Name ______________________________ Date ____________

Left, Left, Left, Right, Left

Compound Probability for Data Displayed in Two-Way Tables

Vocabulary

Write the term that best completes each statement.

1. A two-way table is a table that shows the relationship between two data sets, one organized in ___rows___ and one organized in ___columns___.
2. A ___frequency___ table is a table that shows how often each item, number, or event appears in a sample space.
3. A ___two-way frequency table___, also called a ___contingency table___, shows the number of data points and their frequencies for two variables.
4. Data that can be grouped into categories, such as eye color and gender, are called ___categorical___ or ___qualitative___ data.
5. A relative frequency is the ratio of occurrences within a category to the ___total number___ of occurrences.
6. A two-way relative frequency table displays the ___relative frequencies___ for two categories of data.

Problem Set

Of the students in Molly's homeroom, 11 students have brown hair, 7 have black hair, 5 have auburn hair, 4 have blonde hair, and 3 have red hair. Calculate each relative frequency. Round to the nearest thousandth if necessary.

1. brown hair

   $\frac{11}{30} \approx 0.367$

2. black hair

   $\frac{7}{30} \approx 0.233$

3. auburn hair

   $\frac{5}{30} \approx 0.167$

4. blonde hair

   $\frac{4}{30} \approx 0.133$

5. red hair

   $\frac{3}{30} = 0.1$

6. brown or black hair

   $\frac{11 + 7}{30} = \frac{18}{30} = 0.6$

7. auburn or red hair

   $\frac{5 + 3}{30} = \frac{8}{30} \approx 0.267$

8. not brown hair

   $\frac{7 + 5 + 4 + 3}{30} = \frac{19}{30} \approx 0.633$

20

The two-way frequency table shows the number of students from each grade who plan to attend this year's homecoming football game.

Grade

Are you going to the homecoming game?

	Freshmen	Sophomores	Juniors	Seniors	Total
Attending Homecoming Game	31	28	25	32	116
Not Attending Homecoming Game	17	24	11	6	58
Total	48	52	36	38	174

Calculate the relative frequency of the entries in the two-way table. Round each relative frequency to the nearest tenth of a percent if necessary.

9. a freshman going to the homecoming game
$\frac{31}{174} \approx 17.8\%$

10. a sophomore going to the homecoming game
$\frac{28}{174} \approx 16.1\%$

11. a junior going to the homecoming game
$\frac{25}{174} \approx 14.4\%$

12. a senior going to the homecoming game
$\frac{32}{174} \approx 18.4\%$

13. a freshman not going to the homecoming game
$\frac{17}{174} \approx 9.8\%$

14. a sophomore not going to the homecoming game
$\frac{24}{174} \approx 13.8\%$

15. a junior not going to the homecoming game
$\frac{11}{174} \approx 6.3\%$

16. a senior not going to the homecoming game
$\frac{6}{174} \approx 3.4\%$

17. freshmen students
$\frac{48}{174} \approx 27.6\%$

20

Name ____________________ Date ____________

18. sophomore students

$\frac{52}{174} \approx 29.9\%$

19. junior students

$\frac{36}{174} \approx 20.7\%$

20. senior students

$\frac{38}{174} \approx 21.8\%$

21. students from all grades going to the homecoming game

$\frac{116}{174} \approx 66.7\%$

22. students from all grades not going to the homecoming game

$\frac{58}{174} \approx 33.3\%$

The two-way frequency table shows the current inventory of hardwood that a lumberyard carries. Suppose a board is selected at random from the lumberyard's inventory. Use the table to calculate each probability. Round to the nearest tenth of a percent if necessary.

Size

Type of Hardwood	1 × 2	1 × 3	1 × 4	1 × 6	Total
Oak	20	13	17	12	62
Maple	14	28	9	19	70
Cherry	8	17	28	25	78
Total	42	58	54	56	210

23. $P(\text{oak})$

$\frac{62}{210} \approx 0.295 = 29.5\%$

24. $P(\text{maple})$

$\frac{70}{210} \approx 0.333 = 33.3\%$

25. $P(\text{cherry})$

$\frac{78}{210} \approx 0.371 = 37.1\%$

26. $P(1 \times 2)$

$\frac{42}{210} = 0.2 = 20\%$

27. $P(1 \times 3)$

$\frac{58}{210} \approx 0.276 = 27.6\%$

28. $P(1 \times 4)$

$\frac{54}{210} \approx 0.257 = 25.7\%$

29. $P(1 \times 6)$

$\frac{56}{210} \approx 0.267 = 26.7\%$

30. $P(\text{maple and } 1 \times 3)$

$\frac{28}{210} \approx 0.133 = 13.3\%$

31. $P(\text{oak and } 1 \times 2)$

$\frac{20}{210} \approx 0.095 = 9.5\%$

32. $P(\text{maple or cherry})$

$\frac{70 + 78}{210} = \frac{148}{210} \approx 0.705 = 70.5\%$

33. $P(\text{cherry or } 1 \times 4)$

$\frac{78 + 54 - 28}{210} = \frac{104}{210} \approx 0.495 = 49.5\%$

34. $P(\text{maple or } 1 \times 6)$

$\frac{70 + 56 - 19}{210} = \frac{107}{210} \approx 0.510 = 51.0\%$

The two-way relative frequency table shows the results of a survey on the mayor's job approval. Suppose a member of the sample population is selected at random. Use the table of relative frequencies to calculate each probability. Express each probability as a decimal.

Party Affiliation

Do You Approve of the Mayor's Job Performance?

