 + 8x + y^2 - \frac{4}{3}y = -\frac{49}{3}$$

$$(x^2 + 8x + 16) + \left(y^2 - \frac{4}{3}y + \frac{4}{9}\right) = -\frac{49}{3} + 16 + \frac{4}{9}$$

$$(x + 4)^2 + \left(y - \frac{2}{3}\right)^2 = \frac{1}{9}$$

 center: $\left(-4, \frac{2}{3}\right)$, radius: $\frac{1}{3}$

12. $36x^2 + 36y^2 - 36x + 72y + 29 = 0$

$$36x^2 + 36y^2 - 36x + 72y + 29 = 0$$

$$36x^2 - 36x + 36y^2 + 72y = -29$$

$$x^2 - x + y^2 + 2y = -\frac{29}{36}$$

$$\left(x^2 - x + \frac{1}{4}\right) + (y^2 + 2y + 1) = -\frac{29}{36} + \frac{1}{4} + 1$$

$$\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \frac{4}{9}$$

center: $\left(\frac{1}{2}, -1\right)$, radius: $\frac{2}{3}$

Determine if each equation represents a circle. If so, describe the location of the center and radius.

13. $x^2 + y^2 + 4x + 4y - 17 = 0$

$$x^2 + y^2 + 4x + 4y - 17 = 0$$

$$x^2 + 4x + y^2 + 4y = 17$$

$$(x^2 + 4x + 4) + (y^2 + 4y + 4) = 17 + 4 + 4$$

$$(x + 2)^2 + (y + 2)^2 = 25$$

center: $(-2, -2)$, radius: 5

14. $2x^2 + y^2 + 2x - 6y + 6 = 0$

This equation does not represent a circle because $A \neq C$, or $2 \neq 1$.

15. $x^2 + 4y^2 - 3x + 3y - 9 = 0$

This equations does not represent a circle because $A \neq C$, or $1 \neq 4$.

16. $x^2 + y^2 - 8x - 10y + 5 = 0$

$$x^2 + y^2 - 8x - 10y + 5 = 0$$

$$x^2 - 8x + y^2 - 10y = -5$$

$$(x^2 - 8x + 16) + (y^2 - 10y + 25) = -5 + 16 + 25$$

$$(x - 4)^2 + (y - 5)^2 = 36$$

center: $(4, 5)$, radius: 6

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Determine an equation of the circle that meets the given conditions.

17. Same center as circle A , $(x + 3)^2 + (y + 5)^2 = 9$, but with a circumference that is twice that of circle A
 The radius of circle A is $\sqrt{9}$, or 3. To determine the circumference of A , substitute 3 for r in the formula for the circumference of a circle.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3) \\ C &= 6\pi \end{aligned}$$

A circle with twice the circumference of circle A has circumference $2(6\pi)$, or 12π units. To determine its radius, substitute 12π for C in the formula for the circumference of a circle, and then solve for r .

$$\begin{aligned} C &= 2\pi r \\ 12\pi &= 2\pi r \\ 6 &= r \end{aligned}$$

The radius of the circle is 6. So an equation of the circle with the same center as circle A but with a circumference that is twice that of circle A is

$$(x + 3)^2 + (y + 5)^2 = 6^2, \text{ or } (x + 3)^2 + (y + 5)^2 = 36.$$

18. Same center as circle B , $(x - 6)^2 + (y + 4)^2 = 49$, but with a circumference that is three times that of circle B

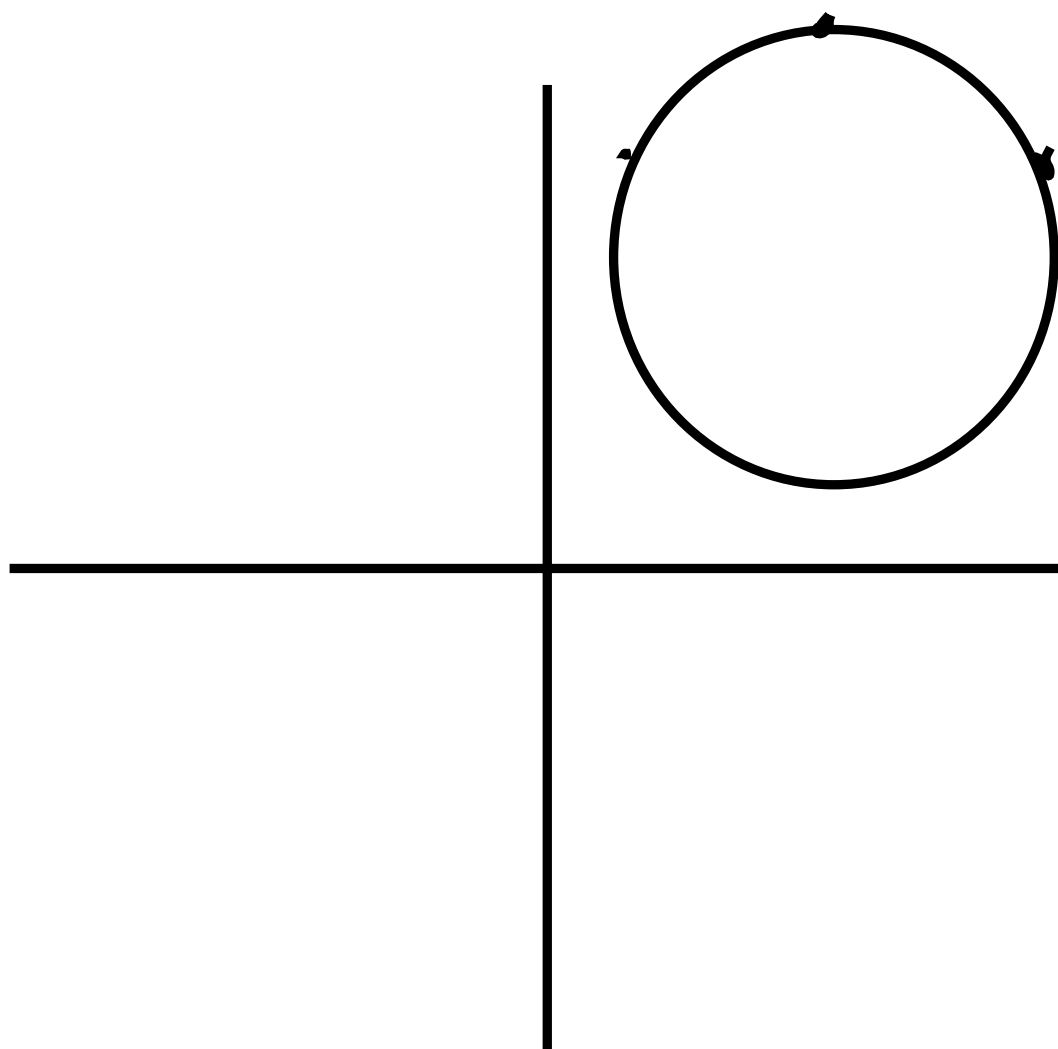
The radius of circle B is $\sqrt{49}$, or 7. To determine the circumference of B , substitute 7 for r in the formula for the circumference of a circle.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(7) \\ C &= 14\pi \end{aligned}$$