	Republican	Democrat	Independent	Total
Approve	0.14	0.25	0.03	0.42
Disapprove	0.22	0.1	0.12	0.44
No Opinion	0.03	0.07	0.04	0.14
Total	0.39	0.42	0.19	1

35. *P*(approve)

0.42

36. *P*(disapprove)

0.44

37. *P*(no opinion)

0.14

38. *P*(republican)

0.39

39. *P*(democrat)

0.42

40. *P*(independent)

0.19

41. *P*(republican and disapprove)

0.22

42. *P*(democrat and no opinion)

0.07

43. *P*(independent and approve)

0.03

44. *P*(disapprove or no opinion)

$0.44 + 0.14 = 0.58$

Name ______________________________ Date ______________

45. P(democrat or independent)

$0.42 + 0.19 = 0.61$

46. P(democrat or disapprove)

$0.42 + 0.44 - 0.1 = 0.76$

47. P(republican or approve)

$0.39 + 0.42 - 0.14 = 0.67$

48. P(independent or disapprove)

$0.19 + 0.44 - 0.12 = 0.51$

20

Name ____ Date ______________

It All Depends
Conditional Probability

Vocabulary

Define the term in your own words.

1. conditional probability

   Conditional probability is the probability of Event *B*, given that Event *A* has already occurred.

Problem Set

A five-sided letter die with faces labeled A, B, C, D, and E is rolled twice. The table shows the sample space of possible outcomes. Use the table to determine each probability. Express your answers as fractions in simplest form.

Second Roll

First Roll	A	B	C	D	E
A	A, A	A, B	A, C	A, D	A, E
B	B, A	B, B	B, C	B, D	B, E
C	C, A	C, B	C, C	C, D	C, E
D	D, A	D, B	D, C	D, D	D, E
E	E, A	E, B	E, C	E, D	E, E

1. *P*(B on the first roll)

   P(*B* on the first roll) $= \frac{1}{5}$

   There are 25 possible outcomes and the 5 cells in row *B* represent getting a *B* on the first roll. So, *P*(*B* on the first roll) $= \frac{15}{25}$, or $\frac{1}{5}$.

2. *P*(vowel on the second roll)

   P(vowel on the second roll) $= \frac{2}{5}$

   There are 25 possible outcomes and the 10 cells in column *A* and column *E* represent getting vowels on the second roll. So, *P*(vowel on the second roll) $= \frac{10}{25}$, or $\frac{2}{5}$.

3. P(consonant on the second roll)

P(consonant on the second roll) $= \frac{3}{5}$

There are 25 possible outcomes and the 15 cells in columns B, C, and D represent getting consonants on the second roll. So, P(consonant on the second roll) $= \frac{15}{25}$, or $\frac{3}{5}$.

4. P(A or B on the first roll)

P(A or B on the first roll) $= \frac{2}{5}$

There are 25 possible outcomes and the 10 cells in rows A and B represent getting an A or B on the first roll. So, P(A or B on the first roll) $= \frac{10}{25}$, or $\frac{2}{5}$.

5. P(A on the first roll and consonant on the second roll)

P(A on the first roll and consonant on the second roll) $= \frac{3}{25}$

The 3 cells located at the intersection of row A and column B, the intersection of row A and column C, and the intersection of row A and column D represent an A on the first roll and consonant on the second roll. There are 25 possible. So, P(A on the first roll and consonant on the second roll) $= \frac{3}{25}$.

6. P(vowel on the first roll and vowel or D on the second roll)

P(vowel on the first roll and vowel or D on the second roll) $= \frac{6}{25}$

Six of the 25 cells represent getting a vowel on the first roll and vowel or D on the second roll. Three of the 6 cells are located at the intersection of row A and column A, the intersection of row A and column E, and the intersection of row A and column D. The other 3 cells are located in row E.

So, P(vowel on the first roll and vowel or D on the second roll) $= \frac{6}{25}$.

7. P(B or D on the second roll, given that the first roll was a consonant)

P(B or D on the second roll, given that the first roll is a consonant) $= \frac{2}{5}$

Fifteen of the cells represent getting a consonant on the first roll. These 15 cells are located in rows B, C, and D.

Six out of those 15 cells represent getting a B or D on the second roll. Two of the 6 cells are located at the intersection of row B and column B, and the intersection of row B and column D. The remaining 4 out of the 6 cells are located in rows C and D.

So, P(B or D on the second roll, given that the first roll is a consonant) $= \frac{6}{15}$, or $\frac{2}{5}$.

Name ____________________ Date ____________

8. P(consonant on the second roll, given that the first roll was a vowel)

P(consonant on the second roll, given that the first roll was a vowel) $= \frac{3}{5}$

Ten of the cells represent getting a vowel on the first roll. These 10 cells are located in rows A and E.

Six out of those 10 cells represent getting a consonant on the second roll. Three of the 6 cells are located at the intersection of row A and column B, the intersection of row A and column C, and the intersection of row A and column D. The remaining 3 out of the 6 cells are located in row E.

So, P(consonant on the second roll, given that the first roll was a vowel) $= \frac{6}{10}$, or $\frac{3}{5}$.

9. P(A on the second roll, given that the first roll was a vowel)

P(A on the second roll, given that the first roll was a vowel) $= \frac{1}{5}$

Ten of the cells represent getting a vowel on the first roll. These 10 cells are located in rows A and E.

Two out of those 10 cells represent getting an A on the second roll. The 2 cells are located at the intersection of row A and column A and the intersection of row E and column A.

So, P(A on the second roll, given that the first roll was a vowel) $= \frac{2}{10}$, or $\frac{1}{5}$.

10. P(vowel on the second roll, given that the first roll was not an A)

P(vowel on the second roll, given that the first roll was not an A) $= \frac{2}{5}$

Twenty of the cells represent not getting an A on the first roll. These 20 cells are located in rows B, C, D, and E.

Eight of those 20 cells represent getting a vowel on the second roll. Two of the 8 cells are located at the intersection of row B and column A, and the intersection of row B and column E. The remaining 6 cells are located in rows C, D, and E.

So, P(vowel on the second roll, given that the first roll was not an A) $= \frac{8}{20}$, or $\frac{2}{5}$.

Determine each conditional probability.

11. Given $P(A) = 0.25$, $P(B) = 0.3$, and $P(A \text{ and } B) = 0.1$, determine $P(B \mid A)$.

$P(B|A) = 0.4$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.1}{0.25} = 0.4$$

20

12. Given $P(A) = \frac{2}{9}$, $P(B) = \frac{3}{10}$, and $P(A \text{ and } B) = \frac{2}{15}$, determine $P(A \mid B)$.

$P(A \mid B) = \frac{4}{9}$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$
$$= \frac{\frac{2}{15}}{\frac{3}{10}}$$
$$= \frac{\frac{2}{15}}{\frac{3}{10}} \times \frac{\frac{10}{3}}{\frac{10}{3}}$$
$$= \frac{2}{15} \times \frac{10}{3}$$
$$= \frac{20}{45}$$
$$= \frac{4}{9}$$

13. Given $P(A) = \frac{5}{6}$, $P(B) = \frac{1}{2}$, and $P(A \text{ and } B) = \frac{1}{12}$, determine $P(B \mid A)$.

$P(B \mid A) = \frac{1}{10}$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$
$$= \frac{\frac{1}{12}}{\frac{5}{6}}$$
$$= \frac{\frac{1}{12}}{\frac{5}{6}} \times \frac{\frac{6}{5}}{\frac{6}{5}}$$
$$= \frac{1}{12} \times \frac{6}{5}$$
$$= \frac{6}{60}$$
$$= \frac{1}{10}$$

14. Given $P(A) = 0.12$, $P(B) = 0.2$, and $P(A \text{ and } B) = 0.05$, determine $P(A \mid B)$.

$P(A \mid B) = 0.25$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$
$$= \frac{0.05}{0.2}$$
$$= 0.25$$

Name ______________________________ Date ____________

The jar on Mrs. Wilson's desk contains 20 green paper clips, 30 red paper clips, 15 white paper clips, and 10 black paper clips. She selects a paper clip without looking, does not replace it, and selects another. Determine each probability. Round each answer to the nearest tenth of a percent if necessary.

15. *P*(both paper clips are red)

The probability of choosing two red paper clips is approximately 15.7%.

$$
\begin{aligned}
P(\text{red 1st and red 2nd}) &= P(\text{red 1st}) \cdot P(\text{red 2nd}) \\
&= \frac{30}{75} \cdot \frac{29}{74} \\
&= \frac{2}{5} \cdot \frac{29}{74} \\
&= \frac{58}{370} \\
&\approx 0.157
\end{aligned}
$$

16. *P*(both paper clips are white)

The probability of choosing two white paper clips is approximately 3.8%.

$$
\begin{aligned}
P(\text{white 1st and white 2nd}) &= P(\text{white 1st}) \cdot P(\text{white 2nd}) \\
&= \frac{15}{75} \cdot \frac{14}{74} \\
&= \frac{1}{5} \cdot \frac{7}{37} \\
&= \frac{7}{185} \\
&\approx 0.038
\end{aligned}
$$

17. *P*(second paper clip is green | first paper clip is black)

The probability of choosing a green paper clip second, given that a black paper clip was chosen first is approximately 27.0%.

$$
\begin{aligned}
P(\text{green 2nd given black 1st}) &= \frac{P(\text{black 1st and green 2nd})}{P(\text{black 1st})} \\
&= \frac{P(\text{black 1st}) \cdot P(\text{green 2nd})}{P(\text{black 1st})} \\
&= \frac{\frac{10}{75} \times \frac{20}{74}}{\frac{10}{75}} \\
&= \frac{20}{74} \\
&\approx 0.270
\end{aligned}
$$

20

18. P(second paper clip is white | first paper clip is red)

The probability of choosing a white paper clip second, given that a red paper clip was chosen first is approximately 20.3%.

$$P(\text{white 2nd given red 1st}) = \frac{P(\text{red 1st and white 2nd})}{P(\text{red 1st})}$$

$$= \frac{P(\text{red 1st}) \cdot P(\text{white 2nd})}{P(\text{red 1st})}$$

$$= \frac{\frac{30}{75} \times \frac{10}{74}}{\frac{30}{75}}$$

$$= \frac{15}{74}$$

$$\approx 0.203$$

19. P(second paper clip is green | first paper clip is not green)

The probability of choosing a green paper clip second, given that a non-green paper clip was chosen first is approximately 27.0%.

$$P(\text{green 2nd given not green 1st}) = \frac{P(\text{not green 1st and green 2nd})}{P(\text{not green 1st})}$$

$$= \frac{P(\text{not green 1st}) \cdot P(\text{green 2nd})}{P(\text{not green 1st})}$$

$$= \frac{\frac{55}{75} \times \frac{20}{74}}{\frac{55}{75}}$$

$$= \frac{20}{74}$$

$$\approx 0.270$$

20. P(second paper clip is not red | first paper clip is red)

The probability of choosing a non-red paper clip second, given that a red paper clip was chosen first is approximately 60.8%.

$$P(\text{not red 2nd given red 1st}) = \frac{P(\text{red 1st and not red 2nd})}{P(\text{red 1st})}$$

$$= \frac{P(\text{red 1st}) \cdot P(\text{not red 2nd})}{P(\text{red 1st})}$$

$$= \frac{\frac{30}{75} \times \frac{45}{74}}{\frac{30}{75}}$$

$$= \frac{45}{74}$$

$$\approx 0.608$$

Name ____ Date ____________

The two-way frequency table shows the results of a study in which a new topical medicine cream was tested for its effectiveness in treating poison ivy. Half of the study participants applied the cream to their poison ivy for 3 days and noted any changes in their symptoms. The other half applied a placebo and noted any changes.