A circle with three times the circumference of circle B has circumference $3(14\pi)$, or 42π units. To determine its radius, substitute 42π for C in the formula for the circumference of a circle, and then solve for r .

$$\begin{aligned} C &= 2\pi r \\ 42\pi &= 2\pi r \\ 21 &= r \end{aligned}$$

The radius of the circle is 21. So an equation of the circle with the same center as circle B but with a circumference that is three times that of circle B is $(x - 6)^2 + (y + 4)^2 = 21^2$, or $(x - 6)^2 + (y + 4)^2 = 441$.



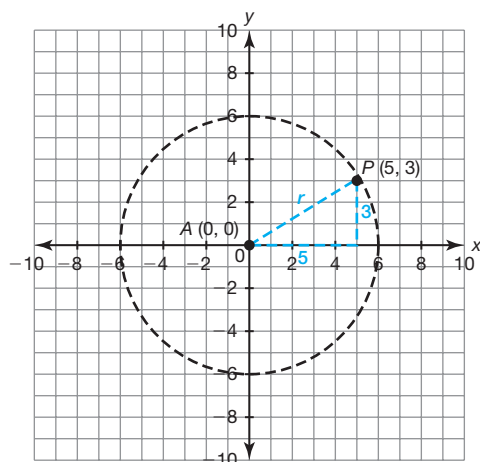
Name _____ Date _____

Is That Point on the Circle? Determining Points on a Circle

Problem Set

For each circle A , determine whether the given point P lies on the circle. Explain your reasoning.

1.



$$a^2 + b^2 = c^2$$

$$5^2 + 3^2 = r^2$$

$$25 + 9 = r^2$$

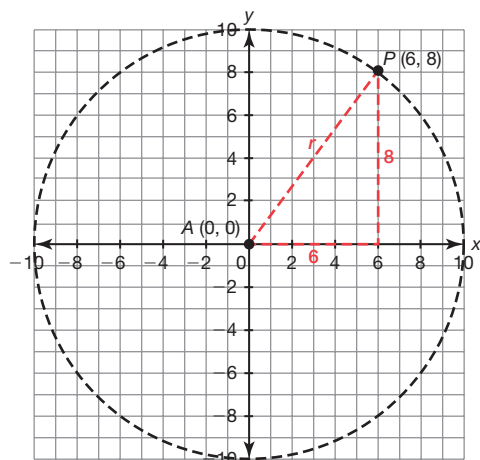
$$34 = r^2$$

$$r = \sqrt{34} \approx 5.8$$

Because the length of line segment AP is approximately 5.8 units instead of 6 units, line segment AP is not a radius of circle A ; therefore, point P does not lie on circle A .

18

2.



$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = r^2$$

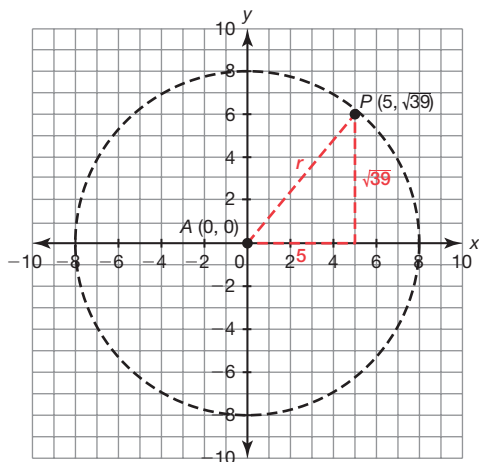
$$36 + 64 = r^2$$

$$100 = r^2$$

$$r = \sqrt{100} = 10$$

Because the length of line segment AP is 10 units, line segment AP is a radius of circle A ; therefore, point P must lie on circle A .

3.