Results

Treatment	Significant improvement	Moderate Improvement	No Improvement	Total
Medicine Cream	21	10	4	35
Placebo	9	14	12	35
Total	30	24	16	70

21. Use the table to determine each probability. Round each answer to the nearest tenth of a percent if necessary.

a. *P*(significant improvement | medicine cream)

P(significant improvement | medicine cream) = 60%

$$P(\text{significant improvement given medicine cream}) = \frac{P(\text{medicine cream and significant improvement})}{P(\text{medicine cream})}$$

$$= \frac{\frac{21}{70}}{\frac{35}{70}}$$

$$= \frac{\frac{21}{70}}{\frac{35}{70}} \times \frac{\frac{70}{35}}{\frac{70}{35}}$$

$$= \frac{21}{70} \times \frac{70}{35}$$

$$= \frac{21}{35}$$

$$= 0.6$$

b. Are significant improvement and medicine cream treatment independent or dependent events? Explain your reasoning.

Significant improvement, *S*, and medicine cream treatment, *M*, are dependent events because the value of *P*(*S* | *T*) is not equal to the value of *P*(*S*).

$$P(S \mid M) = \frac{21}{35} = \frac{3}{5}$$

$$P(S) = \frac{30}{70} = \frac{3}{7}$$

22. Use the table to determine each probability. Round each answer to the nearest tenth of a percent if necessary.

a. P(placebo | no improvement)

P(placebo | no improvement) = 75%

$$P(\text{placebo given no improvement}) = \frac{P(\text{no improvement and placebo})}{P(\text{no improvement})}$$

$$= \frac{\frac{12}{70}}{\frac{16}{70}}$$

$$= \frac{\frac{12}{70}}{\frac{16}{70}} \times \frac{\frac{70}{16}}{\frac{70}{16}}$$

$$= \frac{12}{70} \times \frac{70}{16}$$

$$= \frac{12}{16}$$

$$= 0.75$$

b. Are no improvement and placebo treatment independent or dependent events? Explain your reasoning.

No improvement, N, and placebo treatment, C, are dependent events because the value of $P(C \mid N)$ is not equal to the value of $P(C)$.

$$P(C \mid N) = \frac{12}{16} = 0.75$$

$$P(C) = \frac{35}{70} = 0.50$$

20

Name ____ Date ____________

23. Use the table to determine each probability. Round each answer to the nearest tenth of a percent if necessary.

a. *P*(medicine cream | moderate improvement)

P(medicine cream | moderate improvement) ≈ 41.7%

$$P\text{ (medicine cream given moderate improvement)} = \frac{P\text{(moderate improvement and medicine cream)}}{P\text{(moderate improvement)}}$$

$$= \frac{\frac{10}{70}}{\frac{24}{70}}$$

$$= \frac{\frac{10}{70}}{\frac{24}{70}} \times \frac{\frac{70}{24}}{\frac{70}{24}}$$

$$= \frac{10}{70} \times \frac{70}{24}$$

$$= \frac{10}{24}$$

$$\approx 0.417$$

b. Are medicine cream treatment and moderate improvement independent or dependent events? Explain your reasoning.

Medicine cream treatment, *MT*, and moderate improvement, *MI*, are dependent events because the value of *P*(*MT* | *MI*) is not equal to the value of *P*(*MT*).

$$P(MT \mid MI) = \frac{10}{24} \approx 0.417$$

$$P(MI) = \frac{24}{70} \approx 0.343$$

24. Use the table to determine each probability. Round each answer to the nearest tenth of a percent if necessary.

a. P(placebo | significant improvement)

P(placebo | significant improvement) = 30%

$$P(\text{placebo given significant improvement}) = \frac{P(\text{significant improvement and placebo})}{P(\text{significant improvement})}$$

$$= \frac{\frac{9}{70}}{\frac{30}{70}}$$

$$= \frac{\frac{9}{70}}{\frac{30}{70}} \times \frac{\frac{70}{30}}{\frac{70}{30}}$$

$$= \frac{9}{70} \times \frac{70}{30}$$

$$= \frac{9}{30}$$

$$= 0.3$$

b. Are placebo treatment and significant improvement independent or dependent events? Explain your reasoning.

Placebo treatment, T, and significant improvement, S, are dependent events because the value of $P(T \mid S)$ is not equal to the value of $P(T)$.

$$P(T \mid S) = \frac{9}{30} = 0.3$$

$$P(T) = \frac{35}{70} = 0.5$$

20

Name ____________________ Date __________

Counting
Permutations and Combinations

Vocabulary

Define each term in your own words.

1. factorial

   The factorial of n, which is written with an exclamation mark as $n!$, is the product of all non-negative integers less than or equal to n: $n(n - 1)(n - 2)...$

2. permutation

   An ordered arrangement of items without repetition is called a permutation.

3. combination

   A combination is an unordered collection of items.

4. circular permutation

   A circular permutation is an ordered arrangement of objects around a circle.

Problem Set

Evaluate each expression.

1. ${}_8P_3$

   ${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$

2. ${}_5P_5$

   ${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5 \times 4 \times 3 \times 2 \times 1 = 120$

3. ${}_{10}P_4$

$${}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \times 9 \times 8 \times 7 = 5040$$

4. ${}_{7}P_5$

$${}_{7}P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$$

5. ${}_{6}P_6$

$${}_{6}P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

6. ${}_{9}P_6$

$${}_{9}P_6 = \frac{9!}{(9-6)!} = \frac{9!}{3!} = 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60{,}480$$

Calculate the number of possible outcomes in each of the following situations.

7. A computer code uses 4 randomly selected letters of the alphabet. If no letters are repeated, how many possible codes are there?

There are 358,800 possible codes.

$${}_{26}P_4 = 26 \times 25 \times 24 \times 23 = 358{,}800$$

8. Twelve students are competing in the finals of a spelling bee. The top 3 finishers are awarded a gold, silver, and bronze medal. In how many ways can the medals be won?

There medals can be won in 1320 ways.

$${}_{12}P_3 = 12 \times 11 \times 10 = 1320$$

9. A summer camp offers 12 different afternoon activities. Caleb selects 2 of the activities to do today. How many possible outcomes are there if the order of the activities is important?

There are 132 different ways for Caleb to select 2 activities.

$${}_{12}P_2 = 12 \times 11 = 132$$

10. There are 15 different seminars at a teacher's convention. Mrs. Alvarez will choose 3 of the seminars to attend today. How many possible outcomes are there if the order of the seminars is important?

There are 2730 different arrangements of choosing three seminars.

$${}_{15}P_3 = 15 \times 14 \times 13 = 2730$$

Name ______________________________ Date ______________

Evaluate each expression.

11. ${}_{11}C_4$

${}_{11}C_4 = \frac{11!}{(11-4)!4!} = \frac{11!}{7!4!} = 330$

12. ${}_6C_5$

${}_6C_5 = \frac{6!}{(6-5)!5!} = \frac{6!}{1!5!} = 6$

13. ${}_5C_3$

${}_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = 10$

14. ${}_{12}C_{10}$

${}_{12}C_{10} = \frac{12!}{(12-10)!10!} = \frac{12!}{2!10!} = 66$

15. ${}_8C_4$

${}_8C_4 = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = 70$

16. ${}_7C_2$

${}_7C_2 = \frac{7!}{(7-2)!2!} = \frac{7!}{5!2!} = 21$

Calculate the number of possible outcomes in each of the following situations.

17. A committee of 4 students is to be formed from a homeroom of 25 students. How many different committees are possible?

${}_{25}C_4 = \frac{25!}{21!4!} = 12{,}650$

18. A pizzeria offers 8 different toppings on their pizzas. If a customer wants to order a 3-topping pizza, how many possible options are there?

${}_8C_3 = \frac{8!}{5!3!} = 56$

19. Seven friends are playing musical chairs. In the first round there are 5 chairs, so only 5 of the friends will move on to the second round. How many different groups of friends are possible for the second round of the game?

${}_7C_5 = \frac{7!}{2!5!} = 24$

20. Fran has 4 pennies, 3 nickels, 5 dimes, and 2 quarters in her pocket. In how many ways can she pull 3 coins out of her pocket if the order of the coins is not important?

${}_{14}C_3 = \frac{14!}{11!3!} = 364$

A website requires users to enter a 6-digit personal identification number (PIN) for security. A user's PIN must use the digits 1, 2, 3, 4, 5, and 6, and no digit can be used more than once. Determine each probability. Express your answers as fractions in simplest form.

21. Suppose a user is unable to remember his PIN and enters the digits randomly. What is the probability that he will guess correctly?

The probability of guessing correctly is $\frac{1}{720}$.

There is 1 correct PIN and I determined the total possible number of PINs by calculating ${}_6P_6$, which equals 720.

So, the probability of guessing correctly is $\frac{1}{720}$.

22. Suppose a user randomly selects a PIN. What is the probability that the user's PIN is an even number?

The probability of selecting an even PIN is $\frac{1}{2}$.

An even PIN number must have a 2, 4, or 6 as the last digit. There are 3 choices for the last digit, which means the number of even PINs is $5 \times 4 \times 3 \times 2 \times 1 \times 3 = 360$. I determined the total possible number of PINs by calculating ${}_6P_6$, which equals 720.

So, the probability of selecting an even PIN is $\frac{360}{720}$, or $\frac{1}{2}$.

23. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with 3 even digits and ends with 3 odd digits?

The probability of selecting a PIN that begins with 3 even digits and ends with 3 odd digits is $\frac{1}{20}$.

Three of the digits are even: 2, 4, and 6. There are ${}_3P_3 = 6$ ways to arrange the three even digits. Three of the digits are odd: 1, 3, and 5. There are ${}_3P_3 = 6$ ways to arrange the three odd digits. So, there are $6 \times 6 = 36$ different PINs that begin with 3 even digits and end with 3 odd digits.

I determined the total possible number of PINs by calculating ${}_6P_6$, which equals 720.

So, the probability of selecting a PIN that begins with 3 even digits and ends with 3 odd digits is $\frac{36}{720}$, or $\frac{1}{20}$.

Name ______________________________ Date ____________

24. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with the digits 123?

The probability of selecting a PIN that begins with the digits 123 is $\frac{1}{120}$.

There is one way for the first three digits to be 123, and there are ${}_3P_3 = 6$ ways to arrange the last three digits. So, there are $1 \times 6 = 6$ different PINs that begin with the digits 123.

I determined the total possible number of PINs by calculating ${}_6P_6$, which equals 720.

So, the probability of selecting a PIN that begins with the digits 123 is $\frac{6}{720}$, or $\frac{1}{120}$.

A group of 6 seniors, 5 juniors, and 4 sophomores are running for student council. The council is made up of 6 members. Assume that each student has an equal chance of being elected to student council. Determine each probability and express your answers as fractions in simplest form.

25. What is the probability that 2 seniors, 2 juniors, and 2 sophomores are elected?

The probability of choosing 2 seniors, 2 juniors, and 2 sophomores is $\frac{180}{1001}$.

There are 15 ways to choose 2 seniors: ${}_6C_2 = 15$.
There are 10 ways to choose 2 juniors: ${}_5C_2 = 10$.
There are 6 ways to choose 2 sophomores: ${}_4C_2 = 6$.
So, there are 900 ways to choose 2 seniors, 2 juniors, and 2 sophomores: $15 \times 10 \times 6 = 900$.

There are 5005 ways to choose 6 student council members from a pool of 15 candidates: ${}_{15}C_6 = 5005$.

So, the probability of choosing 2 seniors, 2 juniors, and 2 sophomores is $\frac{900}{5005}$, or $\frac{180}{1001}$

26. What is the probability that the student council is made up of all seniors?

The probability of choosing all seniors is $\frac{1}{5005}$.

There is 1 way to choose 6 seniors: ${}_6C_6 = 1$.
There is 1 way to choose 0 juniors: ${}_5C_0 = 1$.
There is 1 way to choose 0 sophomores: ${}_4C_0 = 1$.
So, there is 1 way to choose 6 seniors: $1 \times 1 \times 1 = 1$.