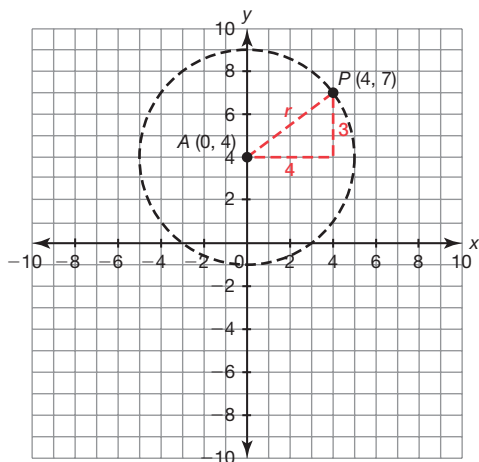


$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + (\sqrt{39})^2 &= r^2 \\ 25 + 39 &= r^2 \\ 64 &= r^2 \\ r &= \sqrt{64} = 8 \end{aligned}$$

Because the length of line segment AP is 8 units, line segment AP is a radius of circle A ; therefore, point P must lie on circle A .

18

4.

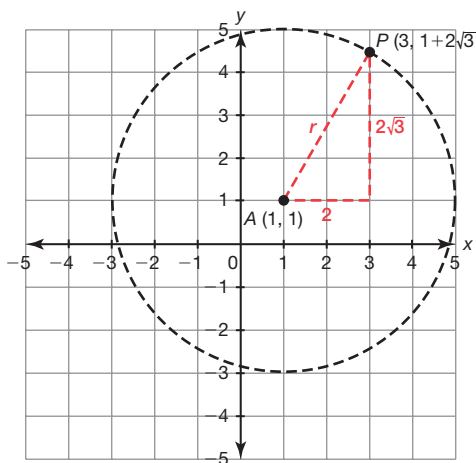


$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 3^2 &= r^2 \\ 16 + 9 &= r^2 \\ 25 &= r^2 \\ r &= \sqrt{25} = 5 \end{aligned}$$

Because the length of line segment AP is 5 units, line segment AP is a radius of circle A ; therefore, point P must lie on circle A .

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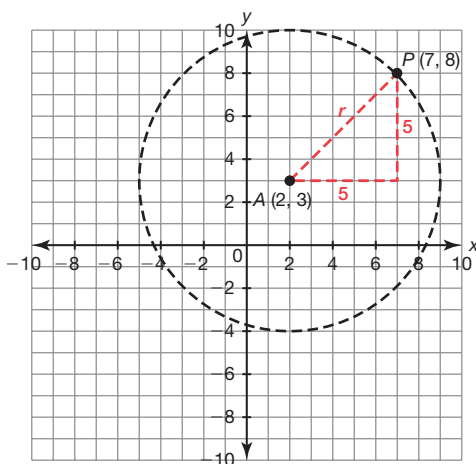
5.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 2^2 + (2\sqrt{3})^2 &= r^2 \\ 4 + 12 &= r^2 \\ 16 &= r^2 \\ r &= \sqrt{16} = 4 \end{aligned}$$

Because the length of line segment AP is 4 units, line segment AP is a radius of circle A ; therefore, point P must lie on circle A .

6.

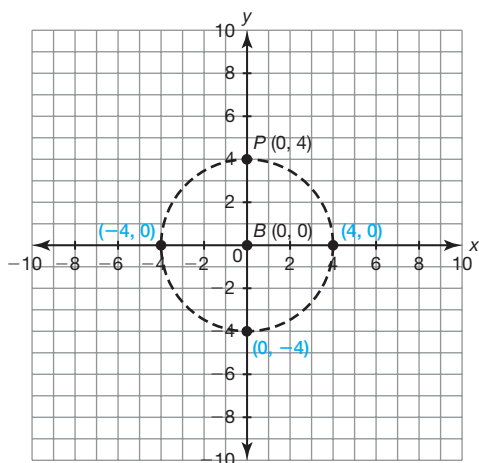


$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 5^2 &= r^2 \\ 25 + 25 &= r^2 \\ 50 &= r^2 \\ r &= \sqrt{50} \approx 7.1 \end{aligned}$$

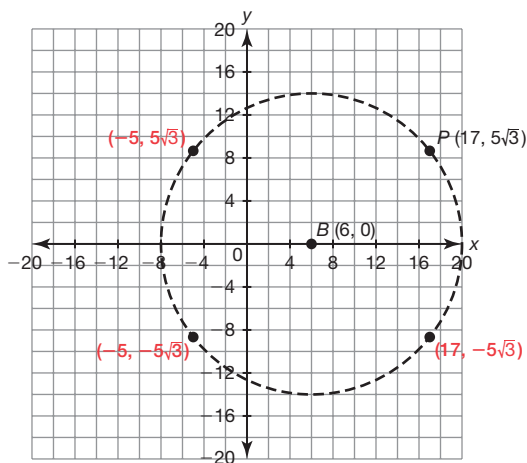
Because the length of line segment AP is approximately 7.1 units instead of 7 units, line segment AP is not a radius of circle A ; therefore, point P does not lie on circle A .

Use symmetry to determine the coordinates of each labeled point on the circle. Give exact values, not approximations.

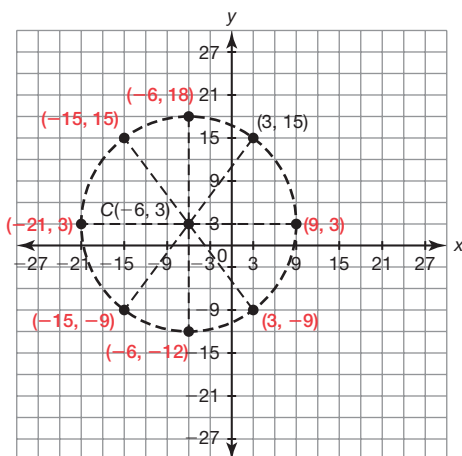
7.