There are 5005 ways to elect a student council that has 6 seniors: ${}_{15}C_6 = 5005$.

So, the probability of choosing all seniors is $\frac{1}{5005}$.

27. What is the probability that 3 seniors, 2 juniors, and 1 sophomore are elected?

The probability of choosing 3 seniors, 2 juniors, and 1 sophomore is $\frac{160}{1001}$.

There are 20 ways to choose 3 seniors: ${}_6C_3 = 20$.
There are 10 ways to choose 2 juniors: ${}_5C_2 = 10$.
There are 4 ways to choose 1 sophomore: ${}_4C_1 = 4$.
So, there are 800 ways to elect a student council that has 3 seniors, 2 juniors, and 1 sophomore:
$20 \times 10 \times 4 = 800$.

There are 5005 ways to choose 6 student council members from a pool of 15 candidates:
${}_{15}C_6 = 5005$.

So, the probability of choosing 3 seniors, 2 juniors, and 1 sophomore is $\frac{800}{5005}$, or $\frac{160}{1001}$.

28. What is the probability that 3 juniors and 3 sophomores are elected?

The probability of choosing 3 juniors and 3 sophomores is $\frac{8}{1001}$.

There is 1 way to choose 0 seniors: ${}_6C_0 = 1$.
There are 10 ways to choose 3 juniors: ${}_5C_3 = 10$.
There are 4 ways to choose 3 sophomores: ${}_4C_3 = 4$.
So, there are 40 ways to elect a student council that has 0 seniors, 3 juniors, and 3 sophomores:
$1 \times 10 \times 4 = 40$.

There are 5005 ways to choose 6 student council members from a pool of 15 candidates:
${}_{15}C_6 = 5005$.

So, the probability of choosing 0 seniors, 3 juniors, and 3 sophomores is $\frac{40}{5005}$, or $\frac{8}{1001}$.

Calculate the number of ways the letters of each word can be arranged.

29. SUNNY

The letters in the word SUNNY can be arranged 60 different ways.

$\frac{5!}{2!} = 60$

30. FACTORIAL

The letters in the word FACTORIAL can be arranged 181,440 different ways.

$\frac{9!}{2!} = 181{,}440$

31. ARRANGE

The letters in the word ARRANGE can be arranged 1260 different ways.

$\frac{7!}{2!2!} = 1260$

32. PROBABILITY

The letters in the word PROBABILITY can be arranged 9,979,200 different ways.

$\frac{11!}{2!2!} = 9{,}979{,}200$

33. PARALLEL

The letters in the word PARALLEL can be arranged 3360 different ways.

$\frac{8!}{3!2!} = 3360$

34. MISSISSIPPI

The letters in the word MISSISSIPPI can be arranged 34,650 different ways.

$\frac{11!}{4!4!2!} = 34{,}650$

Name ____ Date ______________

Calculate the number of ways each arrangement can be made.

35. 12 flowers planted around the base of a tree

The flowers can be arranged around the base of the tree in 39,916,800 different ways.

$(12 - 1)! = 11! = 39{,}916{,}800$

36. 7 dinner guests seated around a table

The dinner guests can be arranged around the table in 720 different ways.

$(7 - 1)! = 6! = 720$

37. 10 candles arranged around the outside of a circulate birthday cake

The candles can be arranged around the birthday cake in 362,880 different ways.

$(10 - 1)! = 9! = 362{,}880$

38. 3 baseball players standing in a circle under a fly ball

The players can be circled around the ball in 3 different ways.

$(3 - 1)! = 2! = 3$

39. 6 teachers seated around a circular table at a conference

The teachers can be arranged around the table in 120 different ways.

$(6 - 1)! = 5! = 120$

40. 5 kittens arranged around a ball of yarn

The kittens can be arranged around the ball of yarn in 24 different ways.

$(5 - 1)! = 4! = 24$

20

Name ______________________________ Date ____________

Trials
Independent Trials

Problem Set

Determine the probability in each situation. Express your answers as fractions in simplest form.

1. On average, Malcolm makes a par on $\frac{2}{3}$ of the golf holes that he plays. What is the probability that he will make a par on each of the next 2 holes that he plays?

The probability that Malcolm makes a par on the next 2 holes is $\frac{4}{9}$.

Let PAR represent making par on one hole.

$$\begin{aligned} P(\text{PAR 1st and PAR 2nd}) &= P(\text{PAR 1st}) \cdot P(\text{PAR 2nd}) \\ &= \frac{2}{3} \cdot \frac{2}{3} \\ &= \frac{4}{9} \end{aligned}$$

2. Clarence rolls 2 number cubes. What is the probability that he will roll a number greater than 4 on both rolls?

The probability that Clarence will roll a number greater than 4 on the next 2 rolls is $\frac{1}{9}$.

Let >4 represent rolling a number greater than 4.

$$\begin{aligned} P(>4 \text{ 1st and } >4 \text{ 2nd}) &= P(>4 \text{ 1st}) \cdot P(>4 \text{ 2nd}) \\ &= \frac{2}{6} \cdot \frac{2}{6} \\ &= \frac{1}{3} \cdot \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

3. For every 5 penalty kicks that Missy attempts, she scores on average 3 goals. What is the probability that Missy will score 2 goals in her next 2 penalty kick attempts?

The probability that Missy will make the next two penalty kicks is $\frac{9}{25}$.

$$\begin{aligned} P(\text{make 1st and make 2nd}) &= P(\text{make 1st}) \cdot P(\text{make 2nd}) \\ &= \frac{3}{5} \cdot \frac{3}{5} \\ &= \frac{9}{25} \end{aligned}$$

4. On average 3 out of 4 students order spaghetti at the cafeteria when it is offered. What is the probability that the first 2 students in line for lunch will have spaghetti if it is offered?

The probability that the next students will order spaghetti is $\frac{9}{16}$.