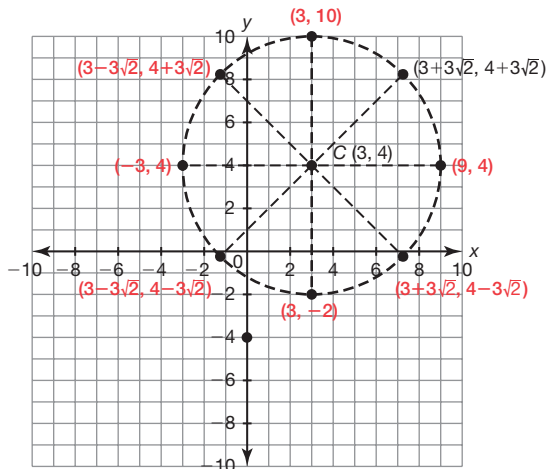
8.



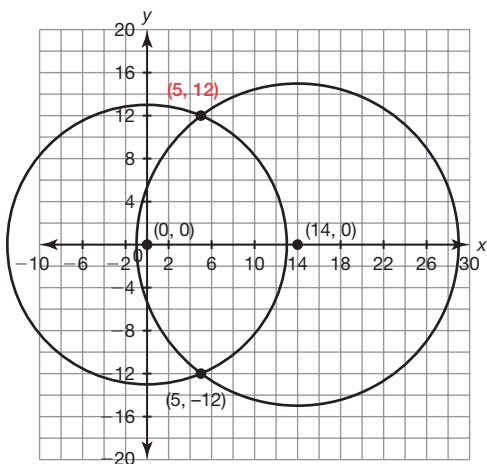
9.



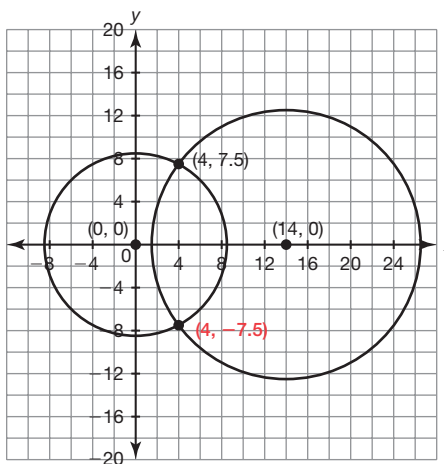
10.



11.



12.



Name _____ Date _____

The Parabola

Equation of a Parabola

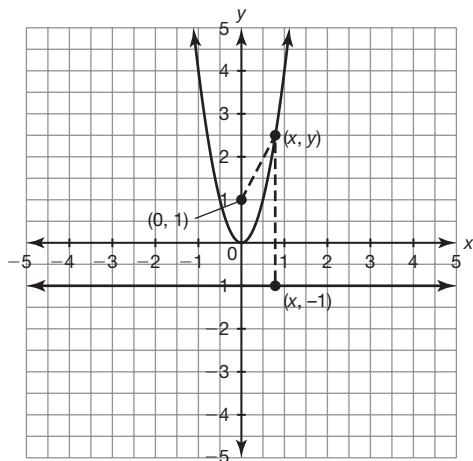
Vocabulary

- | | | |
|--------------------------------|----------|--|
| 1. locus of points | f | a. $Ax^2 + Dy = 0$ or $By^2 + Cx = 0$ |
| 2. parabola | d | b. $x^2 = 4py$ or $y^2 = 4px$ |
| 3. focus of a parabola | h | c. describes the orientation of the curvature of the parabola |
| 4. directrix of a parabola | i | d. a set of points in a plane that are equidistant from a fixed point and a fixed line |
| 5. general form of a parabola | a | e. the maximum or minimum point of a parabola |
| 6. standard form of a parabola | b | f. a set of points that share a property |
| 7. axis of symmetry | g | g. a line that passes through the parabola and divides the parabola into two symmetrical parts that are mirror images of each other |
| 8. vertex of a parabola | e | h. the fixed point from which all points of a parabola are equidistant |
| 9. concavity | c | i. the fixed line from which all points of a parabola are equidistant |

Problem Set

Determine the equation of the parabola.

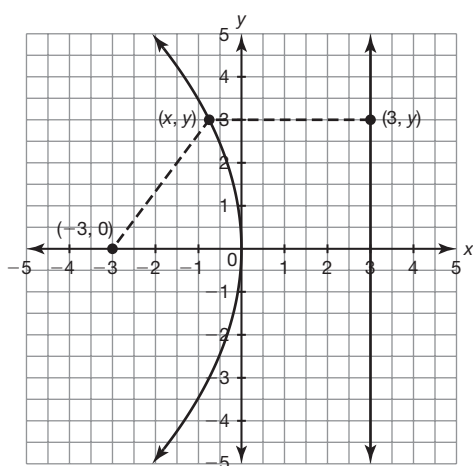
1.



$$\begin{aligned}\sqrt{(0-x)^2 + (1-y)^2} &= \sqrt{(x-x)^2 + (-1-y)^2} \\ \sqrt{x^2 + (1-y)^2} &= \sqrt{(-1-y)^2} \\ x^2 + (1-y)^2 &= (-1-y)^2 \\ x^2 + 1 - 2y + y^2 &= 1 + 2y + y^2 \\ x^2 &= 4y\end{aligned}$$

18

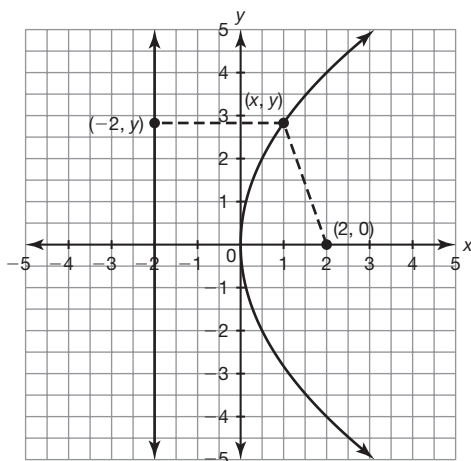
2.



$$\begin{aligned}\sqrt{(-3-x)^2 + (0-y)^2} &= \sqrt{(x-3)^2 + (y-y)^2} \\ \sqrt{(-3-x)^2 + (-y)^2} &= \sqrt{(x-3)^2} \\ (-3-x)^2 + (-y)^2 &= (x-3)^2 \\ 9 + 6x + x^2 + y^2 &= x^2 - 6x + 9 \\ y^2 &= -12x\end{aligned}$$

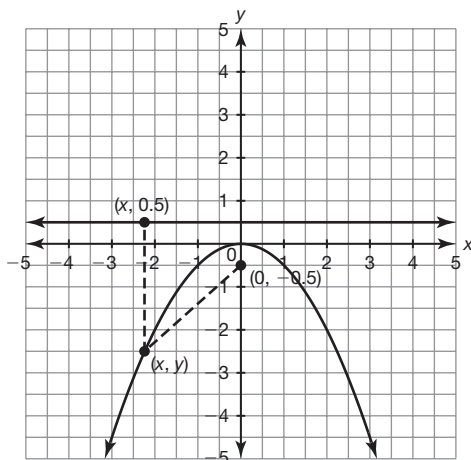
Name _____ Date _____

3.



$$\begin{aligned}\sqrt{(-2-x)^2 + (y-y)^2} &= \sqrt{(x-2)^2 + (y-0)^2} \\ \sqrt{(-2-x)^2} &= \sqrt{(x-2)^2 + y^2} \\ (-2-x)^2 &= (x-2)^2 + y^2 \\ 4 + 4x + x^2 &= x^2 - 4x + 4 + y^2 \\ 8x &= y^2\end{aligned}$$

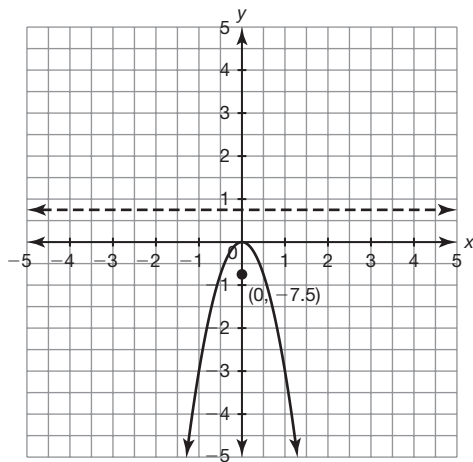
4.



$$\begin{aligned}\sqrt{(x-x)^2 + (0.5-y)^2} &= \sqrt{(x-0)^2 + (y-(-0.5))^2} \\ \sqrt{(0.5-y)^2} &= \sqrt{x^2 + (y+0.5)^2} \\ (0.5-y)^2 &= x^2 + (y+0.5)^2 \\ 0.25 - y + y^2 &= x^2 + y^2 + y + 0.25 \\ -2y &= x^2\end{aligned}$$

Identify the vertex, axis of symmetry, value of p , focus and directrix for each parabola.

5.



Vertex: $(0, 0)$

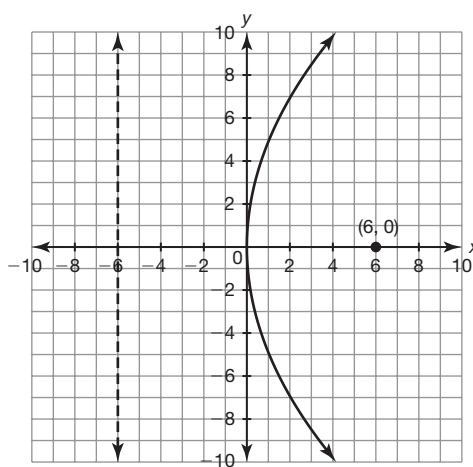
Axis of symmetry: $x = 0$

Value of p : -0.75

Focus: $(0, -0.75)$

Directrix: $y = 0.75$

6.



Vertex: $(0, 0)$

Axis of symmetry: $y = 0$

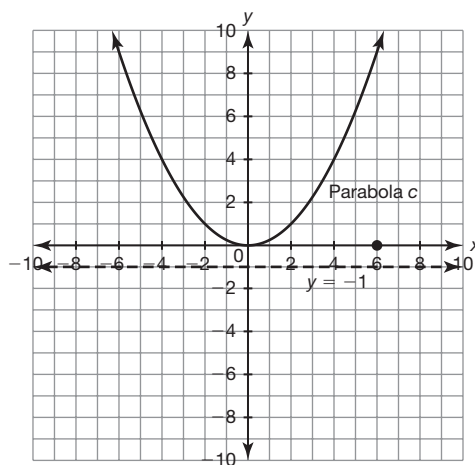
Value of p : 6

Focus: $(6, 0)$

Directrix: $x = -6$

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7.



Vertex: $(0, 0)$

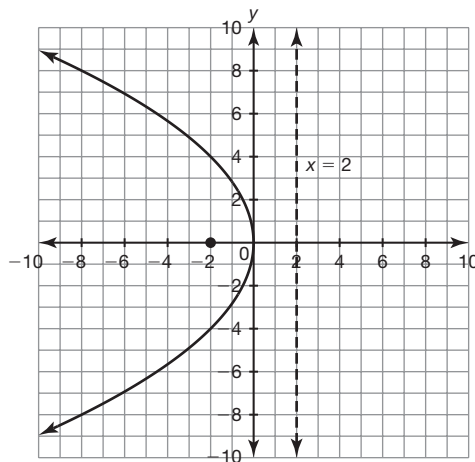
Axis of symmetry: $x = 0$

Value of p : 1

Focus: $(0, 1)$

Directrix: $y = -1$

8.