Let S represent a student that orders spaghetti.

$$P(S \text{ 1st and } S \text{ 2nd}) = P(S \text{ 1st}) \cdot P(S \text{ 2nd})$$
$$= \frac{3}{4} \cdot \frac{3}{4}$$
$$= \frac{9}{16}$$

5. A spinner has 8 equal size spaces with the colors: red, green, yellow, red, blue, white, green, red. If the spinner is spun twice, what is the probability of spinning red on both spins?

The probability of a red result on the next two spins is $\frac{9}{64}$.

$$P(\text{red 1st and red 2nd}) = P(\text{red 1st}) \cdot P(\text{red 2nd})$$
$$= \frac{3}{8} \cdot \frac{3}{8}$$
$$= \frac{9}{64}$$

6. The table shows the results of a survey of likely voters. Suppose 2 likely voters are selected at random. Based on the results of the survey, what is the probability that both likely voters support Issue #1?

Do You Support Issue #1?	
Response	**Frequency**
Strongly Oppose	40
Somewhat Oppose	16
No Opinion	19
Somewhat Support	10
Strongly Support	25

The probability that both voters support Issue #1 is $\frac{48}{484}$.

Let S represent a voter that supports Issue #1.

$$P(S \text{ 1st and } S \text{ 2nd}) = P(S \text{ 1st}) \cdot P(S \text{ 2nd})$$
$$= \frac{35}{110} \cdot \frac{35}{110}$$
$$= \frac{7}{22} \cdot \frac{7}{22}$$
$$= \frac{49}{484}$$

20

Name ______________________________ Date __________

According to a recent survey, 80% of high school students have their own cell phone. Suppose 10 high school students are selected at random. Determine each probability. Round your answers to the nearest tenth of a percent if necessary.

7. P(8 of the students have cell phones)

$_{10}C_8(0.8)^8(0.2)^2 \approx 0.302 = 30.2\%$

8. P(5 of the students have cell phones)

$_{10}C_5(0.8)^5(0.2)^5 \approx 0.026 = 2.6\%$

9. P(3 of the students do not have cell phones)

$_{10}C_3(0.2)^3(0.8)^7 \approx 0.201 = 20.1\%$

10. P(all of the students have cell phones)

$_{10}C_{10}(0.8)^{10}(0.2)^0 \approx 0.107 = 10.7\%$

11. P(9 of the students have cell phones)

$_{10}C_9(0.8)^9(0.2)^1 \approx 0.268 = 26.8\%$

12. P(none of the student shave cell phones)

$_{10}C_0(0.8)^0(0.2)^{10} \approx 0.0\%$

Based on past results, a batter knows that the opposing pitcher throws a fastball 75% of the time and a curveball 25% of the time. Suppose the batter sees 8 pitches during a particular at-bat. Determine each probability. Round your answers to the nearest tenth of a percent if necessary.

13. P(4 fastballs and 4 curveballs)

$_8C_4(0.75)^4(0.25)^4 \approx 0.087 = 8.7\%$

14. P(all fastballs)

$_8C_8(0.75)^8(0.25)^0 \approx 0.100 = 10.0\%$

15. P(5 fastballs and 3 curveballs)

$_8C_5(0.75)^5(0.25)^3 \approx 0.208 = 20.8\%$

16. P(5 curveballs and 3 fastballs)

$_8C_5(0.25)^5(0.75)^3 \approx 0.023 = 2.3\%$

17. P(7 fastballs and 1 curveball)

$_8C_7(0.75)^7(0.25)^1 \approx 0.267 = 26.7\%$

18. P(no fastballs)

$_8C_0(0.75)^0(0.25)^8 \approx 0.0\%$

Name ____________________ Date ____________

To Spin or Not to Spin
Expected Value

Vocabulary

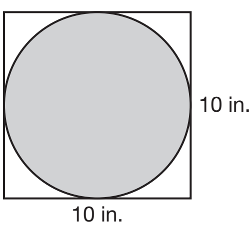
Write the term that best completes each statement.

1. Geometric probability is a ratio of measures such as length, area, and volume.
2. The expected value is the average value when the number of trials is large.

Problem Set

Determine the probability that a dart that lands on a random part of each target will land in the shaded scoring section. Assume that all squares in a figure and all circles in a figure are congruent unless otherwise marked. Round each answer to the nearest tenth of a percent if necessary.

1.



The probability of a dart landing in the shaded area is approximately 78.5%.

Area of Entire Board: 10(10) = 100 in.^{2}

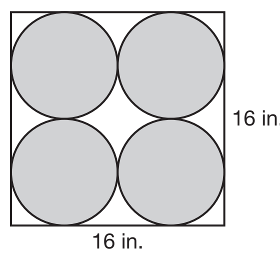
Area of Shaded Section:

$5^2\pi \approx 78.54$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{78.54}{100} \approx 78.5\%$

2.



The probability of a dart landing in the shaded area is approximately 78.5%.

Area of Entire Board: 16(16) = 256 in.^{2}

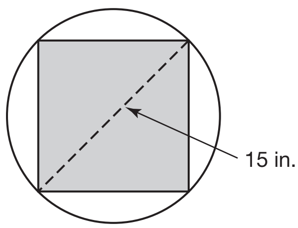
Area of Shaded Section:

$4(4^2\pi) \approx 201.06$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{201.06}{256} \approx 78.5\%$

3.



The probability of a dart landing in the shaded area is approximately 63.7%.

Area of Entire Board: $\left(\frac{15}{2}\right)^2\pi \approx 176.71$ in.^{2}

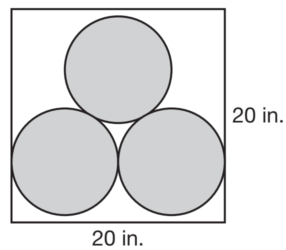
Area of Shaded Section:

$\sqrt{112.5} \times \sqrt{112.5} = 112.5$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{112.5}{176.71} \approx 63.7\%$

4.



The probability of a dart landing in the shaded area is approximately 58.9%.