Vertex: $(0, 0)$

Axis of symmetry: $y = 0$

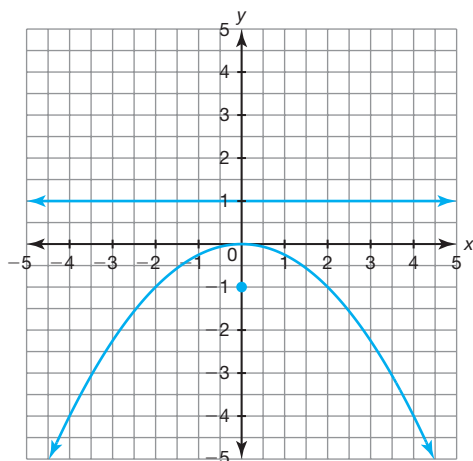
Value of p : -2

Focus: $(-2, 0)$

Directrix: $x = 2$

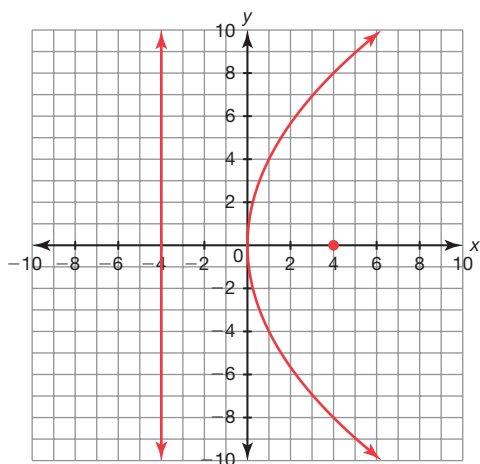
Sketch each parabola.

9. $x^2 = -4y$



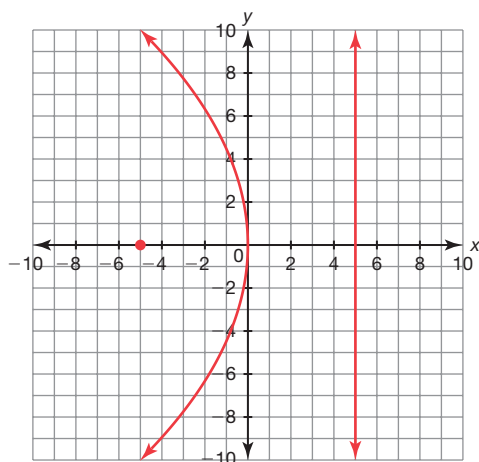
18

10. $y^2 = 16x$

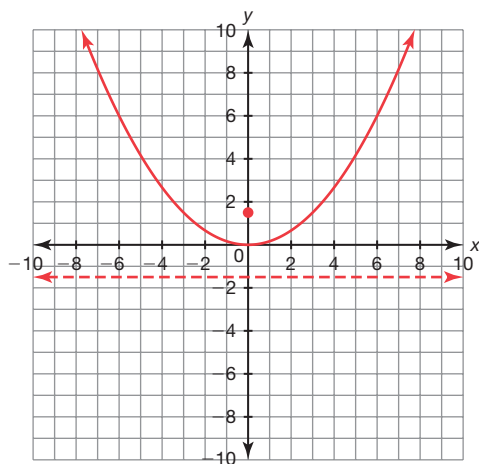


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11. $y^2 = -20x$



12. $x^2 = 6y$



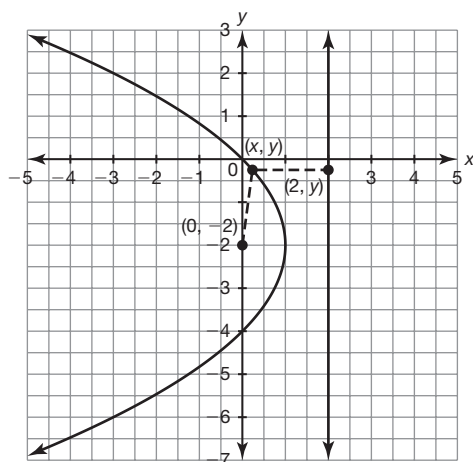
Name _____ Date _____

Simply Parabolic More with Parabolas

Problem Set

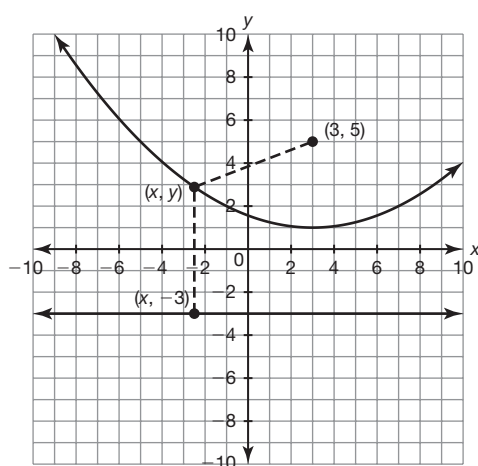
Determine the equation of the parabola.

1.



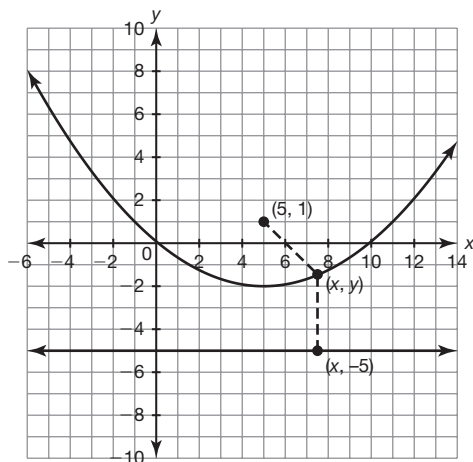
$$\begin{aligned}\sqrt{(0-x)^2 + (-2-y)^2} &= \sqrt{(x-2)^2 + (y-y)^2} \\ \sqrt{x^2 + (-2-y)^2} &= \sqrt{(x-2)^2} \\ x^2 + (-2-y)^2 &= (x-2)^2 \\ x^2 + 4 + 4y + y^2 &= x^2 - 4x + 4 \\ y^2 - 4x + 4y &= 0\end{aligned}$$

2.



$$\begin{aligned}\sqrt{(x-3)^2 + (y-5)^2} &= \sqrt{(x-x)^2 + (y-(-3))^2} \\ \sqrt{(x-3)^2 + (y-5)^2} &= \sqrt{(y+3)^2} \\ (x-3)^2 + (y-5)^2 &= (y+3)^2 \\ x^2 - 6x + 9 + y^2 - 10y + 25 &= y^2 + 6y + 9 \\ x^2 - 6x - 16y + 25 &= 0\end{aligned}$$

3.



$$\sqrt{(5-x)^2 + (1-y)^2} = \sqrt{(x-x)^2 + (y-(-5))^2}$$

$$\sqrt{(5-x)^2 + (1-y)^2} = \sqrt{(y+5)^2}$$

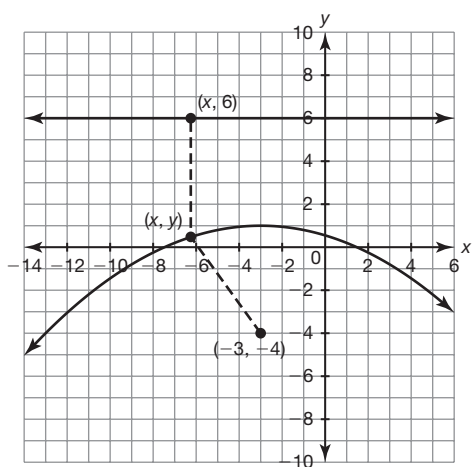
$$(5-x)^2 + (1-y)^2 = (y+5)^2$$

$$25 - 10x + x^2 + 1 - 2y + y^2 = y^2 + 10y + 25$$

$$x^2 - 10x - 12y + 1 = 0$$

18

4.



$$\sqrt{(x-x)^2 + (6-y)^2} = \sqrt{(x-(-3))^2 + (y-(-4))^2}$$

$$\sqrt{(6-y)^2} = \sqrt{(x+3)^2 + (y+4)^2}$$

$$(6-y)^2 = (x+3)^2 + (y+4)^2$$

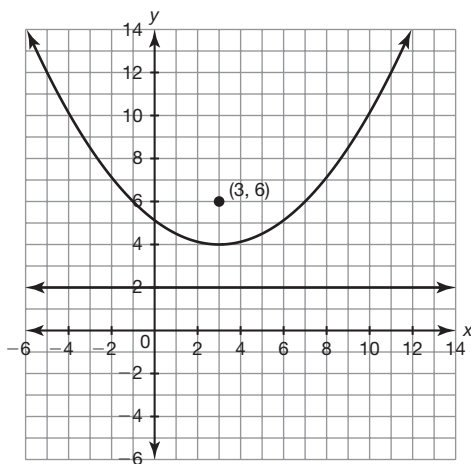
$$36 - 12y + y^2 = x^2 + 6x + 9 + y^2 + 8y + 16$$

$$0 = x^2 + 6x - 20y - 11$$

Name _____ Date _____

Identify the vertex, axis of symmetry, value of p , focus and directrix for each parabola.

5.



Vertex: (3, 4)

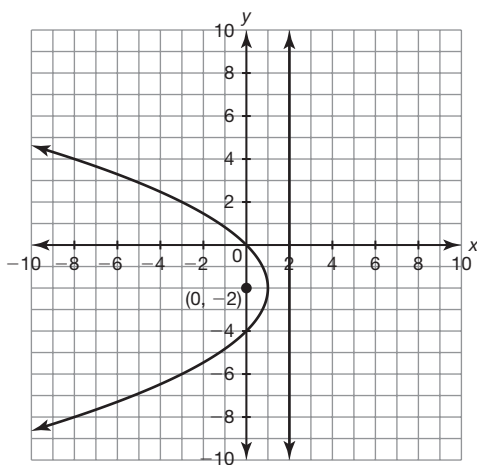
Axis of symmetry: $x = 3$

Value of p : 2

Focus: (3, 6)

Directrix: $y = 2$

6.



Vertex: (1, -2)

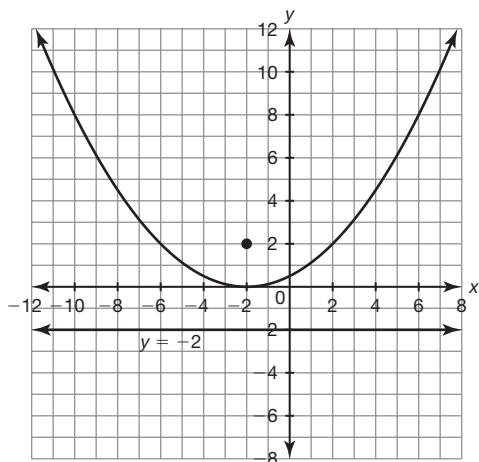
Axis of symmetry: $y = -2$

Value of p : -1

Focus: (0, -2)

Directrix: $x = 2$

7.