Area of Entire Board: $20(20) = 400$ in.^{2}

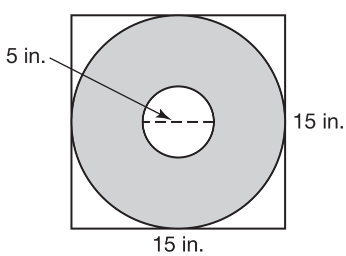
Area of Shaded Section:

$3(5^2\pi) \approx 235.62$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{235.62}{400} \approx 58.9\%$

5.



The probability of a dart landing in the shaded area is approximately 69.8%.

Area of Entire Board: $15(15) = 225$ in.^{2}

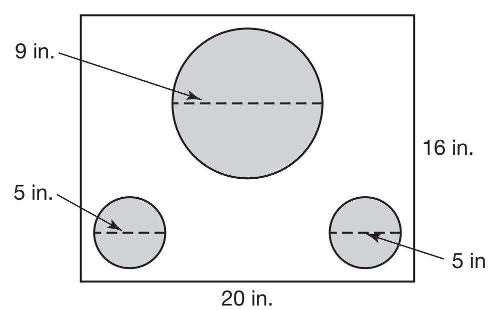
Area of Shaded Section:

$7.5^2\pi - 2.5^2\pi \approx 157.08$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{157.08}{225} \approx 69.8\%$

6.



The probability of a dart landing in the shaded area is approximately 32.2%.

Area of Entire Board: $20(16) = 320$ in.^{2}

Area of Shaded Section:

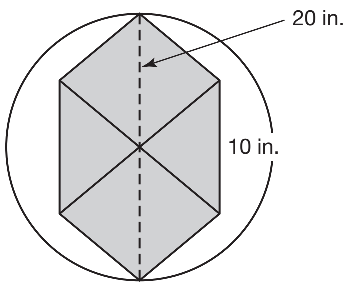
$2(2.5^2\pi) + 4.5^2\pi \approx 102.89$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{102.89}{320} \approx 32.2\%$

Name ______________________________ Date ____________

7.



The probability of a dart landing in the shaded area is approximately 47.7%.

Area of Entire Board: $10^2\pi \approx 314.16$ in.^{2}

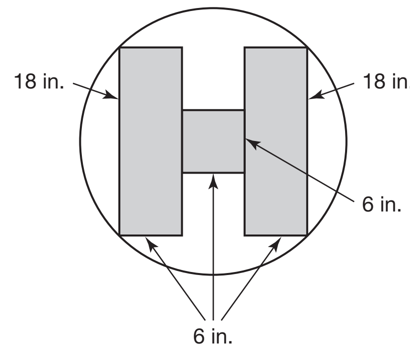
Area of Shaded Section:

$3(5\sqrt{2} \times 5\sqrt{2}) = 150$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{150}{314.16} \approx 47.7\%$

8.



The probability of a dart landing in the shaded area is approximately 49.5%.

Area of Entire Board: $(9\sqrt{2})^2\pi \approx 508.94$ in.^{2}

Area of Shaded Section:

$2(6 \times 18) + 6 \times 6 = 252$ in.^{2}

Probability of Landing in Shaded Section:

$\frac{252}{508.94} \approx 49.5\%$

Benjamin rolls a six-sided number cube 12 times. Determine each expected value and explain your solution method.

9. How many of the outcomes do you expect to result in a 1?

I would expect 2 out of the 12 outcomes to result in a 1.

$\frac{1}{6}(12) = 2$

10. How many of the outcomes do you expect to result in a 1 or a 6?

I would expect 4 out of the 12 outcomes to result in a 1 or a 6.

$\frac{2}{6}(12) = 4$

11. How many of the outcomes do you expect to result in number greater than 2?

I would expect 8 out of the 12 outcomes to result in a number greater than 2.

$\frac{4}{6}(12) = 8$

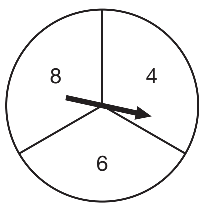
12. How many of the outcomes do you expect to result in an even number?

I would expect 6 out of the 12 outcomes to result in an even number.

$\frac{3}{6}(12) = 6$

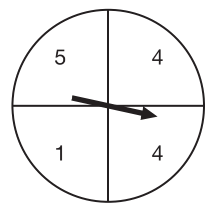
Calculate the expected value of spinning each spinner one time. Round to the nearest hundredth if necessary.

13.



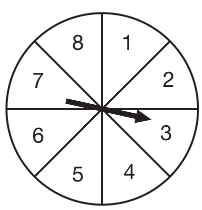
$4\left(\frac{1}{3}\right) + 6\left(\frac{1}{3}\right) + 8\left(\frac{1}{3}\right) = 6$

14.



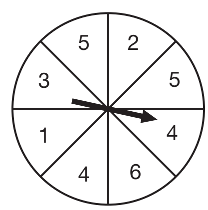
$4\left(\frac{2}{4}\right) + 1\left(\frac{1}{4}\right) + 5\left(\frac{1}{4}\right) = 3.5$

15.



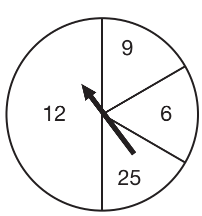
$1\left(\frac{1}{8}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right) + 7\left(\frac{1}{8}\right) + 8\left(\frac{1}{8}\right) = 4.5$

16.



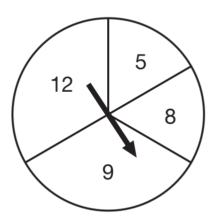
$1\left(\frac{1}{8}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{2}{8}\right) + 5\left(\frac{2}{8}\right) + 6\left(\frac{1}{8}\right) = 3.75$

17.



$9\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 25\left(\frac{1}{6}\right) + 12\left(\frac{3}{6}\right) \approx 12.67$

18.



$5\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + 9\left(\frac{2}{6}\right) + 12\left(\frac{2}{6}\right) \approx 9.17$