Vertex: $(-2, 0)$

Axis of symmetry: $x = -2$

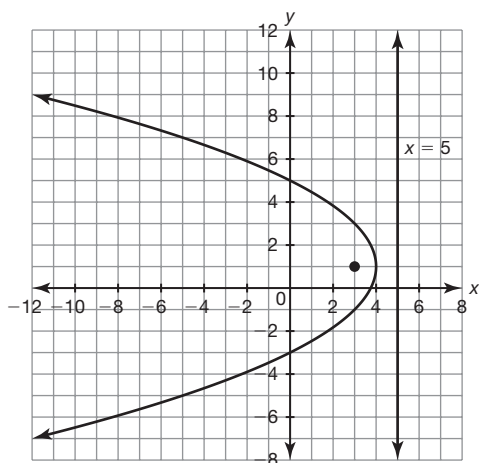
Value of p : 2

Focus: $(-2, 2)$

Directrix: $y = -2$

18

8.



Vertex: $(4, 1)$

Axis of symmetry: $y = 1$

Value of p : -1

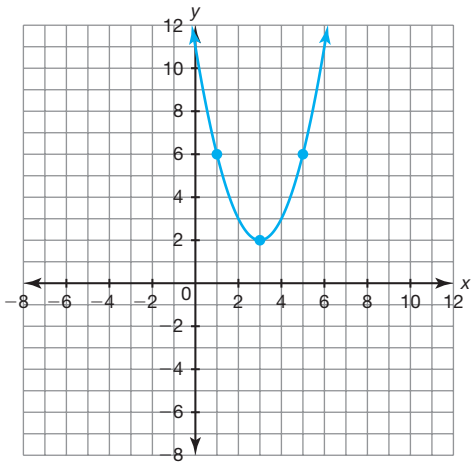
Focus: $(3, 1)$

Directrix: $x = 5$

Name _____ Date _____

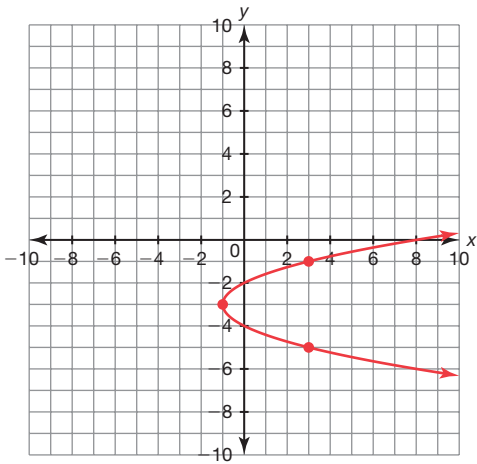
Complete the table for each equation. Then, plot the points and graph the curve on the coordinate plane.

9. $x^2 - 6x - y + 11 = 0$



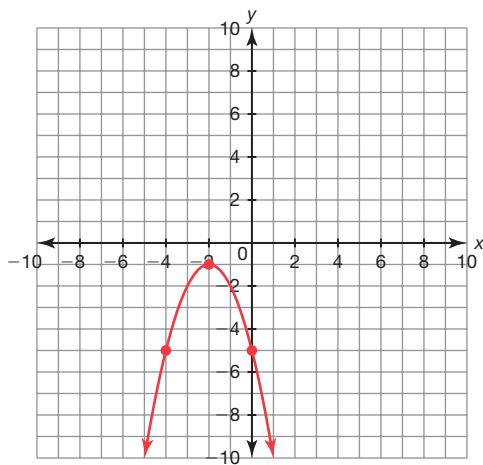
x	y
1	6
3	2
5	6

10. $y^2 - x + 6y + 8 = 0$



x	y
-1	-3
3	-1
3	-5

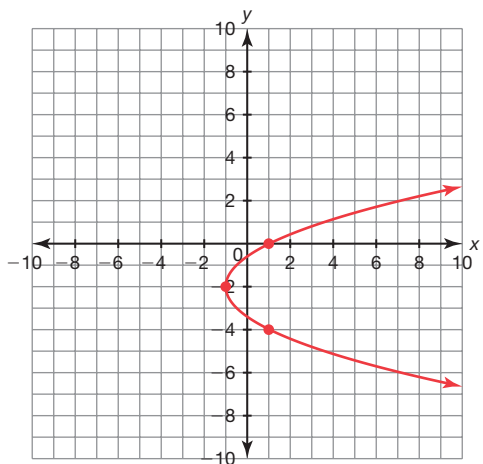
11. $x^2 + 4x + y + 5 = 0$



x	y
-4	-5
-2	-1
0	-5

18

12. $y^2 - 6x - y + 11 = 0$



x	y
-1	-1
1	0
1	-4

Name _____ Date _____

Rewrite the equations in standard form.

13. $x^2 - 4x + 3y + 10 = 0$

$x^2 - 4x + 3y + 10 = 0$

$x^2 - 4x = -3y - 10$

$x^2 - 4x + 4 = -3y - 10 + 4$

$(x - 2)^2 = -3(y + 2)$

14. $y^2 - 6x - 6y + 15 = 0$

$y^2 - 6x - 6y + 15 = 0$

$y^2 - 6y = 6x - 15$

$y^2 - 6y + 9 = 6x - 15 + 9$

$(y - 3)^2 = 6(x + 1)$

15. $x^2 + 2x + 2y + 1 = 0$

$x^2 + 2x + 2y + 1 = 0$

$x^2 + 2x = -2y - 1$

$x^2 + 2x + 1 = -2y - 1 + 1$

$(x + 1)^2 = -2y$

16. $y^2 - 2x + 6y + 3 = 0$

$y^2 - 2x + 6y + 3 = 0$

$y^2 + 6y = 2x - 3$

$x^2 + 6x + 9 = 2x - 3 + 9$

$(x + 3)^2 = 2(x + 3)$

